

# Module 5: Probabilistic Reasoning in the Service of Gambling

All life is six to five against.

– Damon Runyon

**Abstract:** Probabilistic reasoning is applied to several topics in gambling. We begin with the Chevalier de Méré asking the mathematician Blaise Pascal in the early 17th century for help with his gambling interests. Pascal in a series of letters with another mathematician, Pierre de Fermat, laid out what was to be the foundations for a modern theory of probability. Some of this formalization is briefly reviewed; also, to give several numerical examples, the Pascal-Fermat framework is applied to the type of gambles the Chevalier engaged in. Several other gambling related topics are discussed at some length: spread betting, parimutuel betting, and the psychological considerations behind gambling studied by Tversky, Kahneman, and others concerned with the psychology of choice and decision making.

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## 1 Betting, Gaming, and Risk

Antoine Gombaud, better known as the Chevalier de Méré, was a French writer and amateur mathematician from the early 17th century. He is important to the development of probability theory because of one specific thing; he asked a mathematician, Blaise Pascal, about a gambling problem dating from the Middle Ages, named “the problem of points.” The question was one of fairly dividing the stakes among individuals who had agreed to play a certain number of games, but for whatever reason had to stop before they were finished. Pascal in a series of letters with Pierre de Fermat, solved this equitable division task, and in the process laid out the foundations for a modern theory of probability. Pascal and Fermat also provided the Chevalier with a solution to a vexing problem he was having in his own personal gambling. Apparently, the Chevalier had been very successful in making even money bets that a six would be rolled at least once in four throws of a single die. But when he tried a similar bet based on tossing two dice 24 times and looking for a double-six to occur, he was singularly unsuccessful in making any money. The reason for this difference between the Chevalier’s two wagers was clarified by the formalization developed by Pascal and Fermat for such games of chance. This formalization is briefly reviewed below, and then used to discuss the Chevalier’s two gambles as well as those occurring in various other casino-type games.

We begin by defining several useful concepts: a simple experiment, sample space, sample point, event, elementary event:

A *simple experiment* is some process that we engage in that leads

to one single outcome from a set of possible outcomes that could occur. For example, a simple experiment could consist of rolling a single die once, where the set of possible outcomes is  $\{1, 2, 3, 4, 5, 6\}$  (note that curly braces will be used consistently to denote a set). Or, two dice could be tossed and the number of spots occurring on each die noted; here, the possible outcomes are integer number pairs:  $\{(a, b) \mid 1 \leq a \leq 6; 1 \leq b \leq 6\}$ . Flipping a single coin would give the set of outcomes,  $\{H, T\}$ , with “ $H$ ” for “heads” and “ $T$ ” for “tails”; picking a card from a normal deck could give a set of outcomes containing 52 objects, or if we were only interested in the particular suit for a card chosen, the possible outcomes could be  $\{H, D, C, S\}$ , corresponding to heart, diamond, club, and spade, respectively.

The set of possible outcomes for a simple experiment is the *sample space* (which we denote by the script letter  $\mathcal{S}$ ). An object in a sample space is a *sample point*. An *event* is defined as a subset of the sample space, and an event containing just a single sample point is an *elementary event*. A particular event is said to occur when the outcome of the simple experiment is a sample point belonging to the defining subset for that event.

As a simple example, consider the toss of a single die, where  $\mathcal{S} = \{1, 2, 3, 4, 5, 6\}$ . The event of obtaining an even number is the subset  $\{2, 4, 6\}$ ; the event of obtaining an odd number is  $\{1, 3, 5\}$ ; the (elementary) event of tossing a 5 is a subset with a single sample point,  $\{5\}$ , and so on.

For a sample space containing  $K$  sample points, there are  $2^K$  possible events (that is, there are  $2^K$  possible subsets of the sample space). This includes the “impossible event” (usually denoted by

$\emptyset$ ), characterized as that subset of  $\mathcal{S}$  containing no sample points and which therefore can never occur; and the “sure event,” defined as that subset of  $\mathcal{S}$  containing all sample points (that is,  $\mathcal{S}$  itself), which therefore must always occur. In our single die example, there are  $2^6 = 64$  possible events, including  $\emptyset$  and  $\mathcal{S}$ .

The motivation for introducing the idea of a simple experiment and sundry concepts is to use this structure as an intuitively reasonable mechanism for assigning probabilities to the occurrence of events. These probabilities are usually assigned through an assumption that sample points are equally likely to occur, assuming we have characterized appropriately what is to be in  $\mathcal{S}$ . Generally, only the probabilities are needed for the  $K$  elementary events containing single sample points. The probability for any other event is merely the sum of the probabilities for all those elementary events defined by the sample points making up that particular event. This last fact is due to the disjoint set property of probability introduced in the first module. In the specific instance in which the sample points are equally likely to occur, the probability assigned to any event is merely the number of sample points defining the event divided by  $K$ . As special cases, we obtain a probability of 0 for the impossible event, and 1 for the sure event.

The use of the word *appropriately* in characterizing a sample space is important to keep in mind whenever we wish to use the idea of being equally likely to generate the probabilities for all the various events. For example, in throwing two dice and letting the sample space be  $\mathcal{S} = \{(a, b) \mid 1 \leq a \leq 6; 1 \leq b \leq 6\}$ , it makes sense, assuming that the dice are not “loaded,” to consider the 36 integer

number pairs to be equally likely. When the conception of what is being observed changes, however, the equally-likely notion may no longer be “appropriate.” For example, suppose our interest is only in the sum of spots on the two dice being tossed, and let our sample space be  $\mathcal{S} = \{2, 3, \dots, 12\}$ . The eleven integer sample points in this sample space are not equally likely; in fact, it is a common exercise in an elementary statistics course to derive the probability distribution for the objects in this latter sample space based on the idea that the underlying 36 integer number pairs are equally likely. To illustrate, suppose our interest is in the probability that a “sum of seven” appears on the dice. At the level of the sample space containing the 36 integer number pairs, a “sum of seven” corresponds to the event  $\{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$ . Thus, the probability of a “sum of seven” is  $6/36$ ; there are six equally-likely sample points making up the event and there are 36 equally-likely integer pairs in the sample space. Although probably apocryphal, it has been said that many would-be probabilists hired by gambling patrons in the 17th century, came to grief when they believed that every stated sample space had objects that could be considered equally likely, and communicated this fact to their employers as an aid in betting.

One particularly helpful use of the sample space/event concepts is when a simple experiment is carried out multiple times (for, say,  $N$  replications), and the outcomes defining the sample space are the ordered  $N$ -tuples formed from the results obtained for the individual simple experiments. The Chevalier who rolls a single die four times, generates the sample space

$$\{(D_1, D_2, D_3, D_4) \mid 1 \leq D_i \leq 6, 1 \leq i \leq 4\} ,$$

that is, all 4-tuples containing the integers from 1 to 6. Generally, in a replicated simple experiment with  $K$  possible outcomes on each trial, the number of different  $N$ -tuples is  $K^N$  (using a well-known arithmetic multiplication rule). Thus, for the Chevalier example, there are  $6^4 = 1296$  possible 4-tuples, and each such 4-tuple should be equally likely to occur (given the “fairness” of the die being used; so, no “loaded” dice are allowed). To define the event of “no sixes rolled in four replications,” we would use the subset (event)

$$\{(D_1, D_2, D_3, D_4) \mid 1 \leq D_i \leq 5, 1 \leq i \leq 4\} ,$$

containing  $5^4 = 625$  sample points. Thus, the probability of “no sixes rolled in four replications” is  $625/1296 = .4822$ . As we will see formally below, the fact that this latter probability is strictly less than  $1/2$  gives the Chevalier a distinct advantage in playing an even money game defined by his being able to roll at least one six in four tosses of a die.

The other game that was not as successful for the Chevalier, was tossing two dice 24 times and betting on obtaining a double-six somewhere in the sequence. The sample space here is

$$\{(P_1, P_2, \dots, P_{24})\}, \text{ where } P_i = \{(a_i, b_i) \mid 1 \leq a_i \leq 6; 1 \leq b_i \leq 6\},$$

and has  $36^{24}$  possible sample points. The event of “not obtaining a double-six somewhere in the sequence” would look like the sample space just defined except that the  $(6, 6)$  pair would be excluded from each  $P_i$ . Thus, there are  $35^{24}$  members in this event. The probability of “not obtaining a double-six somewhere in the sequence” is

$$\frac{35^{24}}{36^{24}} = \left(\frac{35}{36}\right)^{24} = .5086 .$$

Because this latter value is greater than  $1/2$  (in contrast to the previous gamble), the Chevalier would now be at a disadvantage making an even money bet.

The best way to evaluate the perils or benefits present in a wager is through the device of a discrete random variable. Suppose  $X$  denotes the outcome of some bet; and let  $a_1, \dots, a_T$  represent the  $T$  possible payoffs from one wager, where positive values reflect gain and negative values reflect loss. In addition, we know the probability distribution for  $X$ ; that is,  $P(X = a_t)$  for  $1 \leq t \leq T$ . What one expects to realize from one observation on  $X$  (or from one play of the game) is its expected value,

$$E(X) = \sum_{t=1}^T a_t P(X = a_t).$$

If  $E(X)$  is negative, we would expect to lose this much on each bet; if positive, this is the expected gain on each bet. When  $E(X)$  is 0, the term “fair game” is applied to the gamble, implying that one neither expects to win or lose anything on each trial; one expects to “break even.” When  $E(X) \neq 0$ , the game is “unfair” but it could be unfair in your favor ( $E(X) > 0$ ), or unfair against you ( $E(X) < 0$ ).

To evaluate the Chevalier’s two games, suppose  $X$  takes on the values of  $+1$  and  $-1$  (the winning or losing of one dollar, say). For the single die rolled four times,  $E(X) = (+1)(.5178) + (-1)(.4822) = .0356 \approx .04$ . Thus, the game is unfair in the Chevalier’s favor because he expects to win a little less than four cents on each wager. For the 24 tosses of two dice,  $E(X) = (+1)(.4914) + (-1)(.5086) = -.0172 \approx -.02$ . Here, the Chevalier is at a disadvantage. The game

is unfair against him, and he expects to lose about two cents on each play of the game.

Besides using the expectation of  $X$  as an indication of whether a game is fair or not, and in whose favor, the variance of  $X$  is an important additional characteristic of any gamble. The larger the variance, the more one would expect a “boom or bust” scenario to take over, with the possibility of wild swings in the sizes of the gains or losses. But if one cannot play a game having a large variance multiple times, then it doesn’t make much difference if one has a slight positive favorable expectation. There is another story, probably again apocryphal, of a man with a suitcase of money who for whatever reason needed twice this amount or it really didn’t matter if he lost it all. He goes into a casino and bets it all at once at a roulette table—on red. He either gets twice his money on this one play or loses it all; in the latter case as we noted, maybe it doesn’t matter; for example, because he previously borrowed money, the mob will place a “hit” on him if he can’t come up with twice the amount that he had to begin with. Or recently, consider the hugely successful negative bets that Goldman Sachs and related traders (such as John Paulson) made on the toxic derivatives they had themselves created (in the jargon, they held a “short position” where one expects the price to fall and to thereby make money in the process).

A quotation from the author of the 1995 novel *Casino*, Nicholas Pileggi, states the issue well for casinos and the usual games of chance where skill is irrelevant (for example, roulette, slots, craps, keno, lotto, or blackjack [without card counting]); all are unfair and in the house’s favor:

A casino is a mathematics palace set up to separate players from their money. Every bet in a casino has been calibrated within a fraction of its life to maximize profit while still giving the players the illusion they have a chance.

The negative expectations may not be big in any absolute sense, but given the enormous number of plays made, and the convergent effects of the law of large numbers (to be discussed in a later chapter), casinos don't lose money, period. The next time an acquaintance brags about what a killing he or she made in the casino on a game involving no skill, you can just comment that the game must not have been played long enough.<sup>1</sup>

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<sup>1</sup>We give two short anecdotes that may be helpful in motivating the material in this section:

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Charles Marie de La Condamine (1701–1774) is best known for answering the question as to whether the earth was flat or round. He based his answer (which was “round”) on extensive measurements taken at the equator in Ecuador and in Lapland. For our purposes, however, he will be best known for giving the French philosopher Voltaire a gambling tip that allowed him to win 500,000 francs in a lottery. Condamine noted to Voltaire that through a miscalculation, the sum of all the ticket prices for the lottery was far less than the prize. Voltaire bought all the tickets and won.

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Joseph Jagger (1830–1892) is known as “the man who broke the bank at Monte Carlo.” In reality, he was a British engineer working in the Yorkshire cotton manufacturing industry, and very knowledgeable about spindles that were “untrue.” Jagger speculated that a roulette wheel did not necessarily “turn true,” and the outcomes not purely random but biased toward particular outcomes. We quote a brief part of the Wikipedia entry on Joseph Jagger that tells the story:

Jagger was born in September 1829 in the village of Shelf near Halifax, Yorkshire. Jagger gained his practical experience of mechanics working in Yorkshire's cotton manufacturing industry. He extended his experience to the behaviour of a roulette wheel, speculating that its outcomes were not purely random sequences but that mechanical imbalances might result in biases toward particular outcomes.

In 1873, Jagger hired six clerks to clandestinely record the outcomes of the six roulette wheels at the Beaux-Arts Casino at Monte Carlo, Monaco. He discovered that one of the six wheels showed a clear bias, in that nine of the numbers (7, 8, 9, 17, 18, 19, 22, 28 and 29) occurred more frequently than the others. He therefore placed his first bets on 7 July 1875

## 1.1 Spread Betting

The type of wagering that occurs in roulette or craps is often referred to as fixed-odds betting; you know your chances of winning when you place your bet. A different type of wager is spread betting, invented by a mathematics teacher from Connecticut, Charles McNeil, who became a Chicago bookmaker in the 1940s. Here, a payoff is based on the wager's accuracy; it is no longer a simple "win or lose" situation. Generally, a spread is a range of outcomes, and the bet itself is on whether the outcome will be above or below the spread. In common sports betting (for example, NCAA college basketball), a "point spread" for some contest is typically advertised by a bookmaker. If the gambler chooses to bet on the "underdog," he is said to "take the points" and will win if the underdog's score plus the point spread is greater than that of the favored team; conversely, if the gambler bets on the favorite, he "gives the points" and wins only if the favorite's score minus the point spread is greater than the underdog's score. In general, the announcement of a point spread is an attempt to even out the market for the bookmaker, and to generate an equal amount of money bet on each side. The commission that a bookmaker charges will ensure a livelihood, and thus, the bookmaker

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and quickly won a considerable amount of money, £14,000 (equivalent to around 50 times that amount in 2005, or £700,000, adjusted for inflation). Over the next three days, Jagger amassed £60,000 in earnings with other gamblers in tow emulating his bets. In response, the casino rearranged the wheels, which threw Jagger into confusion. After a losing streak, Jagger finally recalled that a scratch he noted on the biased wheel wasn't present. Looking for this telltale mark, Jagger was able to locate his preferred wheel and resumed winning. Counterattacking again, the casino moved the frets, metal dividers between numbers, around daily. Over the next two days Jagger lost and gave up, but he took his remaining earnings, two million francs, then about £65,000 (around £3,250,000 in 2005), and left Monte Carlo never to return.

can be unconcerned about the actual outcome.

Several of the more notorious sports scandals in United States history have involved a practice of “point shaving,” where the perpetrators of such a scheme try to prevent a favored team from “covering” a published point spread. This usually involves a sports gambler and one or more players on the favored team. They are compensated when their team fails to “cover the spread”; and those individuals who have bet on the underdog, win. Two famous examples of this practice in college basketball are the Boston College point shaving scandal of 1978/9, engineered by the gangsters Henry Hill and Jimmy Burke, and the CCNY scandal of 1950/1 involving organized crime and 33 players from some seven schools (CCNY, Manhattan College, NYU, Long Island University, Bradley University (Peoria), University of Kentucky, and the University of Toledo). More recently, there is the related 2007 NBA betting scandal surrounding a referee, Tim Donaghy.<sup>2</sup>

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<sup>2</sup>When this section on point shaving was being written in June of 2014, an obituary for Gene Melchiorre appeared in the *New York Times* (June 26, 2014), with the title “For Gene Melchiorre, a Regretful Turn Brought a Unique N.B.A. Distinction.” Several paragraphs are given below that shed some personal light on the point-shaving scandal of 1951 mentioned in the text:

At the dead end of a private, wooded road about 20 miles north of Chicago sits a two-story house belonging to Gene Melchiorre, a short, pigeon-toed grandfather of 15 known by his many friends as Squeaky. Family photos decorate his office, but one artifact is unlike the others: a 63-year-old comic book drawing of a giant, youthful Melchiorre wearing a No. 23 basketball jersey, a superhero in short shorts.

Melchiorre, 86, a former two-time all-American at Bradley once called the “greatest little man in basketball,” was the first overall pick in the 1951 N.B.A. draft. But he holds an unusual distinction: He is the only No. 1 pick in N.B.A. history to never play in the league.

There have been plenty of top draft picks who have flamed out, sometimes in spectacular fashion. But there has never been a draft pick like Squeaky Melchiorre. After being chosen first by the Baltimore Bullets, Melchiorre was barred for life from the N.B.A. for his role in

In an attempt to identify widespread corruption in college basketball, Justin Wolfers investigated the apparent tendency for favored NCAA teams nationally not to “cover the spread.” His article in the *American Economic Review* (2006, 96, 279–283) is provoca-

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the point-shaving scandal of 1951. He and more than 30 other players from seven universities were arrested in the scandal.

The trouble began in 1949, while Melchiorre’s team was in New York for the National Invitation Tournament. A gambler from Brooklyn named Nick Englis (widely known as Nick the Greek) intentionally “bumped into” a player inside the team’s hotel, according to an account Melchiorre gave to *Look Magazine* in 1953. Soon, Melchiorre and two teammates were in a room with three gamblers, who “told us the colleges were getting rich on basketball and we ought to be getting something for it.”

The conversation changed Melchiorre’s life dramatically. He could have been an N.B.A. legend – “Melchiorre knows every trick that can shake a man loose,” Kentucky Coach Adolph Rupp declared in 1951. But that never happened.

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When asked about the scandal today, Melchiorre falls silent, then changes the subject. But in a 1953 article in *Look* titled “How I Fell for the Basketball Bribers,” Melchiorre described his downfall.

The gamblers he met in the hotel room told him that point-shaving was widespread and had been going on for years. Players were using the money to start businesses after graduation. “It’s not as if you’re throwing a game,” a gambler said. “All you have to do is win by more points or fewer points than the bookmakers think you’re supposed to.”

They assured the players there was no chance of getting caught.

Melchiorre admitted in the article to accepting money during his career. But he denied ever altering his play to manipulate the point spread.

“Why did we do it?” Melchiorre said in the 1953 article. “Well, none of us had any money. We justified ourselves, I guess, by saying the colleges were making plenty out of us. We argued to ourselves that what we were doing was wrong, but not too wrong, because we weren’t going to throw any games.”

#### A Suspended Sentence

In February and March 1951, the Manhattan district attorney’s office arrested several players from City College and Long Island University on bribery charges. In July, Melchiorre and several other Bradley players were arrested.

Melchiorre eventually pleaded guilty to a misdemeanor and received a suspended sentence. The scandal ended the careers of two N.B.A. All-Stars and the nation’s leading scorer, Sherman White, who served nine months on Rikers Island. As for Melchiorre, the N.B.A. barred him for life.

tively entitled “Point Shaving: Corruption in NCAA Basketball.” We quote the discussion section of this article to give a sense of what Wolfers claims he found in the data:

These data suggest that point shaving *may* be quite widespread, with an indicative, albeit rough, estimate suggesting that around 6 percent of strong favorites have been willing to manipulate their performance. Given that around one-fifth of all games involve a team favored to win by at least 12 points, this suggests that around 1 percent of all games (or nearly 500 games through my 16-year sample) involve gambling related corruption. This estimate derives from analyzing the extent to which observed patterns in the data are consistent with the incentives for corruption derived from spread betting; other forms of manipulation may not leave this particular set of footprints in the data, and so this is a lower bound estimate of the extent of corruption. Equally, the economic model suggests a range of other testable implications, which are the focus of ongoing research.

My estimate of rates of corruption receives some rough corroboration in anonymous self-reports. Eight of 388 Men’s Division I basketball players surveyed by the NCAA reported either having taken money for playing poorly or having knowledge of teammates who had done so.

A shortcoming of the economic approach to identifying corruption is that it relies on recognizing systematic patterns emerging over large samples, making it difficult to pinpoint specific culprits. Indeed, while the discussion so far has proceeded as if point shaving reflected a conspiracy between players and gamblers, these results might equally reflect selective manipulation by coaches of playing time for star players. Further, there need not be any shadowy gamblers offering bribes, as the players can presumably place bets themselves, rendering a coconspirator an unnecessary added expense.

The advantage of the economic approach is that it yields a clear understanding of the incentives driving corrupt behavior, allowing policy conclusions that extend beyond the usual platitudes that “increased education, prevention, and awareness programs” are required. The key incentive driving point shaving is that bet pay-offs are discontinuous at a point—the spread—that is (or should be) essentially irrelevant to the players. Were gamblers

restricted to bets for which the pay-off was a linear function of the winning margin, their incentive to offer bribes would be sharply reduced. Similarly, restricting wagers to betting on which team wins the game sharply reduces the incentive of basketball players to accept any such bribes. This conclusion largely repeats a finding that is now quite well understood in the labor literature and extends across a range of contexts—that highly nonlinear pay-off structures can yield rather perverse incentives and, hence, undesirable behaviors. (p. 283)

Another more recent article on this same topic is by Dan Bernhardt and Steven Heston (*Economic Inquiry*, 2010, 48, 14–25) entitled “Point Shaving in College Basketball: A Cautionary Tale for Forensic Economics.” As this title might suggest, an alarmist position about the rampant corruption present in NCAA basketball is not justified. An alternative explanation for the manifest “point shaving” is the use of strategic end-game efforts by a basketball team trying to maximize its probability of winning (for example, when a favored team is ahead late in the game, the play may move from a pure scoring emphasis to one that looks to “wind down the clock”). The first paragraph of the conclusion section of the Bernhardt and Heston article follows:

Economists must often resort to indirect methods and inference to uncover the level of illegal activity in the economy. Methodologically, our article highlights the care with which one must design indirect methods in order to distinguish legal from illegal behavior. We first show how a widely reported interpretation of the patterns in winning margins in college basketball can lead a researcher to conclude erroneously that there is an epidemic of gambling-related corruption. We uncover decisive evidence that this conclusion is misplaced and that the patterns in winning margins are driven by factors intrinsic to the game of basketball itself. (p. 24)

The use of spreads in betting has moved somewhat dramatically to the world financial markets, particularly in the United Kingdom.

We suggest the reader view an article from the *Times (London)* (April 10, 2009) by David Budworth entitled “Spread-Betting Fails Investors in Trouble.” Even though it emphasizes what is occurring in the United Kingdom, it still provides a cautionary tale for the United States as well. The moral might be that just because someone can create something to bet on (think CDOs [Collateralized Debt Obligations] and Goldman Sachs) doesn’t mean that it is necessarily a good idea to do so.

## 1.2 Parimutuel Betting

The term *parimutuel betting* (based on the French for “mutual betting”) characterizes the type of wagering system used in horse racing, dog tracks, jai alai, and similar contests where the participants end up in a rank order. It was devised in 1867 by Joseph Oller, a Catalan impresario (he was also a bookmaker and founder of the Paris Moulin Rouge in 1889). Very simply, all bets of a particular type are first pooled together; the house then takes its commission and the taxes it has to pay from this aggregate; finally, the payoff odds are calculated by sharing the residual pool among the winning bets. To explain using some notation, suppose there are  $T$  contestants and bets are made of  $W_1, W_2, \dots, W_T$  on an outright “win.” The total pool is  $T_{pool} = \sum_{t=1}^T W_t$ . If the commission and tax rate is a proportion,  $R$ , the residual pool,  $R_{pool}$ , to be allocated among the winning bettors is  $R_{pool} = T_{pool}(1 - R)$ . If the winner is denoted by  $t^*$ , and the money bet on the winner is  $W_{t^*}$ , the payoff per dollar for a successful bet is  $R_{pool}/W_{t^*}$ . We refer to the odds on outcome  $t^*$  as

$$\left(\frac{R_{pool}}{W_{t^*}} - 1\right) \text{ to } 1.$$

For example, if  $\frac{R_{pool}}{W_{t*}}$  had a value of 9.0, the odds would be 8 to 1: you get 8 dollars back for every dollar bet plus the original dollar.

Because of the extensive calculations involved in a parimutuel system, a specialized mechanical calculating machine, named a totalizator, was invented by the mechanical engineer George Julius, and first installed at Ellerslie Race Track in New Zealand in 1913. In the 1930s, totalizators were installed at many of the race tracks in the United States (for example, Hialeah Park in Florida and Arlington Race Track and Sportsman's Park in Illinois). All totalizators came with "tote" boards giving the running payoffs for each horse based on the money bet up to a given time. After the pools for the various categories of bets were closed, the final payoffs (and odds) were then determined for all winning bets.

In comparison with casino gambling, parimutuel betting pits one gambler against other gamblers, and not against the house. Also, the odds are not fixed but calculated only after the betting pools have closed (thus, odds cannot be turned into real probabilities legitimately; they are empirically generated based on the amounts of money bet). A skilled horse player (or "handicapper") can make a steady income, particularly in the newer Internet "rebate" shops that return to the bettor some percentage of every bet made. Because of lower overhead, these latter Internet gaming concerns can reduce their "take" considerably (from, say, 15% to 2%), making a good handicapper an even better living than before.

### 1.3 Psychological Considerations in Gambling

As shown in the work of Tversky and Kahneman (for example, Tversky & Kahneman, 1981), the psychology of choice is dictated to a great extent by the framing of a decision problem; that is, the context into which a particular decision problem is placed. The power of framing in how decision situations are assessed, can be illustrated well through an example and the associated discussion provided by Tversky and Kahneman (1981, p. 453):

Problem 1 [ $N = 152$ ]: Imagine that the United States is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:

If Program A is adopted, 200 people will be saved. [72 percent]

If Program B is adopted, there is  $1/3$  probability that 600 people will be saved, and  $2/3$  probability that no people will be saved. [28 percent]

Which of the two programs would you favor?

The majority choice in this problem is risk averse: the prospect of certainly saving 200 lives is more attractive than a risky prospect of equal expected value, that is, a one-in-three chance of saving 600 lives.

A second group of respondents was given the cover story of problem 1 with a different formulation of the alternative programs, as follows:

Problem 2 [ $N = 155$ ]:

If Program C is adopted, 400 people will die. [22 percent]

If Program D is adopted, there is  $1/3$  probability that nobody will die, and  $2/3$  probability that 600 people will die. [78 percent]

Which of the two programs would you favor?

The majority choice in problem 2 is risk taking: the certain death of 400 people is less acceptable than the two-in-three chance that 600 will die. The preferences in problems 1 and 2 illustrate a common pattern: choices involving gains are often risk averse and choices involving losses are often risk

taking. However, it is easy to see that the two problems are effectively identical. The only difference between them is that the outcomes are described in problem 1 by the number of lives saved and in problem 2 by the number of lives lost. The change is accompanied by a pronounced shift from risk aversion to risk taking. (p. 453)

The effects of framing can be very subtle when certain conscious or unconscious (coded) words are used to provide a salient context that influences decision processes. A recent demonstration of this in the framework of our ongoing climate-change debate is given by Hardisty, Johnson, and Weber (2010) in *Psychological Science*. The article has the interesting title, “A Dirty Word or a Dirty World? Attribute Framing, Political Affiliation, and Query Theory.” The abstract follows:

We explored the effect of attribute framing on choice, labeling charges for environmental costs as either an earmarked tax or an offset. Eight hundred ninety-eight Americans chose between otherwise identical products or services, where one option included a surcharge for emitted carbon dioxide. The cost framing changed preferences for self-identified Republicans and Independents, but did not affect Democrats’ preferences. We explain this interaction by means of query theory and show that attribute framing can change the order in which internal queries supporting one or another option are posed. The effect of attribute labeling on query order is shown to depend on the representations of either taxes or offsets held by people with different political affiliations. (p. 86)

Besides emphasizing the importance of framing in making decisions, Tversky and Kahneman developed a theory of decision making, called prospect theory, to model peoples’ real-life choices, which are not necessarily the optimal ones (Kahneman & Tversky, 1979). Prospect theory describes decisions between risky alternatives with

uncertain outcomes when the probabilities are generally known. One particular phenomenon discussed at length in prospect theory is loss aversion, or the tendency to strongly avoid loss as opposed to acquiring gains. In turn, loss aversion leads to risk aversion, or the reluctance of people to choose gambles with an uncertain payoff rather than another with a more certain but possibly lower expected payoff. For example, an investor who is risk averse might choose to put money into a fixed-interest bank account or a certificate-of-deposit rather than into some stock with the potential of high returns but also with a chance of becoming worthless.

The notion of risk aversion has been around since antiquity. Consider the legend of Scylla and Charybdis, two sea monsters of Greek mythology situated on opposite sides of the Strait of Messina in Italy, between Calabria and Sicily. They were placed close enough to each other that they posed an inescapable threat to passing ships, so avoiding Scylla meant passing too close to Charybdis and conversely. In Homer's *Odyssey*, Odysseus is advised by Circe to follow the risk-adverse strategy of sailing closer to Scylla and losing a few men rather than sailing closer to the whirlpools created by Charybdis that could sink his ship. Odysseus sailed successfully past Scylla and Charybdis, losing six sailors to Scylla —

they writhed  
gasping as Scylla swung them up her cliff and there  
at her cavern's mouth she bolted them down raw —  
screaming out, flinging their arms toward me,  
lost in that mortal struggle.

The phrase of being “between a rock and a hard place” is a more modern version of being “between Scylla and Charybdis.”

The most relevant aspect of any decision-making proposition involving risky alternatives is the information one has, both on the probabilities that might be associated with the gambles and what the payoffs might be. In the 1987 movie, *Wall Street*, the character playing Gordon Gekko states: “The most valuable commodity I know of is information.” The value that information has is reflected in a great many ways: by laws against “insider trading” (think Martha Stewart); the mandatory injury reports and the not-likely-to-play announcements by the sports leagues before games are played; the importance of counting cards in blackjack to obtain some idea of the number of high cards remaining in the deck (and to make blackjack an unfair game in your favor); massive speed-trading on Wall Street designed to obtain a slight edge in terms of what the market is doing currently (and to thereby “beat out” one’s competitors with this questionably obtained edge); the importance of correct assessments by the credit rating agencies (think of all the triple-A assessments for the Goldman Sachs toxic collateralized debt obligations and what that meant to the buyers of these synthetic financial instruments); and finally, in the case against Goldman Sachs, the bank supposedly knew about the toxicity of what it sold to their clients and then made a huge profit betting against what they sold (the proverbial “short position”). A movie quotation from *Dirty Harry* illustrates the crucial importance of who has information and who doesn’t – “I know what you’re thinkin’. ‘Did he fire six shots or only five?’ Well, to tell you the truth, in all this excitement I kind of lost track myself.” At the end of this Harry Callahan statement to the bank robber as to whether he felt lucky, the bank robber says: “I gots to know!” Harry puts the .44 Magnum to the robber’s head and pulls

the trigger; Harry knew that he had fired six shots and not five.

The availability of good information is critical in all the decisions we make under uncertainty and risk, both financially and in terms of our health. When buying insurance, for example, we knowingly engage in loss-averse behavior. The information we have on the possible downside of not having insurance usually outweighs any consideration that insurance companies have an unfair game going in their favor. When deciding to take new drugs or undergo various medical procedures, information is again crucial in weighing risks and possible benefits—ask your doctor if he or she has some information that is right for you—and coming to a decision that is “best” for us (consider, for example, the previous discussion about undergoing screenings for various kinds of cancer).

At the same time that we value good information, it is important to recognize when available “information” really isn’t of much value and might actually be counterproductive, for example, when we act because of what is most likely just randomness or “noise” in a system. An article by Jeff Sommer in the *New York Times* (March 13, 2010) has the intriguing title, “How Men’s Overconfidence Hurts Them as Investors.” Apparently, men are generally more prone to act (trade) on short-term financial news that is often only meaningless “noise.” Men are also more confident in their abilities to make good decisions, and are more likely to make many more high-risk gambles.

For many decades, the financial markets have relied on rating agencies, such as Moody’s, Standard & Poor’s, and Fitch, to provide impeccable information to guide wise investing, and for assessing re-

alistically the risk being incurred. We are now learning that we can no longer be secure in the data the rating agencies produce. Because rating agencies have made public the computer programs and algorithms they use, banks have learned how to “reverse-engineer” the process to see how the top ratings might be obtained (or better, scammed). In the Goldman Sachs case, for example, the firm profited from the misery it helped create through the inappropriate high ratings given to its toxic CDOs. As Carl Levin noted as Chair of the Senate Permanent Subcommittee on Investigations: “A conveyor belt of high-risk securities, backed by toxic mortgages, got AAA ratings that turned out not to be worth the paper they were printed on.” The rating agencies have been in the position of the “fox guarding the hen house.” The reader is referred to an informative editorial that appeared in the *New York Times* (“What About the Raters?”, May 1, 2010) dealing with rating agencies and the information they provide.

By itself, the notion of “insurance” is psychologically interesting; the person buying insurance is willingly giving away a specific amount of money to avoid a more catastrophic event that might happen even though the probability of it occurring might be very small. Thus, we have a bookie “laying off” bets made with him or her to some third party; a blackjack player buying insurance on the dealer having a “blackjack” when the dealer has an ace showing (it is generally a bad idea for a player to buy insurance); or individuals purchasing catastrophic health insurance but paying the smaller day-to-day medical costs themselves. Competing forces are always at work between the insurer and the insured. The insurer wishes his “pool” to be as large as possible (so the central limit theorem discussed later can operate),

and relatively “safe”; thus, the push to exclude high-risk individuals is the norm, and insuring someone with pre-existing conditions is always problematic. The insured, on the other hand, wants to give away the least money to buy the wanted protection. As one final item to keep in mind, we should remember that insurance needs to be purchased before and not after the catastrophic event occurs. In late 2010, there was a national cable news story about the person whose house burned down as the county firetrucks stood by. The person felt very put upon and did not understand why they just let his house burn down; he had offered to pay the \$75 fire protection fee (but only after the house started to burn). The cable news agencies declared a “duty to rescue,” and the failure of the fire trucks to act was “manifestly immoral.” Well, we doubt it because no life was lost, only the property, and all because of a failure to pay the small insurance premium “up front.” For a discussion of this incident, see the article by Robert Mackey, “Tennessee Firefighters Watch Home Burn” (*New York Times*, October 6, 2010)

A second aspect of insurance purchase with psychological interest is how to estimate the probability of some catastrophic event. Insurers commonly have a database giving an estimated value over those individuals they may consider insuring. This is where the actuaries and statisticians make their worth known; how much should the insurance companies charge for a policy so the company would continue to make money. The person to be insured has no easy access to any comparable database and merely guesses a value or more usually, acts on some vague “gut feeling” as to what one should be willing to pay to avoid the catastrophic downside. The person being insured has no personal relative frequency estimate on which to rely.

Assessing risks when no database is available to an insuring body is more problematic. If every one were honest about these situations, it might be labeled as subjectively obtained, or more straightforwardly, a “guess.” This may be “gussied up” slightly with the phrase “engineering judgment,” but at its basis it is still a guess. Richard Feynman, in his role on the Rogers Commission investigating the Challenger accident of 1986, commented that “engineering judgment” was making up numbers according to the hallowed tradition of the “dry lab.” Here, one makes up data as opposed to observation and experimentation. You work backwards to the beginning from the result you want to obtain at the end. For shuttle risk, the management started with a level of risk that was acceptable and worked backwards until they got the probability estimate that gave this final “acceptable” risk level.

## References

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