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Global Optimization Toolbox User's Guide


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<td>New for Version 1.0 (Release 13SP1+)</td>
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<td>June 2004</td>
<td>First printing</td>
<td>Revised for Version 1.0.1 (Release 14)</td>
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Types of Problems and Solvers

Global Optimization Toolbox provides methods that search for global solutions to problems that contain multiple maxima or minima. It includes global search, multistart, pattern search, genetic algorithm, and simulated annealing solvers. You can use these solvers to solve optimization problems where the objective or constraint function is continuous, discontinuous, stochastic, does not possess derivatives, or includes simulations or black-box functions with undefined values for some parameter settings.

Genetic algorithm and pattern search solvers support algorithmic customization. You can create a custom genetic algorithm variant by modifying initial population and fitness scaling options or by defining parent selection, crossover, and mutation functions. You can customize pattern search by defining polling, searching, and other functions.

Key Features

- Interactive tools for defining and solving optimization problems and monitoring solution progress
- Global search and multistart solvers for finding single or multiple global optima
- Genetic algorithm solver that supports linear, nonlinear, mixed integer, and bound constraints
- Multiobjective genetic algorithm with Pareto-front identification, including linear and bound constraints
- Pattern search solver that supports linear, nonlinear, and bound constraints
- Simulated annealing tools that implement a random search method, with options for defining annealing process, temperature schedule, and acceptance criteria
- Parallel computing support in multistart, genetic algorithm, and pattern search solver
- Custom data type support in genetic algorithm, multiobjective genetic algorithm, and simulated annealing solvers

**Implementation Notes**

Global Optimization Toolbox solvers repeatedly attempt to locate a global solution. However, no solver employs an algorithm that can certify a solution as global.

You can extend the capabilities of Global Optimization Toolbox functions by writing your own files, by using them in combination with other toolboxes, or with the MATLAB® or Simulink® environments.
Example: Comparing Several Solvers

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Function to Optimize

This example shows how to minimize Rastrigin’s function with four solvers. Each solver has its own characteristics. The characteristics lead to different solutions and run times. The results, examined in “Comparison of Syntax and Solutions” on page 1-10, can help you choose an appropriate solver for your own problems.

Rastrigin’s function has many local minima, with a global minimum at (0,0):

$$\text{Ras}(x) = 20 + x_1^2 + x_2^2 - 10(\cos 2\pi x_1 + \cos 2\pi x_2)$$.

Usually you don’t know the location of the global minimum of your objective function. To show how the solvers look for a global solution, this example starts all the solvers around the point [20,30], which is far from the global minimum.

The `rastriginsfcn.m` file implements Rastrigin’s function. This file comes with Global Optimization Toolbox software. This example employs a scaled version of Rastrigin’s function with larger basins of attraction. For information, see “Basins of Attraction” on page 1-13.

```matlab
rf2 = @(x)rastriginsfcn(x/10);
```
This example minimizes \( rf2 \) using the default settings of \texttt{fminunc} (an Optimization Toolbox™ solver), \texttt{patternsearch}, and \texttt{GlobalSearch}. It also uses \texttt{ga} with a nondefault setting, to obtain an initial population around the point \([20,30]\).

\textbf{Four Solution Methods}

- “\texttt{fminunc}” on page 1-6
- “\texttt{patternsearch}” on page 1-7
- “\texttt{ga}” on page 1-8
Introducing Global Optimization Toolbox Functions

- “GlobalSearch” on page 1-9

**fminunc**

To solve the optimization problem using the fminunc Optimization Toolbox solver, enter:

```
rf2 = @(x)rastriginsfcn(x/10); % objective
x0 = [20,30]; % start point away from the minimum
[xf ff flf of] = fminunc(rf2,x0)
```

fminunc returns

Warning: Gradient must be provided for trust-region algorithm; using line-search algorithm instead.
> In fminunc at 347

Local minimum found.

Optimization completed because the size of the gradient is less than the default value of the function tolerance.

```
xf =
   19.8991  29.8486
ff =
   12.9344
flf =
    1
of =
    iterations: 3
    funcCount: 15
    stepsize: 1
    firstorderopt: 5.9907e-009
    algorithm: 'medium-scale: Quasi-Newton line search'
    message: [1x438 char]
```

- xf is the minimizing point.
- ff is the value of the objective, rf2, at xf.
- flf is the exit flag. An exit flag of 1 indicates xf is a local minimum.
• of is the output structure, which describes the fminunc calculations leading to the solution.

**patternsearch**

To solve the optimization problem using the patternsearch Global Optimization Toolbox solver, enter:

```matlab
rf2 = @(x)rastriginsfcn(x/10); % objective
x0 = [20,30]; % start point away from the minimum
[xp fp flp op] = patternsearch(rf2,x0)
```

**patternsearch returns**

```matlab
Optimization terminated: mesh size less than options.TolMesh.
xp =
    19.8991   -9.9496
fp =
    4.9748
flp =
    1
op =
    function: @(x)rastriginsfcn(x/10)
    problemtype: 'unconstrained'
    pollmethod: 'gpspositivebasis2n'
    searchmethod: []
    iterations: 48
    funccount: 174
    meshsize: 9.5367e-007
    message: 'Optimization terminated: mesh size less than options.TolMesh.'
```

• xp is the minimizing point.
• fp is the value of the objective, rf2, at xp.
• flp is the exit flag. An exit flag of 1 indicates xp is a local minimum.
• op is the output structure, which describes the patternsearch calculations leading to the solution.
To solve the optimization problem using the ga Global Optimization Toolbox solver, enter:

```matlab
rf2 = @(x)rastriginsfcn(x/10); % objective
x0 = [20,30]; % start point away from the minimum
initpop = 10*randn(20,2) + repmat([10 30],20,1);
opts = gaoptimset('InitialPopulation',initpop);
[xga fga flga oga] = ga(rf2,2,[],[],[],[],[],[],opts)
```

`initpop` is a 20-by-2 matrix. Each row of `initpop` has mean [10,30], and each element is normally distributed with standard deviation 10. The rows of `initpop` form an initial population matrix for the `ga` solver.

`opts` is an optimization structure setting `initpop` as the initial population.

The final line calls `ga`, using the optimization structure.

`ga` uses random numbers, and produces a random result. In this case `ga` returns:

```
Optimization terminated: average change in the fitness value
less than options.TolFun.
```

- `xga` is the minimizing point.
- `fga` is the value of the objective, `rf2`, at `xga`.
- `flga` is the exit flag. An exit flag of 1 indicates `xga` is a local minimum.
• oga is the output structure, which describes the ga calculations leading to the solution.

GlobalSearch
To solve the optimization problem using the GlobalSearch solver, enter:

```matlab
rf2 = @(x)rastriginsfcn(x/10); % objective
x0 = [20,30]; % start point away from the minimum
problem = createOptimProblem('fmincon','objective',rf2,...
'x0',x0);
gs = GlobalSearch;
[xg fg flg og] = run(gs,problem)
```

problem is an optimization problem structure. problem specifies the fmincon solver, the rf2 objective function, and x0=[20,30]. For more information on using createOptimProblem, see “Create a Problem Structure” on page 3-4.

**Note** You must specify fmincon as the solver for GlobalSearch, even for unconstrained problems.

gs is a default GlobalSearch object. The object contains options for solving the problem. Calling run(gs,problem) runs problem from multiple start points. The start points are random, so the following result is also random.

In this case, the run returns:

```
GlobalSearch stopped because it analyzed all the trial points.

All 6 local solver runs converged with a positive local solver exit flag.
```

```matlab
xg =
    1.0e-007 *
    -0.1405  -0.1405
fg =
    0
flg =
    1
og =
```
Introducing Global Optimization Toolbox Functions

• $x_g$ is the minimizing point.
• $f_g$ is the value of the objective, $rf2$, at $x_g$.
• $flg$ is the exit flag. An exit flag of 1 indicates all $fmincon$ runs converged properly.
• $og$ is the output structure, which describes the $GlobalSearch$ calculations leading to the solution.

Comparison of Syntax and Solutions

One solution is better than another if its objective function value is smaller than the other. The following table summarizes the results, accurate to one decimal.

<table>
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<th>$ga$</th>
<th>$GlobalSearch$</th>
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<tr>
<td>solution</td>
<td>[19.9 29.9]</td>
<td>[19.9 -9.9]</td>
<td>[0 19.9]</td>
<td>[0 0]</td>
</tr>
<tr>
<td>objective</td>
<td>12.9</td>
<td>5</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td># Fevals</td>
<td>15</td>
<td>174</td>
<td>1040</td>
<td>2312</td>
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These results are typical:

• $fminunc$ quickly reaches the local solution within its starting basin, but does not explore outside this basin at all. $fminunc$ has simple calling syntax.
• $patternsearch$ takes more function evaluations than $fminunc$, and searches through several basins, arriving at a better solution than $fminunc$. $patternsearch$ calling syntax is the same as that of $fminunc$.
• $ga$ takes many more function evaluations than $patternsearch$. By chance it arrived at a slightly better solution. $ga$ is stochastic, so its results change
with every run. ga has simple calling syntax, but there are extra steps to have an initial population near [20,30].

- **GlobalSearch** run takes many more function evaluations than **patternsearch**, searches many basins, and arrives at an even better solution. In this case, **GlobalSearch** found the global optimum. Setting up **GlobalSearch** is more involved than setting up the other solvers. As the example shows, before calling **GlobalSearch**, you must create both a **GlobalSearch** object (gs in the example), and a problem structure (problem). Then, call the run method with gs and problem. For more details on how to run **GlobalSearch**, see “How to Optimize with **GlobalSearch** and MultiStart” on page 3-2.
What Is Global Optimization?

In this section...

“Local vs. Global Optima” on page 1-12
“Basins of Attraction” on page 1-13

Local vs. Global Optima

Optimization is the process of finding the point that minimizes a function. More specifically:

- A local minimum of a function is a point where the function value is smaller than or equal to the value at nearby points, but possibly greater than at a distant point.

- A global minimum is a point where the function value is smaller than or equal to the value at all other feasible points.

Generally, Optimization Toolbox solvers find a local optimum. (This local optimum can be a global optimum.) They find the optimum in the basin of attraction of the starting point. For more information, see “Basins of Attraction” on page 1-13.

In contrast, Global Optimization Toolbox solvers are designed to search through more than one basin of attraction. They search in various ways:

- GlobalSearch and MultiStart generate a number of starting points. They then use a local solver to find the optima in the basins of attraction of the starting points.
• **ga** uses a set of starting points (called the population) and iteratively generates better points from the population. As long as the initial population covers several basins, **ga** can examine several basins.

• **simulannealbnd** performs a random search. Generally, **simulannealbnd** accepts a point if it is better than the previous point. **simulannealbnd** occasionally accepts a worse point, in order to reach a different basin.

• **patternsearch** looks at a number of neighboring points before accepting one of them. If some neighboring points belong to different basins, **patternsearch** in essence looks in a number of basins at once.

### Basins of Attraction

If an objective function $f(x)$ is smooth, the vector $-\nabla f(x)$ points in the direction where $f(x)$ decreases most quickly. The equation of steepest descent, namely

$$\frac{d}{dt} x(t) = -\nabla f(x(t)),$$

yields a path $x(t)$ that goes to a local minimum as $t$ gets large. Generally, initial values $x(0)$ that are close to each other give steepest descent paths that tend to the same minimum point. The *basin of attraction* for steepest descent is the set of initial values leading to the same local minimum.

The following figure shows two one-dimensional minima. The figure shows different basins of attraction with different line styles, and it shows directions of steepest descent with arrows. For this and subsequent figures, black dots represent local minima. Every steepest descent path, starting at a point $x(0)$, goes to the black dot in the basin containing $x(0)$.
The following figure shows how steepest descent paths can be more complicated in more dimensions.

The following figure shows even more complicated paths and basins of attraction.
Constraints can break up one basin of attraction into several pieces. For example, consider minimizing $y$ subject to:

- $y \geq |x|$
- $y \geq 5 - 4(x-2)^2$.

The figure shows the two basins of attraction with the final points.
The steepest descent paths are straight lines down to the constraint boundaries. From the constraint boundaries, the steepest descent paths travel down along the boundaries. The final point is either $(0,0)$ or $(11/4,11/4)$, depending on whether the initial $x$-value is above or below 2.
Optimization Workflow

To solve an optimization problem:

1 Decide what type of problem you have, and whether you want a local or global solution (see “Local vs. Global Optima” on page 1-12). Choose a solver per the recommendations in “Choosing a Solver” on page 1-18.

2 Write your objective function and, if applicable, constraint functions per the syntax in “Computing Objective Functions” on page 2-2 and “Constraints” on page 2-6.

3 Set appropriate options with `psoptimset`, `gaoptimset`, or `saoptimset`, or prepare a `GlobalSearch` or `MultiStart` problem as described in “How to Optimize with GlobalSearch and MultiStart” on page 3-2. For details, see “Pattern Search Options” on page 9-9, “Genetic Algorithm Options” on page 9-31, or “Simulated Annealing Options” on page 9-56.

4 Run the solver.

5 Examine the result. For information on the result, see “Examining Results” in the Optimization Toolbox documentation or “Examining Results” on page 3-23 for `GlobalSearch` or `MultiStart`.

6 If the result is unsatisfactory, change options or start points or otherwise update your optimization and rerun it. For information, see “Improving Results” on page 3-53, or see “When the Solver Fails”, “When the Solver Might Have Succeeded”, or “When the Solver Succeeds” in the Optimization Toolbox documentation.
Choosing a Solver

In this section...

- “Table for Choosing a Solver” on page 1-18
- “Solver Characteristics” on page 1-23
- “Why Are Some Solvers Objects?” on page 1-25

Table for Choosing a Solver

There are six Global Optimization Toolbox solvers:

- `ga` (Genetic Algorithm)
- `GlobalSearch`
- `MultiStart`
- `patternsearch`, also called direct search
- `simulannealbnd` (Simulated Annealing)
- `gamultiobj`, which is not a minimizer; see Chapter 7, “Multiobjective Optimization”

Choose an optimizer based on problem characteristics and on the type of solution you want. “Solver Characteristics” on page 1-23 contains more information that can help you decide which solver is likely to be most suitable.

<table>
<thead>
<tr>
<th>Desired Solution</th>
<th>Smooth Objective and Constraints</th>
<th>Nonsmooth Objective or Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single local solution</td>
<td>Optimization Toolbox functions; see “Optimization Decision Table” in the Optimization Toolbox documentation</td>
<td><code>fminbnd, patternsearch, fminsearch, ga, simulannealbnd</code></td>
</tr>
<tr>
<td>Multiple local solutions</td>
<td><code>GlobalSearch, MultiStart</code></td>
<td></td>
</tr>
</tbody>
</table>
### Choosing a Solver

#### Desired Solution

<table>
<thead>
<tr>
<th>Desired Solution</th>
<th>Smooth Objective and Constraints</th>
<th>Nonsmooth Objective or Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single global solution</td>
<td>GlobalSearch, MultiStart, patternsearch, ga, simulannealbnd</td>
<td>patternsearch, ga, simulannealbnd</td>
</tr>
<tr>
<td>Single local solution using parallel processing</td>
<td>MultiStart, Optimization Toolbox functions</td>
<td>patternsearch, ga</td>
</tr>
<tr>
<td>Multiple local solutions using parallel processing</td>
<td>MultiStart</td>
<td></td>
</tr>
<tr>
<td>Single global solution using parallel processing</td>
<td>MultiStart</td>
<td>patternsearch, ga</td>
</tr>
</tbody>
</table>

### Explanation of “Desired Solution”

To understand the meaning of the terms in “Desired Solution,” consider the example

\[
f(x) = 100x^2(1-x)^2-x,
\]

which has local minima \(x_1\) near 0 and \(x_2\) near 1:
Introducing Global Optimization Toolbox Functions

The minima are located at:

\[
x_1 = \text{fminsearch(@(x) (100*x^2*(x - 1)^2 - x), 0)}
\]
\[
x_1 = 0.0051
\]

\[
x_2 = \text{fminsearch(@(x) (100*x^2*(x - 1)^2 - x), 1)}
\]
\[
x_2 = 1.0049
\]

Description of the Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single local solution</td>
<td>Find one local solution, a point ( x ) where the objective function ( f(x) ) is a local minimum. For more details, see “Local vs. Global Optima” on</td>
</tr>
</tbody>
</table>
Description of the Terms (Continued)

<table>
<thead>
<tr>
<th>Term</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>page 1-12. In the example, both (x_1) and (x_2) are local solutions.</td>
<td></td>
</tr>
<tr>
<td>Multiple local solutions</td>
<td>Find a set of local solutions. In the example, the complete set of local solutions is ({x_1, x_2}).</td>
</tr>
<tr>
<td>Single global solution</td>
<td>Find the point (x) where the objective function (f(x)) is a global minimum. In the example, the global solution is (x_2).</td>
</tr>
</tbody>
</table>

Choosing Between Solvers for Smooth Problems

Single Global Solution.

1 Try GlobalSearch first. It is most focused on finding a global solution, and has an efficient local solver, `fmincon`.

2 Try MultiStart second. It has efficient local solvers, and can search a wide variety of start points.

3 Try patternsearch third. It is less efficient, since it does not use gradients. However, patternsearch is robust and is more efficient than the remaining local solvers.

4 Try ga fourth. It can handle all types of constraints, and is usually more efficient than `simulannealbnd`.

5 Try `simulannealbnd` last. It can handle problems with no constraints or bound constraints. `simulannealbnd` is usually the least efficient solver. However, given a slow enough cooling schedule, it can find a global solution.

Multiple Local Solutions. GlobalSearch and MultiStart both provide multiple local solutions. For the syntax to obtain multiple solutions, see “Multiple Solutions” on page 3-24. GlobalSearch and MultiStart differ in the following characteristics:
MultiStart can find more local minima. This is because GlobalSearch rejects many generated start points (initial points for local solution). Essentially, GlobalSearch accepts a start point only when it determines that the point has a good chance of obtaining a global minimum. In contrast, MultiStart passes all generated start points to a local solver. For more information, see “GlobalSearch Algorithm” on page 3-45.

MultiStart offers a choice of local solver: fmincon, fminunc, lsqcurvefit, or lsqnonlin. The GlobalSearch solver uses only fmincon as its local solver.

GlobalSearch uses a scatter-search algorithm for generating start points. In contrast, MultiStart generates points uniformly at random within bounds, or allows you to provide your own points.

MultiStart can run in parallel. See “How to Use Parallel Processing” on page 8-11.

**Choosing Between Solvers for Nonsmooth Problems**

Choose the applicable solver with the lowest number. For problems with integer constraints, use ga.

1. Use `fminbnd` first on one-dimensional bounded problems only. `fminbnd` provably converges quickly in one dimension.

2. Use `patternsearch` on any other type of problem. `patternsearch` provably converges, and handles all types of constraints.

3. Try `fminsearch` next for low-dimensional unbounded problems. `fminsearch` is not as general as `patternsearch` and can fail to converge. For low-dimensional problems, `fminsearch` is simple to use, since it has few tuning options.

4. Try `ga` next. `ga` has little supporting theory and is often less efficient than `patternsearch`. It handles all types of constraints. `ga` is the only solver that handles integer constraints.

5. Try `simulannealbnd` last for unbounded problems, or for problems with bounds. `simulannealbnd` provably converges only for a logarithmic cooling schedule, which is extremely slow. `simulannealbnd` takes only bound constraints, and is often less efficient than `ga`. 
## Solver Characteristics

<table>
<thead>
<tr>
<th>Solver</th>
<th>Convergence</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>GlobalSearch</td>
<td>Fast convergence to local optima for smooth problems.</td>
<td>Deterministic iterates</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gradient-based</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Automatic stochastic start points</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Removes many start points heuristically</td>
</tr>
<tr>
<td>MultiStart</td>
<td>Fast convergence to local optima for smooth problems.</td>
<td>Deterministic iterates</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Can run in parallel; see “How to Use Parallel Processing” on page 8-11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gradient-based</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stochastic or deterministic start points, or combination of both</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Automatic stochastic start points</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Runs all start points</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Choice of local solver: <code>fmincon</code>, <code>fminunc</code>, <code>lsqcurvefit</code>, or <code>lsqnonlin</code></td>
</tr>
<tr>
<td>patternsearch</td>
<td>Proven convergence to local optimum, slower than gradient-based solvers.</td>
<td>Deterministic iterates</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Can run in parallel; see “How to Use Parallel Processing” on page 8-11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No gradients</td>
</tr>
<tr>
<td></td>
<td></td>
<td>User-supplied start point</td>
</tr>
</tbody>
</table>
Introducing Global Optimization Toolbox Functions

<table>
<thead>
<tr>
<th>Solver</th>
<th>Convergence</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>ga</td>
<td>No convergence proof.</td>
<td>Stochastic iterates</td>
</tr>
<tr>
<td></td>
<td>Can run in parallel; see “How to Use Parallel Processing” on page 8-11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Population-based</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No gradients</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Allows integer constraints; see “Mixed Integer Optimization” on page 5-28</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Automatic start population, or user-supplied population, or combination of both</td>
<td></td>
</tr>
<tr>
<td>simulannealbnd</td>
<td>Proven to converge to global optimum for bounded problems with very slow cooling schedule.</td>
<td>Stochastic iterates</td>
</tr>
<tr>
<td></td>
<td>No gradients</td>
<td></td>
</tr>
<tr>
<td></td>
<td>User-supplied start point</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Only bound constraints</td>
<td></td>
</tr>
</tbody>
</table>

Explanation of some characteristics:

- **Convergence** — Solvers can fail to converge to any solution when started far from a local minimum. When started near a local minimum, gradient-based solvers converge to a local minimum quickly for smooth problems. `patternsearch` provably converges for a wide range of problems, but the convergence is slower than gradient-based solvers. Both `ga` and `simulannealbnd` can fail to converge in a reasonable amount of time for some problems, although they are often effective.

- **Iterates** — Solvers iterate to find solutions. The steps in the iteration are iterates. Some solvers have deterministic iterates. Others use random numbers and have stochastic iterates.

- **Gradients** — Some solvers use estimated or user-supplied derivatives in calculating the iterates. Other solvers do not use or estimate derivatives, but use only objective and constraint function values.

- **Start points** — Most solvers require you to provide a starting point for the optimization. One reason they require a start point is to obtain the
Choosing a Solver

dimension of the decision variables. ga does not require any starting points, because it takes the dimension of the decision variables as an input. ga can generate its start population automatically.

Compare the characteristics of Global Optimization Toolbox solvers to Optimization Toolbox solvers.

<table>
<thead>
<tr>
<th>Solver</th>
<th>Convergence</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>fmincon, fminunc, fseminf, lsqcurvefit, lsqnonlin</td>
<td>Proven quadratic convergence to local optima for smooth problems</td>
<td>Deterministic iterates</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gradient-based</td>
</tr>
<tr>
<td></td>
<td></td>
<td>User-supplied starting point</td>
</tr>
<tr>
<td>fminsearch</td>
<td>No convergence proof — counterexamples exist.</td>
<td>Deterministic iterates</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No gradients</td>
</tr>
<tr>
<td></td>
<td></td>
<td>User-supplied start point</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No constraints</td>
</tr>
<tr>
<td>fminbnd</td>
<td>Proven convergence to local optima for smooth problems, slower than quadratic.</td>
<td>Deterministic iterates</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No gradients</td>
</tr>
<tr>
<td></td>
<td></td>
<td>User-supplied start point</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Only one-dimensional problems</td>
</tr>
</tbody>
</table>

All these Optimization Toolbox solvers:

- Have deterministic iterates
- Start from one user-supplied point
- Search just one basin of attraction

**Why Are Some Solvers Objects?**

GlobalSearch and MultiStart are objects. What does this mean for you?

- You create a GlobalSearch or MultiStart object before running your problem.
- You can reuse the object for running multiple problems.
• GlobalSearch and MultiStart objects are containers for algorithms and global options. You use these objects to run a local solver multiple times. The local solver has its own options.

For more information, see the Object-Oriented Programming documentation.
Writing Files for Optimization Functions

- “Computing Objective Functions” on page 2-2
- “Constraints” on page 2-6
Computing Objective Functions

<table>
<thead>
<tr>
<th>In this section...</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Objective (Fitness) Functions” on page 2-2</td>
</tr>
<tr>
<td>“Example: Writing a Function File” on page 2-2</td>
</tr>
<tr>
<td>“Example: Writing a Vectorized Function” on page 2-3</td>
</tr>
<tr>
<td>“Gradients and Hessians” on page 2-4</td>
</tr>
<tr>
<td>“Maximizing vs. Minimizing” on page 2-5</td>
</tr>
</tbody>
</table>

Objective (Fitness) Functions

To use Global Optimization Toolbox functions, you must first write a file (or an anonymous function) that computes the function you want to optimize. This function is called an objective function for most solvers or a fitness function for `ga`. The function should accept a vector whose length is the number of independent variables, and should return a scalar. For vectorized solvers, the function should accept a matrix (where each row represents one input vector), and return a vector of objective function values. This section shows how to write the file.

Example: Writing a Function File

The following example shows how to write a file for the function you want to optimize. Suppose that you want to minimize the function

\[ f(x) = x_1^2 - 2x_1x_2 + 6x_1 + 4x_2^2 - 3x_2. \]

The file that computes this function must accept a vector \( x \) of length 2, corresponding to the variables \( x_1 \) and \( x_2 \), and return a scalar equal to the value of the function at \( x \). To write the file, do the following steps:

1. Select **New > Script (Ctrl+N)** from the MATLAB **File** menu. This opens a new file in the editor.

2. In the file, enter the following two lines of code:

   ```matlab
   function z = my_fun(x)
   ```
3 Save the file in a folder on the MATLAB path.

To check that the file returns the correct value, enter

```matlab
my_fun([2 3])
```

ans =
31

**Example: Writing a Vectorized Function**

The **ga** and **patternsearch** solvers optionally compute the objective functions of a collection of vectors in one function call. This method can take less time than computing the objective functions of the vectors serially. This method is called a vectorized function call.

To compute in vectorized fashion:

- Write your objective function to:
  - Accept a matrix with an arbitrary number of rows
  - Return the vector of function values of each row
- If you have a nonlinear constraint, be sure to write the constraint in a vectorized fashion. For details, see “Vectorized Constraints” on page 2-7.
- Set the `Vectorized` option to `'on'` with `gaoptimset` or `psoptimset`, or set **User function evaluation > Evaluate objective/fitness and constraint functions** to vectorized in the Optimization Tool. For **patternsearch**, also set `CompletePoll` to `'on'`. Be sure to pass the options structure to the solver.

For example, to write the objective function of “Example: Writing a Function File” on page 2-2 in a vectorized fashion,

```matlab
function z = my_fun(x)
    z = x(:,1).^2 - 2*x(:,1).*x(:,2) + 6*x(:,1) + 4*x(:,2).^2 - 3*x(:,2);
```

To use `my_fun` as a vectorized objective function for `patternsearch`:
Writing Files for Optimization Functions

```matlab
options = psoptimset('CompletePoll','on','Vectorized','on');
[x fval] = patternsearch(@my_fun,[1 1],[],[],[],[],[],...
[],options);
```

To use `my_fun` as a vectorized objective function for `ga`:

```matlab
options = gaoptimset('Vectorized','on');
[x fval] = ga(@my_fun,2,[],[],[],[],[],[],[
],options);
```

For more information on writing vectorized functions for `patternsearch`, see “Vectorizing the Objective and Constraint Functions” on page 4-83. For more information on writing vectorized functions for `ga`, see “Vectorizing the Fitness Function” on page 5-92.

**Gradients and Hessians**

If you use `GlobalSearch` or `MultiStart`, your objective function can return derivatives (gradient, Jacobian, or Hessian). For details on how to include this syntax in your objective function, see “Writing Objective Functions” in Optimization Toolbox documentation. Use `optimset` to set options so that your solver uses the derivative information:

**Local Solver = fmincon, fminunc**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Option Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective function contains gradient</td>
<td>'GradObj' = 'on'</td>
</tr>
<tr>
<td>Objective function contains Hessian</td>
<td>'Hessian' = 'on'</td>
</tr>
<tr>
<td>Constraint function contains gradient</td>
<td>'GradConstr' = 'on'</td>
</tr>
<tr>
<td>Calculate Hessians of Lagrangian in an extra function</td>
<td>'Hessian' = 'on', 'HessFcn' = function handle</td>
</tr>
</tbody>
</table>

For more information about Hessians for `fmincon`, see “Hessian”.

**Local Solver = lsqcurvefit, lsqnonlin**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Option Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective function contains Jacobian</td>
<td>'Jacobian' = 'on'</td>
</tr>
</tbody>
</table>
**Maximizing vs. Minimizing**

Global Optimization Toolbox optimization functions minimize the objective or fitness function. That is, they solve problems of the form

$$\min_{x} f(x).$$

If you want to maximize $f(x)$, minimize $-f(x)$, because the point at which the minimum of $-f(x)$ occurs is the same as the point at which the maximum of $f(x)$ occurs.

For example, suppose you want to maximize the function

$$f(x) = x_1^2 - 2x_1x_2 + 6x_1 + 4x_2^2 - 3x_2.$$

Write your function file to compute

$$g(x) = -f(x) = -x_1^2 + 2x_1x_2 - 6x_1 - 4x_2^2 + 3x_2,$$

and minimize $g(x)$. 


Constraints

In this section...

“Links to Optimization Toolbox Documentation” on page 2-6
“Set Bounds” on page 2-6
“Gradients and Hessians” on page 2-7
“Vectorized Constraints” on page 2-7

Links to Optimization Toolbox Documentation

Many Global Optimization Toolbox functions accept bounds, linear constraints, or nonlinear constraints. To see how to include these constraints in your problem, see “Writing Constraints” in the Optimization Toolbox documentation. Try consulting these pertinent links to sections:

- “Bound Constraints”
- “Linear Inequality Constraints” (linear equality constraints have the same form)
- “Nonlinear Constraints”

Set Bounds

It is more important to set bounds for global solvers than for local solvers. Global solvers use bounds in a variety of ways:

- GlobalSearch requires bounds for its scatter-search point generation. If you do not provide bounds, GlobalSearch bounds each component below by -9999 and above by 10001. However, these bounds can easily be inappropriate.
- If you do not provide bounds and do not provide custom start points, MultiStart bounds each component below by -1000 and above by 1000. However, these bounds can easily be inappropriate.
- ga uses bounds and linear constraints for its initial population generation. For unbounded problems, ga uses a default of 0 as the lower bound and 1 as the upper bound for each dimension for initial point generation. For
bounded problems, and problems with linear constraints, ga uses the bounds and constraints to make the initial population.

- `simulannealbnd` and `patternsearch` do not require bounds, although they can use bounds.

**Gradients and Hessians**

If you use `GlobalSearch` or `MultiStart` with `fmincon`, your nonlinear constraint functions can return derivatives (gradient or Hessian). For details, see “Gradients and Hessians” on page 2-4.

**Vectorized Constraints**

The `ga` and `patternsearch` solvers optionally compute the nonlinear constraint functions of a collection of vectors in one function call. This method can take less time than computing the objective functions of the vectors serially. This method is called a vectorized function call.

For the solver to compute in a vectorized manner, you must vectorize both your objective (fitness) function and nonlinear constraint function. For details, see “Vectorizing the Objective and Constraint Functions” on page 4-83.

As an example, suppose your nonlinear constraints for a three-dimensional problem are

\[
\frac{x_1^2}{4} + \frac{x_2^2}{9} + \frac{x_3^2}{25} \leq 6 \\
x_3 \geq \cosh(x_1 + x_2) \\
x_1x_2x_3 = 2.
\]

The following code gives these nonlinear constraints in a vectorized fashion, assuming that the rows of your input matrix `x` are your population or input vectors:

```matlab
function [c ceq] = nlinconst(x)

c(:,1) = x(:,1).^2/4 + x(:,2).^2/9 + x(:,3).^2/25 - 6;
c(:,2) = cosh(x(:,1) + x(:,2)) - x(:,3);
ceq = x(:,1).*x(:,2).*x(:,3) - 2;
```

2-7
For example, minimize the vectorized quadratic function

```matlab
function y = vfun(x)
    y = -x(:,1).^2 - x(:,2).^2 - x(:,3).^2;
```

over the region with constraints `nlinconst` using `patternsearch`:

```matlab
options = psoptimset('CompletePoll','on','Vectorized','on');
[x fval] = patternsearch(@vfun,[1,1,2],[],[],[],[],[],..., @nlinconst,options)
```

Optimization terminated: mesh size less than options.TolMesh and constraint violation is less than options.TolCon.

```
x =
    0.2191   0.7500   12.1712

fval =
    -148.7480
```

Using `ga`:

```matlab
options = gaoptimset('Vectorized','on');
[x fval] = ga(@vfun,3,[],[],[],[],[],[],@nlinconst,options)
```

Optimization terminated: maximum number of generations exceeded.

```
x =
    -1.4098  -0.1216   11.6664

fval =
    -138.1066
```

For this problem `patternsearch` computes the solution far more quickly and accurately.
Using GlobalSearch and MultiStart

- “How to Optimize with GlobalSearch and MultiStart” on page 3-2
- “Examining Results” on page 3-23
- “How GlobalSearch and MultiStart Work” on page 3-43
- “Improving Results” on page 3-53
- “GlobalSearch and MultiStart Examples” on page 3-70
How to Optimize with GlobalSearch and MultiStart

In this section...

| Problems You Can Solve with GlobalSearch and MultiStart” on page 3-2 |
| Outline of Steps” on page 3-2 |
| Determining Inputs for the Problem” on page 3-4 |
| Create a Problem Structure” on page 3-4 |
| Create a Solver Object” on page 3-13 |
| Set Start Points for MultiStart” on page 3-16 |
| Run the Solver” on page 3-19 |

Problems You Can Solve with GlobalSearch and MultiStart

The GlobalSearch and MultiStart solvers apply to problems with smooth objective and constraint functions. The solvers search for a global minimum, or for a set of local minima. For more information on which solver to use, see “Choosing a Solver” on page 1-18.

GlobalSearch and MultiStart work by starting a local solver, such as fmincon, from a variety of start points. Generally the start points are random. However, for MultiStart you can provide a set of start points. For more information, see “How GlobalSearch and MultiStart Work” on page 3-43.

To find out how to use these solvers, see “Outline of Steps” on page 3-2. For examples using these solvers, see the example index.

Outline of Steps

To find a global or multiple local solutions:

1 “Create a Problem Structure” on page 3-4
2 “Create a Solver Object” on page 3-13
3 (Optional, MultiStart only) “Set Start Points for MultiStart” on page 3-16
4 “Run the Solver” on page 3-19

The following figure illustrates these steps.
Determining Inputs for the Problem

A problem structure defines a local optimization problem using a local solver, such as `fmincon`. Therefore, most of the documentation on choosing a solver or preparing the inputs is in the Optimization Toolbox documentation.

**Required Inputs**

<table>
<thead>
<tr>
<th>Input</th>
<th>More Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local solver</td>
<td>“Optimization Decision Table” in the Optimization Toolbox documentation.</td>
</tr>
<tr>
<td>Objective function</td>
<td>“Computing Objective Functions” on page 2-2</td>
</tr>
<tr>
<td>Start point x0</td>
<td>Gives the dimension of points for the objective function.</td>
</tr>
</tbody>
</table>

**Optional Inputs**

<table>
<thead>
<tr>
<th>Input</th>
<th>More Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraint functions</td>
<td>“Constraints” on page 2-6</td>
</tr>
<tr>
<td>Local options structure</td>
<td>“Setting Options”</td>
</tr>
</tbody>
</table>

Create a Problem Structure

To use the `GlobalSearch` or `MultiStart` solvers, you must first create a problem structure. There are two ways to create a problem structure:

- “Using the createOptimProblem Function” on page 3-4
- “Exporting from the Optimization Tool” on page 3-8

**Using the createOptimProblem Function**

Follow these steps to create a problem structure using the `createOptimProblem` function.

1. Define your objective function as a file or anonymous function. For details, see “Computing Objective Functions” on page 2-2. If your solver
How to Optimize with GlobalSearch and MultiStart

is lsqcurvefit or lsqnonlin, ensure the objective function returns a vector, not scalar.

2 If relevant, create your constraints, such as bounds and nonlinear constraint functions. For details, see “Constraints” on page 2-6.

3 Create a start point. For example, to create a three-dimensional random start point xstart:

   xstart = randn(3,1);

4 (Optional) Create an options structure using optimset. For example,

   options = optimset('Algorithm','interior-point');

5 Enter

   problem = createOptimProblem(solver, 

where solver is the name of your local solver:
• For GlobalSearch: 'fmincon'
• For MultiStart the choices are:
  – 'fmincon'
  – 'fminunc'
  – 'lsqcurvefit'
  – 'lsqnonlin'

For help choosing, see “Optimization Decision Table”.

6 Set an initial point using the 'x0' parameter. If your initial point is xstart, and your solver is fmincon, your entry is now

   problem = createOptimProblem('fmincon','x0',xstart,

7 Include the function handle for your objective function in objective:

   problem = createOptimProblem('fmincon','x0',xstart, ... 
   'objective',@objfun,

8 Set bounds and other constraints as applicable.
Using GlobalSearch and MultiStart

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>lower bounds</td>
<td>'lb'</td>
</tr>
<tr>
<td>upper bounds</td>
<td>'ub'</td>
</tr>
<tr>
<td>matrix Aineq for linear inequalities Aineq x ≤ bineq</td>
<td>'Aineq'</td>
</tr>
<tr>
<td>vector bineq for linear inequalities Aineq x ≤ bineq</td>
<td>'bineq'</td>
</tr>
<tr>
<td>matrix Aeq for linear equalities Aeq x = beq</td>
<td>'Aeq'</td>
</tr>
<tr>
<td>vector beq for linear equalities Aeq x = beq</td>
<td>'beq'</td>
</tr>
<tr>
<td>nonlinear constraint function</td>
<td>'nonlcon'</td>
</tr>
</tbody>
</table>

9 If using the lsqcurvefit local solver, include vectors of input data and response data, named 'xdata' and 'ydata' respectively.

10 Best practice: validate the problem structure by running your solver on the structure. For example, if your local solver is fmincon:

   \[
   [x \ fval \ eflag \ output] = \text{fmincon}(\text{problem});
   \]

Note Specify fmincon as the solver for GlobalSearch, even if you have no constraints. However, you cannot run fmincon on a problem without constraints. Add an artificial constraint to the problem to validate the structure:

   \[
   \text{problem}\.lb = -\text{Inf};
   \]

Example: Creating a Problem Structure with createOptimProblem.
This example minimizes the function from “Run the Solver” on page 3-19, subject to the constraint \(x_1 + 2x_2 \geq 4\). The objective is

   \[
   \text{sixmin} = 4x^2 - 2.1x^4 + x^{6/3} + xy - 4y^2 + 4y^4.
   \]

Use the interior-point algorithm of fmincon, and set the start point to [2;3].
1 Write a function handle for the objective function.

\[
\text{sixmin} = @(x)(4*x(1)^2 - 2.1*x(1)^4 + x(1)^6/3 \ldots \\
+ x(1)*x(2) - 4*x(2)^2 + 4*x(2)^4);
\]

2 Write the linear constraint matrices. Change the constraint to “less than” form:

\[
A = \begin{bmatrix} -1 & -2 \end{bmatrix}; \\
b = -4;
\]

3 Create the local options structure to use the interior-point algorithm:

\[
\text{opts} = \text{optimset('Algorithm','interior-point');}
\]

4 Create the problem structure with createOptimProblem:

\[
\text{problem} = \text{createOptimProblem('fmincon', \ldots \\
'x0',[2;3],'objective','sixmin', \ldots \\
'Aineq',A,'bineq',b,'options',opts)}
\]

5 The resulting structure:

\[
\text{problem} = \\
\begin{align*}
\text{objective}: & \text{[function_handle]} \\
x0: & \text{[2x1 double]} \\
Aineq: & \text{[-1 -2]} \\
bineq: & -4 \\
Aeq: & [] \\
beq: & [] \\
lb: & [] \\
ub: & [] \\
nonlcon: & [] \\
solver: & \text{'fmincon'} \\
options: & \text{[1x1 struct]}
\end{align*}
\]

6 Best practice: validate the problem structure by running your solver on the structure:

\[
[x \ fval \ eflag \ output] = \text{fmincon(problem)};
\]
Note Specify \texttt{fmincon} as the solver for \texttt{GlobalSearch}, even if you have no constraints. However, you cannot run \texttt{fmincon} on a problem without constraints. Add an artificial constraint to the problem to validate the structure:

\begin{verbatim}
problem.lb = -Inf;
\end{verbatim}

Exporting from the Optimization Tool
Follow these steps to create a problem structure using the Optimization Tool.

1 Define your objective function as a file or anonymous function. For details, see “Computing Objective Functions” on page 2-2. If your solver is \texttt{lsqcurvefit} or \texttt{lsqnonlin}, ensure the objective function returns a vector, not scalar.

2 If relevant, create nonlinear constraint functions. For details, see “Nonlinear Constraints”.

3 Create a start point. For example, to create a three-dimensional random start point \texttt{xstart}:

\begin{verbatim}
 xstart = randn(3,1);
\end{verbatim}

4 Open the Optimization Tool by entering \texttt{optimtool} at the command line, or by choosing Start > Toolboxes > Optimization > Optimization Tool.

5 Choose the local \texttt{Solver}.

- For \texttt{GlobalSearch} \texttt{fmincon} (default).
- For \texttt{MultiStart}:
  - \texttt{fmincon} (default)
How to Optimize with GlobalSearch and MultiStart

- fminunc
- lsqcurvefit
- lsqnonlin

For help choosing, see “Optimization Decision Table”.

6 Choose an appropriate **Algorithm**. For help choosing, see “Choosing the Algorithm”.

7 Set an initial point (**Start point**).

8 Include the function handle for your objective function in **Objective function**, and, if applicable, include your **Nonlinear constraint function**. For example,

9 Set bounds, linear constraints, or local **Options**. For details on constraints, see “Writing Constraints”.
10 **Best practice: run the problem to verify the setup.**

![Image of Run solver and view results]

**Note** You must specify `fmincon` as the solver for `GlobalSearch`, even if you have no constraints. However, you cannot run `fmincon` on a problem without constraints. Add an artificial constraint to the problem to verify the setup.

![Image of constraints]

11 Choose **File > Export to Workspace** and select **Export problem and options to a MATLAB structure named**

![Image of Export To Workspace dialog box]
Example: Creating a Problem Structure with the Optimization Tool.
This example minimizes the function from “Run the Solver” on page 3-19, subject to the constraint $x_1 + 2x_2 \geq 4$. The objective is

$$\text{sixmin} = 4x^2 - 2.1x^4 + x^6/3 + xy - 4y^2 + 4y^4.$$ 

Use the interior-point algorithm of \texttt{fmincon}, and set the start point to [2;3].

1. Write a function handle for the objective function.

$$\text{sixmin} = @(x)(4*x(1)^2 - 2.1*x(1)^4 + x(1)^6/3 \ldots + x(1)*x(2) - 4*x(2)^2 + 4*x(2)^4);$$

2. Write the linear constraint matrices. Change the constraint to “less than” form:

$$A = [-1, -2];$$
$$b = -4;$$

3. Launch the Optimization Tool by entering \texttt{optimtool} at the MATLAB command line.

4. Set the solver, algorithm, objective, start point, and constraints.
5 Best practice: run the problem to verify the setup.

The problem runs successfully.
Choose File > Export to Workspace and select Export problem and options to a MATLAB structure named

Create a Solver Object
A solver object contains your preferences for the global portion of the optimization.

You do not need to set any preferences. Create a GlobalSearch object named gs with default settings as follows:

```matlab
gs = GlobalSearch;
```
Similarly, create a MultiStart object named `ms` with default settings as follows:

```matlab
ms = MultiStart;
```

**Properties (Global Options) of Solver Objects**

Global options are properties of a GlobalSearch or MultiStart object.

**Properties for both GlobalSearch and MultiStart**

<table>
<thead>
<tr>
<th>Property Name</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Display</td>
<td>Detail level of iterative display. Set to 'off' for no display, 'final' (default) for a report at the end of the run, or 'iter' for reports as the solver progresses. For more information and examples, see “Iterative Display” on page 3-28.</td>
</tr>
<tr>
<td>TolFun</td>
<td>Solvers consider objective function values within TolFun of each other to be identical (not distinct). Default: 1e-6. Solvers group solutions when the solutions satisfy both TolFun and TolX tolerances.</td>
</tr>
<tr>
<td>TolX</td>
<td>Solvers consider solutions within TolX distance of each other to be identical (not distinct). Default: 1e-6. Solvers group solutions when the solutions satisfy both TolFun and TolX tolerances.</td>
</tr>
<tr>
<td>MaxTime</td>
<td>Solvers halt if the run exceeds MaxTime seconds, as measured by a clock (not processor seconds). Default: Inf</td>
</tr>
<tr>
<td>StartPointsToRun</td>
<td>Choose whether to run 'all' (default) start points, only those points that satisfy 'bounds', or only those points that are feasible with respect to bounds and inequality constraints with 'bounds-ineqs'. For an example, see “Example: Using Only Feasible Start Points” on page 3-77.</td>
</tr>
</tbody>
</table>
Properties for both GlobalSearch and MultiStart (Continued)

<table>
<thead>
<tr>
<th>Property Name</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>OutputFcns</td>
<td>Functions to run after each local solver run. See “Output Functions for GlobalSearch and MultiStart” on page 3-34. Default: []</td>
</tr>
<tr>
<td>PlotFcns</td>
<td>Plot functions to run after each local solver run. See “Plot Functions for GlobalSearch and MultiStart” on page 3-38. Default: []</td>
</tr>
</tbody>
</table>

Properties for GlobalSearch

<table>
<thead>
<tr>
<th>Property Name</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>NumTrialPoints</td>
<td>Number of trial points to examine. Default: 1000</td>
</tr>
<tr>
<td>BasinRadiusFactor</td>
<td></td>
</tr>
<tr>
<td>DistanceThresholdFactor</td>
<td></td>
</tr>
<tr>
<td>MaxWaitCycle</td>
<td></td>
</tr>
<tr>
<td>NumStageOnePoints</td>
<td></td>
</tr>
<tr>
<td>PenaltyThresholdFactor</td>
<td>See “Properties” on page 12-35 for detailed descriptions of these properties.</td>
</tr>
</tbody>
</table>

Properties for MultiStart

<table>
<thead>
<tr>
<th>Property Name</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>UseParallel</td>
<td>When 'always', MultiStart attempts to distribute start points to multiple processors for the local solver. Disable by setting to 'never' (default). For details, see “How to Use Parallel Processing” on page 8-11. For an example, see “Example: Parallel MultiStart” on page 3-81.</td>
</tr>
</tbody>
</table>

Example: Creating a Nondefault GlobalSearch Object

Suppose you want to solve a problem and:
• Consider local solutions identical if they are within 0.01 of each other and the function values are within the default TolFun tolerance.
• Spend no more than 2000 seconds on the computation.

To solve the problem, create a GlobalSearch object `gs` as follows:

```matlab
gs = GlobalSearch('TolX',0.01,'MaxTime',2000);
```

**Example: Creating a Nondefault MultiStart Object**

Suppose you want to solve a problem such that:

• You consider local solutions identical if they are within 0.01 of each other and the function values are within the default TolFun tolerance.
• You spend no more than 2000 seconds on the computation.

To solve the problem, create a MultiStart object `ms` as follows:

```matlab
ms = MultiStart('TolX',0.01,'MaxTime',2000);
```

**Set Start Points for MultiStart**

There are four ways you tell MultiStart which start points to use for the local solver:

• Pass a positive integer `k`. MultiStart generates `k - 1` start points as if using a `RandomStartPointSet` object and the problem structure. MultiStart also uses the `x0` start point from the problem structure, for a total of `k` start points.
• Pass a `RandomStartPointSet` object.
• Pass a `CustomStartPointSet` object.
• Pass a cell array of `RandomStartPointSet` and `CustomStartPointSet` objects. Pass a cell array if you have some specific points you want to run, but also want MultiStart to use other random start points.
Note You can control whether MultiStart uses all start points, or only those points that satisfy bounds or other inequality constraints. For more information, see “Filter Start Points (Optional)” on page 3-50.

Positive Integer for Start Points
The syntax for running MultiStart for k start points is

\[
[x_{\text{min}}, f_{\text{min}}, \text{flag}, \text{outpt}, \text{allmins}] = \text{run}(\text{ms}, \text{problem}, k);
\]

The positive integer k specifies the number of start points MultiStart uses. MultiStart generates random start points using the dimension of the problem and bounds from the problem structure. MultiStart generates \(k-1\) random start points, and also uses the \(x_0\) start point from the problem structure.

RandomStartPointSet Object for Start Points
Create a RandomStartPointSet object as follows:

\[
\text{stpoints} = \text{RandomStartPointSet};
\]

By default a RandomStartPointSet object generates 10 start points. Control the number of start points with the NumStartPoints property. For example, to generate 40 start points:

\[
\text{stpoints} = \text{RandomStartPointSet}('\text{NumStartPoints}', 40);
\]

You can set an ArtificialBound for a RandomStartPointSet. This ArtificialBound works in conjunction with the bounds from the problem structure:

- If a component has no bounds, RandomStartPointSet uses a lower bound of -ArtificialBound, and an upper bound of ArtificialBound.
- If a component has a lower bound \(lb\) but no upper bound, RandomStartPointSet uses an upper bound of \(lb + 2 \times \text{ArtificialBound}\).
- Similarly, if a component has an upper bound \(ub\) but no lower bound, RandomStartPointSet uses a lower bound of \(ub - 2 \times \text{ArtificialBound}\).

For example, to generate 100 start points with an ArtificialBound of 50:
using GlobalSearch and MultiStart

```
stpoints = RandomStartPointSet('NumStartPoints',100, ...  
    'ArtificialBound',50);
```

A RandomStartPointSet object generates start points with the same dimension as the x0 point in the problem structure; see list.

**CustomStartPointSet Object for Start Points**

To use a specific set of starting points, package them in a CustomStartPointSet as follows:

1. Place the starting points in a matrix. Each row of the matrix represents one starting point. MultiStart runs all the rows of the matrix, subject to filtering with the StartPointsToRun property. For more information, see “MultiStart Algorithm” on page 3-49.

2. Create a CustomStartPointSet object from the matrix:

   ```matlab```
   tpoints = CustomStartPointSet(ptmatrix);
   ```matlab```

For example, create a set of 40 five-dimensional points, with each component of a point equal to 10 plus an exponentially distributed variable with mean 25:

```
pts = -25*log(rand(40,5)) + 10;
tpoints = CustomStartPointSet(pts);
```

To get the original matrix of points from a CustomStartPointSet object, use the list method:

```
pts = list(tpoints); % Assumes tpoints is a CustomStartPointSet
```

A CustomStartPointSet has two properties: DimStartPoints and NumStartPoints. You can use these properties to query a CustomStartPointSet object. For example, the tpoints object in the example has the following properties:

```
tpoints.DimStartPoints
ans =
    5

tpoints.NumStartPoints
ans =
```
Cell Array of Objects for Start Points

To use a specific set of starting points along with some randomly generated points, pass a cell array of RandomStartPointSet or CustomStartPointSet objects.

For example, to use both the 40 specific five-dimensional points of “CustomStartPointSet Object for Start Points” on page 3-18 and 40 additional five-dimensional points from RandomStartPointSet:

```plaintext
gpts = -25*log(rand(40,5)) + 10;
tpoints = CustomStartPointSet(pts);
rpts = RandomStartPointSet('NumStartPoints',40);
allpts = {tpoints,rpts};
```

Run MultiStart with the allpts cell array:

```plaintext
% Assume ms and problem exist
[xmin,fmin,flag,outpt,allmins] = run(ms,problem,allpts);
```

Run the Solver

Running a solver is nearly identical for GlobalSearch and MultiStart. The only difference in syntax is MultiStart takes an additional input describing the start points.

For example, suppose you want to find several local minima of the sixmin function

\[
sixmin = 4x^2 - 2.1x^4 + x^6/3 + xy - 4y^2 + 4y^4.
\]
Using GlobalSearch and MultiStart

This function is also called the six-hump camel back function [3]. All the local minima lie in the region $-3 \leq x, y \leq 3$.

**Example of Run with GlobalSearch**

To find several local minima of the problem described in “Run the Solver” on page 3-19 using GlobalSearch, enter:

```matlab
 gs = GlobalSearch; 
 opts = optimset('Algorithm','interior-point'); 
 sixmin = @(x)(4*x(1)^2 - 2.1*x(1)^4 + x(1)^6/3 ...  
 + x(1)*x(2) - 4*x(2)^2 + 4*x(2)^4);  
 problem = createOptimProblem('fmincon','x0',[-1,2],... 
 'objective',sixmin,'lb',[-3,-3],'ub',[3,3],... 
 'options',opts);  
 [xming,fming,flagg,outptg,manyminsg] = run(gs,problem); 
```

The output of the run (which varies, based on the random seed):

```
xming,fming,flagg,outptg,manyminsg
```
Example of Run with MultiStart

To find several local minima of the problem described in “Run the Solver” on page 3-19 using 50 runs of fmincon with MultiStart, enter:

```matlab
% % Set the random stream to get exactly the same output
% rng(14,'twister')
ms = MultiStart;
opts = optimset('Algorithm','interior-point');
sixmin = @(x)(4*x(1)^2 - 2.1*x(1)^4 + x(1)^6/3 ... + x(1)*x(2) - 4*x(2)^2 + 4*x(2)^4);
problem = createOptimProblem('fmincon','x0',[-1,2],... 'objective',sixmin,'lb',[-3,-3],'ub',[3,3],...
'options', opts);
[xminm, fminm, flagm, outptm] = run(ms, problem, 50);

The output of the run (which varies based on the random seed):

xminm, fminm, flagm, outptm

xminm =
  -0.0898   0.7127

fminm =
  -1.0316

flagm =
  1

outptm =
  
    funcCount: 2035
    localSolverTotal: 50
    localSolverSuccess: 50
    localSolverIncomplete: 0
    localSolverNoSolution: 0
    message: [1x128 char]

In this case, MultiStart located all six local minima, while GlobalSearch located four. For pictures of the MultiStart solutions, see “Example: Visualizing the Basins of Attraction” on page 3-32.
Examining Results

In this section...

“Single Solution” on page 3-23
“Multiple Solutions” on page 3-24
“Iterative Display” on page 3-28
“Global Output Structures” on page 3-31
“Example: Visualizing the Basins of Attraction” on page 3-32
“Output Functions for GlobalSearch and MultiStart” on page 3-34
“Plot Functions for GlobalSearch and MultiStart” on page 3-38

Single Solution

You obtain the single best solution found during the run by calling run with the syntax

\[
\begin{align*}
[x \ fval \ eflag \ output] &= \text{run}(...) ;
\end{align*}
\]

- \(x\) is the location of the local minimum with smallest objective function value.
- \(fval\) is the objective function value evaluated at \(x\).
- \(eflag\) is an exit flag for the global solver. Values:

Global Solver Exit Flags

<table>
<thead>
<tr>
<th>Exit Flag</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>At least one local minimum found. Some runs of the local solver converged (had positive exit flag).</td>
</tr>
<tr>
<td>1</td>
<td>At least one local minimum found. All runs of the local solver converged (had positive exit flag).</td>
</tr>
<tr>
<td>0</td>
<td>No local minimum found. Local solver called at least once, and at least one local solver exceeded the MaxIter or MaxFunEvals tolerances.</td>
</tr>
<tr>
<td>-1</td>
<td>Solver stopped by output function or plot function.</td>
</tr>
</tbody>
</table>
Global Solver Exit Flags (Continued)

-2 No feasible local minimum found.
-5 MaxTime limit exceeded.
-8 No solution found. All runs had local solver exit flag -1 or smaller.
-10 Failures encountered in user-provided functions.

- output is a structure with details about the multiple runs of the local solver. For more information, see “Global Output Structures” on page 3-31.

The list of outputs is for the case eflag > 0. If eflag <= 0, then x is the following:

- If some local solutions are feasible, x represents the location of the lowest objective function value. “Feasible” means the constraint violations are smaller than problem.options.TolCon.
- If no solutions are feasible, x is the solution with lowest infeasibility.
- If no solutions exist, x, fval, and output are empty ([]).

Multiple Solutions
You obtain multiple solutions in an object by calling run with the syntax

```
[x fval eflag output manymins] = run(...);
```

manymins is a vector of solution objects; see GlobalOptimSolution. The vector is in order of objective function value, from lowest (best) to highest (worst). Each solution object contains the following properties (fields):

- X — a local minimum
- Fval — the value of the objective function at X
- Exitflag — the exit flag for the global solver (described here)
- Output — an output structure (described here)
- X0 — a cell array of start points that led to the solution point X
There are several ways to examine the vector of solution objects:

- In the MATLAB Workspace Browser. Double-click the solution object, and then double-click the resulting display in the Variable Editor.

- Using dot addressing. `GlobalOptimSolution` properties are capitalized. Use proper capitalization to access the properties.

For example, to find the vector of function values, enter:

```
fcnvals = [manymins.Fval]
```
Using GlobalSearch and MultiStart

\[ fcnvals = \begin{array}{ccc}
-1.0316 & -0.2155 & 0 \\
\end{array} \]

To get a cell array of all the start points that led to the lowest function value (the first element of `manymins`), enter:

\[ smallX0 = manymins(1).X0 \]

- Plot some field values. For example, to see the range of resulting `Fval`, enter:

\[ hist([manymins.Fval]) \]

This results in a histogram of the computed function values. (The figure shows a histogram from a different example than the previous few figures.)

![Graph showing histogram of computed function values](image)

**Example: Changing the Definition of Distinct Solutions**

You might find out, after obtaining multiple local solutions, that your tolerances were not appropriate. You can have many more local solutions...
than you want, spaced too closely together. Or you can have fewer solutions than you want, with GlobalSearch or MultiStart clumping together too many solutions.

To deal with this situation, run the solver again with different tolerances. The TolX and TolFun tolerances determine how the solvers group their outputs into the GlobalOptimSolution vector. These tolerances are properties of the GlobalSearch or MultiStart object.

For example, suppose you want to use the active-set algorithm in fmincon to solve the problem in “Example of Run with MultiStart” on page 3-21. Further suppose that you want to have tolerances of 0.01 for both TolX and TolFun. The run method groups local solutions whose objective function values are within TolFun of each other, and which are also less than TolX apart from each other. To obtain the solution:

```matlab
ms = MultiStart('TolFun',0.01,'TolX',0.01);
opts = optimset('Algorithm','active-set');
sixmin = @(x)(4*x(1)^2 - 2.1*x(1)^4 + x(1)^6/3 ... + x(1)*x(2) - 4*x(2)^2 + 4*x(2)^4);
problem = createOptimProblem('fmincon','x0',[-1,2],...
    'objective',sixmin,'lb',[-3,-3],'ub',[3,3],...
    'options',opts);
[xminm,fminm,flagm,outptm,someminsm] = run(ms,problem,50);
```

MultiStart completed the runs from all start points.

All 50 local solver runs converged with a positive local solver exit flag.

```
someminsm

someminsm =

1x6 GlobalOptimSolution

Properties:
    X
    Fval
    Exitflag
```
In this case, `MultiStart` generated six distinct solutions. Here “distinct” means that the solutions are more than 0.01 apart in either objective function value or location.

**Iterative Display**

Iterative display gives you information about the progress of solvers during their runs.

There are two types of iterative display:

- Global solver display
- Local solver display

Both types appear at the command line, depending on global and local options.

Obtain local solver iterative display by setting the `Display` option in the `problem.options` structure to `'iter'` or `'iter-detailed'` with `optimset`. For more information, see “Iterative Display” in the Optimization Toolbox documentation.

Obtain global solver iterative display by setting the `Display` property in the `GlobalSearch` or `MultiStart` object to `'iter'`.

Global solvers set the default `Display` option of the local solver to `'off'`, unless the problem structure has a value for this option. Global solvers do not override any setting you make for local options.

**Note** Setting the local solver `Display` option to anything other than `'off'` can produce a great deal of output. The default `Display` option created by `optimset(@solver)` is `'final'`.  

---

3-28
Example: Types of Iterative Display

Run the example described in “Run the Solver” on page 3-19 using GlobalSearch with GlobalSearch iterative display:

```matlab
gs = GlobalSearch('Display','iter');
opts = optimset('Algorithm','interior-point');
sixmin = @(x)(4*x(1)^2 - 2.1*x(1)^4 + x(1)^6/3 ... 
             + x(1)*x(2) - 4*x(2)^2 + 4*x(2)^4);
problem = createOptimProblem('fmincon','x0',[-1,2],...
    'objective',sixmin,'lb',[-3,-3],'ub',[3,3],...
    'options',opts);
[xming,fming,flagg,outptg,manymins] = run(gs,problem);
```

<table>
<thead>
<tr>
<th>Num Pts</th>
<th>F-count</th>
<th>Best F(x)</th>
<th>Current F(x)</th>
<th>Threshold F(x)</th>
<th>Local F(x)</th>
<th>exitflag</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>34</td>
<td>-1.032</td>
<td></td>
<td>-1.032</td>
<td>1</td>
<td>Initial Point</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>1441</td>
<td>-1.032</td>
<td></td>
<td>0</td>
<td>1</td>
<td>Stage 1 Local</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>1641</td>
<td>-1.032</td>
<td>290.7</td>
<td>0.07006</td>
<td>1</td>
<td>Stage 2 Search</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>1841</td>
<td>-1.032</td>
<td>101</td>
<td>0.8017</td>
<td>1</td>
<td>Stage 2 Search</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>2041</td>
<td>-1.032</td>
<td>51.23</td>
<td>3.483</td>
<td>1</td>
<td>Stage 2 Search</td>
<td></td>
</tr>
<tr>
<td>541</td>
<td>2157</td>
<td>-1.032</td>
<td>5.348</td>
<td>5.456</td>
<td>1</td>
<td>Stage 2 Local</td>
<td></td>
</tr>
<tr>
<td>542</td>
<td>2190</td>
<td>-1.032</td>
<td>2.993</td>
<td>5.348</td>
<td>1</td>
<td>Stage 2 Local</td>
<td></td>
</tr>
<tr>
<td>544</td>
<td>2227</td>
<td>-1.032</td>
<td>1.807</td>
<td>2.993</td>
<td>1</td>
<td>Stage 2 Local</td>
<td></td>
</tr>
<tr>
<td>545</td>
<td>2260</td>
<td>-1.032</td>
<td>1.609</td>
<td>1.807</td>
<td>1</td>
<td>Stage 2 Local</td>
<td></td>
</tr>
<tr>
<td>546</td>
<td>2296</td>
<td>-1.032</td>
<td>0.9061</td>
<td>1.609</td>
<td>1</td>
<td>Stage 2 Local</td>
<td></td>
</tr>
<tr>
<td>552</td>
<td>2348</td>
<td>-1.032</td>
<td>0.6832</td>
<td>0.9061</td>
<td>-0.2155</td>
<td>Stage 2 Local</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>2444</td>
<td>-1.032</td>
<td>1.079</td>
<td>0.1235</td>
<td>1</td>
<td>Stage 2 Search</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>2644</td>
<td>-1.032</td>
<td>0.4859</td>
<td>-0.4791</td>
<td>1</td>
<td>Stage 2 Search</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>2844</td>
<td>-1.032</td>
<td>136.9</td>
<td>0.8202</td>
<td>1</td>
<td>Stage 2 Search</td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>3044</td>
<td>-1.032</td>
<td>37.77</td>
<td>0.4234</td>
<td>1</td>
<td>Stage 2 Search</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>3244</td>
<td>-1.032</td>
<td>-0.05542</td>
<td>-0.598</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

GlobalSearch stopped because it analyzed all the start points.

All 8 local solver runs converged with a positive local solver exit flag.

Run the same example without GlobalSearch iterative display, but with fmincon iterative display:
gs.Display = 'final';
problem.options.Display = 'iter';
[xming,fming,flagg,outptg,manyminsg] = run(gs,problem);

<table>
<thead>
<tr>
<th>Iter</th>
<th>F-count</th>
<th>f(x)</th>
<th>Feasibility</th>
<th>optimality</th>
<th>step</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>4.823333e+001</td>
<td>0.000e+000</td>
<td>1.088e+002</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>2.020476e+000</td>
<td>0.000e+000</td>
<td>2.176e+000</td>
<td>2.488e+000</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>6.525252e-001</td>
<td>0.000e+000</td>
<td>1.937e+000</td>
<td>1.886e+000</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>-8.776121e-001</td>
<td>0.000e+000</td>
<td>9.076e-001</td>
<td>8.539e-001</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>-9.121907e-001</td>
<td>0.000e+000</td>
<td>9.076e-001</td>
<td>1.655e-001</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
<td>-1.009367e+000</td>
<td>0.000e+000</td>
<td>7.326e-001</td>
<td>8.558e-002</td>
</tr>
<tr>
<td>6</td>
<td>22</td>
<td>-1.030423e+000</td>
<td>0.000e+000</td>
<td>2.172e-001</td>
<td>6.670e-002</td>
</tr>
<tr>
<td>7</td>
<td>25</td>
<td>-1.031578e+000</td>
<td>0.000e+000</td>
<td>4.278e-002</td>
<td>1.444e-002</td>
</tr>
<tr>
<td>8</td>
<td>28</td>
<td>-1.031628e+000</td>
<td>0.000e+000</td>
<td>8.777e-003</td>
<td>2.306e-003</td>
</tr>
<tr>
<td>9</td>
<td>31</td>
<td>-1.031628e+000</td>
<td>0.000e+000</td>
<td>8.845e-005</td>
<td>2.750e-004</td>
</tr>
<tr>
<td>10</td>
<td>34</td>
<td>-1.031628e+000</td>
<td>0.000e+000</td>
<td>8.744e-007</td>
<td>1.354e-006</td>
</tr>
</tbody>
</table>

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible directions, to within the selected value of the function tolerance, and constraints were satisfied to within the selected value of the constraint tolerance.

<stopping criteria details>

<table>
<thead>
<tr>
<th>Iter</th>
<th>F-count</th>
<th>f(x)</th>
<th>Feasibility</th>
<th>optimality</th>
<th>step</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>-5.372419e-001</td>
<td>0.000e+000</td>
<td>1.913e+000</td>
<td></td>
</tr>
</tbody>
</table>

... MANY ITERATIONS DELETED ...

| 9    | 39      | -2.154638e-001 | 0.000e+000  | 2.425e-007 | 6.002e-008  |

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible directions, to within the selected value of the function tolerance, and constraints were satisfied to within the selected value of the constraint tolerance.

<stopping criteria details>
GlobalSearch stopped because it analyzed all the start points.

All 7 local solver runs converged with a positive local solver exit flag.

Setting GlobalSearch iterative display, as well as fmincon iterative display, yields both displays intermingled.

For an example of iterative display in a parallel environment, see “Example: Parallel MultiStart” on page 3-81.

**Global Output Structures**

A run can produce two types of output structures:

- A global output structure. This structure contains information about the overall run from multiple starting points. Details follow.

- Local solver output structures. The vector of GlobalOptimSolution objects contains one such structure in each element of the vector. For a description of this structure, see “Output Structures” in the Optimization Toolbox documentation, or the function reference pages for the local solvers: fmincon, fminunc, lsqcurvefit, or lsqnonlin.

**Global Output Structure**

<table>
<thead>
<tr>
<th>Field</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>funcCount</td>
<td>Total number of calls to user-supplied functions (objective or nonlinear constraint)</td>
</tr>
<tr>
<td>localSolverTotal</td>
<td>Number of local solver runs started</td>
</tr>
<tr>
<td>localSolverSuccess</td>
<td>Number of local solver runs that finished with a positive exit flag</td>
</tr>
<tr>
<td>localSolverIncomplete</td>
<td>Number of local solver runs that finished with a 0 exit flag</td>
</tr>
<tr>
<td>localSolverNoSolution</td>
<td>Number of local solver runs that finished with a negative exit flag</td>
</tr>
<tr>
<td>message</td>
<td>GlobalSearch or MultiStart exit message</td>
</tr>
</tbody>
</table>
A positive exit flag from a local solver generally indicates a successful run. A negative exit flag indicates a failure. A 0 exit flag indicates that the solver stopped by exceeding the iteration or function evaluation limit. For more information, see “Exit Flags and Exit Messages” or “Tolerances and Stopping Criteria” in the Optimization Toolbox documentation.

Example: Visualizing the Basins of Attraction

Which start points lead to which basin? For a steepest descent solver, nearby points generally lead to the same basin; see “Basins of Attraction” on page 1-13. However, for Optimization Toolbox solvers, basins are more complicated.

Plot the MultiStart start points from the example, “Example of Run with MultiStart” on page 3-21, color-coded with the basin where they end.

```matlab
% rng(14,'twister')
% Uncomment the previous line to get the same output
ms = MultiStart;
opts = optimset('Algorithm','interior-point');
sixmin = @(x)(4*x(1)^2 - 2.1*x(1)^4 + x(1)^6/3 ... + x(1)*x(2) - 4*x(2)^2 + 4*x(2)^4);
problem = createOptimProblem('fmincon','x0',[-1,2],... 'objective',sixmin,'lb',[-3,-3],'ub',[3,3],... 'options',opts);
xminm,fminm,flagm,outptm,manyminsm = run(ms,problem,50);
possColors = 'kbgcrm';
hold on
for i = 1:size(manyminsm,2)
    % Color of this line
    cIdx = rem(i-1, length(possColors)) + 1;
    color = possColors(cIdx);
    % Plot start points
    u = manyminsm(i).X0;
x0ThisMin = reshape([u{:}], 2, length(u));
plot(x0ThisMin(1, :), x0ThisMin(2, :), '.','Color',color,'MarkerSize',25);
```
% Plot the basin with color i
plot(manyminsm(i).X(1), manyminsm(i).X(2), '*', ...
    'Color', color, 'MarkerSize',25);
end % basin center marked with a *, start points with dots
hold off

The figure shows the centers of the basins by colored * symbols. Start points with the same color as the * symbol converge to the center of the * symbol.

Start points do not always converge to the closest basin. For example, the red points are closer to the green basin center than to the red basin center. Also, many black and blue start points are closer to the opposite basin centers.
The magenta and red basins are shallow, as you can see in the following contour plot.

**Output Functions for GlobalSearch and MultiStart**

- “What Are Output Functions?” on page 3-35
- “Example: GlobalSearch Output Function” on page 3-35
- “No Parallel Output Functions” on page 3-37
What Are Output Functions?

Output functions allow you to examine intermediate results in an optimization. Additionally, they allow you to halt a solver programmatically.

There are two types of output functions, like the two types of output structures:

- Global output functions run after each local solver run. They also run when the global solver starts and ends.
- Local output functions run after each iteration of a local solver. See “Output Functions” in the Optimization Toolbox documentation.

To use global output functions:

- Write output functions using the syntax described in “OutputFcns” on page 9-3.
- Set the OutputFcns property of your GlobalSearch or MultiStart solver to the function handle of your output function. You can use multiple output functions by setting the OutputFcns property to a cell array of function handles.

Example: GlobalSearch Output Function

This output function stops GlobalSearch after it finds five distinct local minima with positive exit flags, or after it finds a local minimum value less than 0.5. The output function uses a persistent local variable, foundLocal, to store the local results. foundLocal enables the output function to determine whether a local solution is distinct from others, to within a tolerance of 1e-4.

To store local results using nested functions instead of persistent variables, see “Example of a Nested Output Function” in the MATLAB Mathematics documentation.

1 Write the output function using the syntax described in “OutputFcns” on page 9-3.

```matlab
function stop = StopAfterFive(optimValues, state)
persistent foundLocal
stop = false;
```
switch state
    case 'init'
        foundLocal = []; % initialized as empty
    case 'iter'
        newf = optimValues.localsolution.Fval;
        eflag = optimValues.localsolution.Exitflag;
        % Now check if the exit flag is positive and
        % the new value differs from all others by at least 1e-4
        % If so, add the new value to the newf list
        if eflag > 0 && all(abs(newf - foundLocal) > 1e-4)
            foundLocal = [foundLocal;newf];
        end
        % Now check if the latest value added to foundLocal
        % is less than 1/2
        % Also check if there are 5 local minima in foundLocal
        % If so, then stop
        if foundLocal(end) < 0.5 || length(foundLocal) >= 5
            stop = true;
        end
    end

2 Save StopAfterFive.m as a file in a folder on your MATLAB path.

3 Write the objective function and create an optimization problem structure as
   in “Example: Finding Global or Multiple Local Minima” on page 3-70.

    function f = sawtoothxy(x,y)
        [t r] = cart2pol(x,y); % change to polar coordinates
        h = cos(2*t - 1/2)/2 + cos(t) + 2;
        g = (sin(r) - sin(2*r)/2 + sin(3*r)/3 - sin(4*r)/4 + 4)...
            .*r.^2./(r+1);
        f = g.*h;
    end

4 Save sawtoothxy.m as a file in a folder on your MATLAB path.

5 At the command line, create the problem structure:

    problem = createOptimProblem('fmincon',...
        'objective', @(x)sawtoothxy(x(1),x(2)),...
        'x0',[100,-50],'options',...)
optimset('Algorithm','sqp'));

6 Create a GlobalSearch object with @StopAfterFive as the output function, and set the iterative display property to 'iter'.

    gs = GlobalSearch('OutputFcns',@StopAfterFive,'Display','iter');

7 (Optional) To get the same answer as this example, set the default random number stream.

    rng('default')

8 Run the problem.

    [x fval] = run(gs,problem)

<table>
<thead>
<tr>
<th>Num Pts</th>
<th>Best</th>
<th>Current</th>
<th>Threshold</th>
<th>Local</th>
<th>Local</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analyzed</td>
<td>F-count</td>
<td>f(x)</td>
<td>Penalty</td>
<td>Penalty</td>
<td>f(x)</td>
</tr>
<tr>
<td>0</td>
<td>465</td>
<td>422.9</td>
<td>2</td>
<td></td>
<td>422.9</td>
</tr>
<tr>
<td>200</td>
<td>1744</td>
<td>1.547e-015</td>
<td></td>
<td></td>
<td>1.547e-015</td>
</tr>
</tbody>
</table>

GlobalSearch stopped by the output or plot function.

All 2 local solver runs converged with a positive local solver exit flag.

    x =
    1.0e-007 *
    0.0414  0.1298

    fval =
    1.5467e-015

The run stopped early because GlobalSearch found a point with a function value less than 0.5.

**No Parallel Output Functions**

While MultiStart can run in parallel, it does not support global output functions and plot functions in parallel. Furthermore, while local output
functions and plot functions run on workers when MultiStart runs in parallel, the effect differs from running serially. Local output and plot functions do not create a display when running on workers. You do not see any other effects of output and plot functions until the worker passes its results to the client (the originator of the MultiStart parallel jobs).

For information on running MultiStart in parallel, see Chapter 8, “Parallel Processing”.

**Plot Functions for GlobalSearch and MultiStart**

- “What Are Plot Functions?” on page 3-38
- “Example: MultiStart Plot Function” on page 3-39
- “No Parallel Plot Functions” on page 3-42

**What Are Plot Functions?**
The `PlotFcns` field of the `options` structure specifies one or more functions that an optimization function calls at each iteration. Plot functions plot various measures of progress while the algorithm executes. Pass a function handle or cell array of function handles. The structure of a plot function is the same as the structure of an output function. For more information on this structure, see “OutputFcns” on page 9-3.

Plot functions are specialized output functions (see “Output Functions for GlobalSearch and MultiStart” on page 3-34). There are two predefined plot functions:

- `@gsplotbestf` plots the best objective function value.
- `@gsplotfuncount` plots the number of function evaluations.

Plot function windows have Pause and Stop buttons. By default, all plots appear in one window.

To use global plot functions:

- Write plot functions using the syntax described in “OutputFcns” on page 9-3.
• Set the PlotFcns property of your GlobalSearch or MultiStart object to the function handle of your plot function. You can use multiple plot functions by setting the PlotFcns property to a cell array of function handles.

**Details of Built-In Plot Functions.** The built-in plot functions have characteristics that can surprise you.

• `@gsplotbestf` can have plots that are not strictly decreasing. This is because early values can result from local solver runs with negative exit flags (such as infeasible solutions). A subsequent local solution with positive exit flag is better even if its function value is higher. Once a local solver returns a value with a positive exit flag, the plot is monotone decreasing.

• `@gsplotfunccount` might not plot the total number of function evaluations. This is because GlobalSearch can continue to perform function evaluations after it calls the plot function for the last time. For more information, see “GlobalSearch Algorithm” on page 3-45 Properties for GlobalSearch.

**Example: MultiStart Plot Function**

This example plots the number of local solver runs it takes to obtain a better local minimum for MultiStart. The example also uses a built-in plot function to show the current best function value.

The example problem is the same as in “Example: Finding Global or Multiple Local Minima” on page 3-70, with additional bounds.

The example uses persistent variables to store previous best values. The plot function examines the best function value after each local solver run, available in the bestfval field of the optimValues structure. If the value is not lower than the previous best, the plot function adds 1 to the number of consecutive calls with no improvement and draws a bar chart. If the value is lower than the previous best, the plot function starts a new bar in the chart with value 1. Before plotting, the plot function takes a logarithm of the number of consecutive calls. The logarithm helps keep the plot legible, since some values can be much larger than others.
To store local results using nested functions instead of persistent variables, see “Example of a Nested Output Function” in the MATLAB Mathematics documentation.

1. Write the objective function:

```matlab
function f = sawtoothxy(x,y)
    [t r] = cart2pol(x,y); % change to polar coordinates
    h = cos(2*t - 1/2)/2 + cos(t) + 2;
    g = (sin(r) - sin(2*r)/2 + sin(3*r)/3 - sin(4*r)/4 + 4) ... 
        .*r.^2./(r+1);
    f = g.*h;
```

2. Save `sawtoothxy.m` as a file in a folder on your MATLAB path.

3. Write the plot function:

```matlab
function stop = NumberToNextBest(optimValues, state)
    persistent bestfv bestcounter
    stop = false;
    switch state
        case 'init'
            % Initialize variable to record best function value.
            bestfv = [];
            % Initialize counter to record number of
            % local solver runs to find next best minimum.
            bestcounter = 1;
            % Create the histogram.
            bar(log(bestcounter),'tag','NumberToNextBest');
            xlabel('Number of New Best Fval Found');
            ylabel('Log Number of Local Solver Runs');
            title('Number of Local Solver Runs to Find Lower Minimum');
        case 'iter'
            % Find the axes containing the histogram.
            NumToNext = ...
                findobj(get(gca,'Children'),'Tag','NumberToNextBest');
```
% Update the counter that records number of local
% solver runs to find next best minimum.
if ~isequal(optimValues.bestfval, bestfv)
    bestfv = optimValues.bestfval;
    bestcounter = [bestcounter 1];
else
    bestcounter(end) = bestcounter(end) + 1;
end

% Update the histogram.
set(NumToNext,'Ydata',log(bestcounter))
end

4 Save NumberToNextBest.m as a file in a folder on your MATLAB path.

5 Create the problem structure and global solver. Set lower bounds of
[-3e3,-4e3], upper bounds of [4e3,3e3] and set the global solver to use
the plot functions:

    problem = createOptimProblem('fmincon',...
        'objective', @(x)sawtoothxy(x(1),x(2)),...
        'x0',[100,-50], 'lb',[-3e3 -4e3],...
        'ub',[4e3,3e3], 'options',...
        optimset('Algorithm','sqp'));

    ms = MultiStart('PlotFcns',{@NumberToNextBest,@gsplotbestf});

6 Run the global solver for 100 local solver runs:

    [x fv] = run(ms,problem,100);

7 The plot functions produce the following figure (your results can differ, since
the solution process is stochastic):
No Parallel Plot Functions

While MultiStart can run in parallel, it does not support global output functions and plot functions in parallel. Furthermore, while local output functions and plot functions run on workers when MultiStart runs in parallel, the effect differs from running serially. Local output and plot functions do not create a display when running on workers. You do not see any other effects of output and plot functions until the worker passes its results to the client (the originator of the MultiStart parallel jobs).

For information on running MultiStart in parallel, see Chapter 8, “Parallel Processing”.
How GlobalSearch and MultiStart Work

In this section...

| “Multiple Runs of a Local Solver” on page 3-43 |
| “Differences Between the Solver Objects” on page 3-43 |
| “GlobalSearch Algorithm” on page 3-45 |
| “MultiStart Algorithm” on page 3-49 |
| “Bibliography” on page 3-52 |

**Multiple Runs of a Local Solver**

GlobalSearch and MultiStart have similar approaches to finding global or multiple minima. Both algorithms start a local solver (such as `fmincon`) from multiple start points. The algorithms use multiple start points to sample multiple basins of attraction. For more information, see “Basins of Attraction” on page 1-13.

**Differences Between the Solver Objects**

GlobalSearch and MultiStart Algorithm Overview on page 3-44 contains a sketch of the GlobalSearch and MultiStart algorithms.
GlobalSearch Algorithm

- Run `fmincon` from `x0`
- Generate trial points (potential start points)
- Stage 1: Run best start point among the first `NumStageOnePoints` trial points
- Stage 2: Loop through remaining trial points, run `fmincon` if point satisfies basin, score, and constraint filters
- Create `GlobalOptimSolutions` vector

MultiStart Algorithm

- Generate start points
- Run start points
- Create `GlobalOptimSolutions` vector

GlobalSearch and MultiStart Algorithm Overview

The main differences between GlobalSearch and MultiStart are:

- **GlobalSearch** uses a scatter-search mechanism for generating start points. **MultiStart** uses uniformly distributed start points within bounds, or user-supplied start points.

- **GlobalSearch** analyzes start points and rejects those points that are unlikely to improve the best local minimum found so far. **MultiStart** runs all start points (or, optionally, all start points that are feasible with respect to bounds or inequality constraints).

- **MultiStart** gives a choice of local solver: `fmincon`, `fminunc`, `lsqcurvefit`, or `lsqnonlin`. The **GlobalSearch** algorithm uses `fmincon`.

- **MultiStart** can run in parallel, distributing start points to multiple processors for local solution. To run **MultiStart** in parallel, see “How to Use Parallel Processing” on page 8-11.
Deciding Which Solver to Use
The differences between these solver objects boil down to the following decision on which to use:

- Use GlobalSearch to find a single global minimum most efficiently on a single processor.
- Use MultiStart to:
  - Find multiple local minima.
  - Run in parallel.
  - Use a solver other than fmincon.
  - Search thoroughly for a global minimum.
  - Explore your own start points.

GlobalSearch Algorithm
For a description of the algorithm, see Ugray et al. [1].

When you run a GlobalSearch object, the algorithm performs the following steps:

- “Run fmincon from x0” on page 3-46
- “Generate Trial Points” on page 3-46
- “Obtain Stage 1 Start Point, Run” on page 3-46
- “Initialize Basins, Counters, Threshold” on page 3-46
- “Begin Main Loop” on page 3-47
- “Examine Stage 2 Trial Point to See if fmincon Runs” on page 3-47
- “When fmincon Runs:” on page 3-47
- “When fmincon Does Not Run:” on page 3-49
- “Create GlobalOptimSolution” on page 3-49
Run fmincon from x0

GlobalSearch runs fmincon from the start point you give in the problem structure. If this run converges, GlobalSearch records the start point and end point for an initial estimate on the radius of a basin of attraction. Furthermore, GlobalSearch records the final objective function value for use in the score function (see “Obtain Stage 1 Start Point, Run” on page 3-46).

The score function is the sum of the objective function value at a point and a multiple of the sum of the constraint violations. So a feasible point has score equal to its objective function value. The multiple for constraint violations is initially 1000. GlobalSearch updates the multiple during the run.

Generate Trial Points

GlobalSearch uses the scatter search algorithm to generate a set of NumTrialPoints trial points. Trial points are potential start points. For a description of the scatter search algorithm, see Glover [2]. The trial points have components in the range (–1e4+1,1e4+1). This range is not symmetric about the origin so that the origin is not in the scatter search.

Obtain Stage 1 Start Point, Run

GlobalSearch evaluates the score function of a set of NumStageOnePoints trial points. It then takes the point with the best score and runs fmincon from that point. GlobalSearch removes the set of NumStageOnePoints trial points from its list of points to examine.

Initialize Basins, Counters, Threshold

The localSolverThreshold is initially the smaller of the two objective function values at the solution points. The solution points are the fmincon solutions starting from x0 and from the Stage 1 start point. If both of these solution points do not exist or are infeasible, localSolverThreshold is initially the penalty function value of the Stage 1 start point.

The GlobalSearch heuristic assumption is that basins of attraction are spherical. The initial estimate of basins of attraction for the solution point from x0 and the solution point from Stage 1 are spheres centered at the solution points. The radius of each sphere is the distance from the initial point to the solution point. These estimated basins can overlap.
There are two sets of counters associated with the algorithm. Each counter is the number of consecutive trial points that:

- Lie within a basin of attraction. There is one counter for each basin.
- Have score function greater than localSolverThreshold. For a definition of the score, see “Run fmincon from x0” on page 3-46.

All counters are initially 0.

**Begin Main Loop**
GlobalSearch repeatedly examines a remaining trial point from the list, and performs the following steps. It continually monitors the time, and stops the search if elapsed time exceeds MaxTime seconds.

**Examine Stage 2 Trial Point to See if fmincon Runs**
Call the trial point p. Run fmincon from p if the following conditions hold:

- p is not in any existing basin. The criterion for every basin i is:
  
  $$|p - \text{center}(i)| > \text{DistanceThresholdFactor} \times \text{radius}(i).$$

  DistanceThresholdFactor is an option (default value 0.75).

  radius is an estimated radius that updates in Update Basin Radius and Threshold and React to Large Counter Values.

- score(p) < localSolverThreshold.
- (optional) p satisfies bound and/or inequality constraints. This test occurs if you set the StartPointsToRun property of the GlobalSearch object to 'bounds' or 'bounds-ineqs'.

**When fmincon Runs:**

1 **Reset Counters**
   Set the counters for basins and threshold to 0.

2 **Update Solution Set**
If \texttt{fmincon} runs starting from \( p \), it can yield a positive exit flag, which indicates convergence. In that case, \texttt{GlobalSearch} updates the vector of \texttt{GlobalOptimSolution} objects. Call the solution point \( xp \) and the objective function value \( fp \). There are two cases:

- For every other solution point \( xq \) with objective function value \( fq \),
  \[
  |xq - xp| > \text{TolX} \times \max(1,|xp|)
  \]
  or
  \[
  |fq - fp| > \text{TolFun} \times \max(1,|fp|).
  \]
  In this case, \texttt{GlobalSearch} creates a new element in the vector of \texttt{GlobalOptimSolution} objects. For details of the information contained in each object, see \texttt{GlobalOptimSolution}.

- For some other solution point \( xq \) with objective function value \( fq \),
  \[
  |xq - xp| \leq \text{TolX} \times \max(1,|xp|)
  \]
  and
  \[
  |fq - fp| \leq \text{TolFun} \times \max(1,|fp|).
  \]
  In this case, \texttt{GlobalSearch} regards \( xp \) as equivalent to \( xq \). The \texttt{GlobalSearch} algorithm modifies the \texttt{GlobalOptimSolution} of \( xq \) by adding \( p \) to the cell array of \( X0 \) points.

There is one minor tweak that can happen to this update. If the exit flag for \( xq \) is greater than 1, and the exit flag for \( xp \) is 1, then \( xp \) replaces \( xq \). This replacement can lead to some points in the same basin being more than a distance of \text{TolX} from \( xp \).

3 Update Basin Radius and Threshold

If the exit flag of the current \texttt{fmincon} run is positive:

- Set threshold to the score value at start point \( p \).
- Set basin radius for \( xp \) equal to the maximum of the existing radius (if any) and the distance between \( p \) and \( xp \).

4 Report to Iterative Display
When the GlobalSearch Display property is 'iter', every point that fmincon runs creates one line in the GlobalSearch iterative display.

**When fmincon Does Not Run:**

1 **Update Counters**
   Increment the counter for every basin containing p. Reset the counter of every other basin to 0.

   Increment the threshold counter if score(p) >= localSolverThreshold. Otherwise, reset the counter to 0.

2 **React to Large Counter Values**
   For each basin with counter equal to MaxWaitCycle, multiply the basin radius by 1 – BasinRadiusFactor. Reset the counter to 0. (Both MaxWaitCycle and BasinRadiusFactor are settable properties of the GlobalSearch object.)

   If the threshold counter equals MaxWaitCycle, increase the threshold:

   \[
   \text{new threshold} = \text{threshold} + \text{PenaltyThresholdFactor} \times (1 + \text{abs(threshold)}). \]

   Reset the counter to 0.

3 **Report to Iterative Display**
   Every 200th trial point creates one line in the GlobalSearch iterative display.

**Create GlobalOptimSolution**
After reaching MaxTime seconds or running out of trial points, GlobalSearch creates a vector of GlobalOptimSolution objects. GlobalSearch orders the vector by objective function value, from lowest (best) to highest (worst). This concludes the algorithm.

**MultiStart Algorithm**
When you run a MultiStart object, the algorithm performs the following steps:
Generate Start Points
If you call MultiStart with the syntax

\[ [x \ fval] = \text{run}(ms,\text{problem},k) \]

for an integer \( k \), MultiStart generates \( k - 1 \) start points exactly as if you used a RandomStartPointSet object. The algorithm also uses the \( x_0 \) start point from the \text{problem} structure, for a total of \( k \) start points.

A RandomStartPointSet object does not have any points stored inside the object. Instead, MultiStart calls the \text{list} method, which generates random points within the bounds given by the \text{problem} structure. If an unbounded component exists, \text{list} uses an artificial bound given by the ArtificialBound property of the RandomStartPointSet object.

If you provide a CustomStartPointSet object, MultiStart does not generate start points, but uses the points in the object.

Filter Start Points (Optional)
If you set the \text{StartPointsToRun} property of the MultiStart object to 'bounds' or 'bounds-ineqs', MultiStart does not run the local solver from infeasible start points. In this context, “infeasible” means start points that do not satisfy bounds, or start points that do not satisfy both bounds and inequality constraints.

The default setting of \text{StartPointsToRun} is 'all'. In this case, MultiStart does not discard infeasible start points.
Run Local Solver
MultiStart runs the local solver specified in problem.solver, starting at the points that pass the StartPointsToRun filter. If MultiStart is running in parallel, it sends start points to worker processors one at a time, and the worker processors run the local solver.

The local solver checks whether MaxTime seconds have elapsed at each of its iterations. If so, it exits that iteration without reporting a solution.

When the local solver stops, MultiStart stores the results and continues to the next step.

Report to Iterative Display. When the MultiStart Display property is 'iter', every point that the local solver runs creates one line in the MultiStart iterative display.

Check Stopping Conditions
MultiStart stops when it runs out of start points. It also stops when it exceeds a total run time of MaxTime seconds.

Create GlobalOptimSolution Object
After MultiStart reaches a stopping condition, the algorithm creates a vector of GlobalOptimSolution objects as follows:

1. Sort the local solutions by objective function value (Fval) from lowest to highest. For the lsqnonlin and lsqcurvefit local solvers, the objective function is the norm of the residual.

2. Loop over the local solutions j beginning with the lowest (best) Fval.

3. Find all the solutions k satisfying both:

   \[ |Fval(k) - Fval(j)| \leq TolFun*\max(1,|Fval(j)|) \]

   \[ |x(k) - x(j)| \leq TolX*\max(1,|x(j)|) \]

4. Record j, Fval(j), the local solver output structure for j, and a cell array of the start points for j and all the k. Remove those points k
from the list of local solutions. This point is one entry in the vector of `GlobalOptimSolution` objects.

The resulting vector of `GlobalOptimSolution` objects is in order by `Fval`, from lowest (best) to highest (worst).

**Report to Iterative Display.** After examining all the local solutions, `MultiStart` gives a summary to the iterative display. This summary includes the number of local solver runs that converged, the number that failed to converge, and the number that had errors.

**Bibliography**


Improving Results

In this section...

“Can You Certify a Solution Is Global?” on page 3-53
“Refining the Start Points” on page 3-56
“Changing Options” on page 3-63
“Reproducing Results” on page 3-67

Can You Certify a Solution Is Global?

How can you tell if you have located the global minimum of your objective function? The short answer is that you cannot; you have no guarantee that the result of a Global Optimization Toolbox solver is a global optimum.

However, you can use the following strategies for investigating solutions:

- “Check if a Solution Is a Local Solution with patternsearch” on page 3-53
- “Identify a Bounded Region That Contains a Global Solution” on page 3-54
- “Use MultiStart with More Start Points” on page 3-55

Check if a Solution Is a Local Solution with patternsearch

Before you can determine if a purported solution is a global minimum, first check that it is a local minimum. To do so, run patternsearch on the problem.

To convert the problem to use patternsearch instead of fmincon or fminunc, enter

\[
\text{problem.solver} = '\text{patternsearch}';
\]

Also, change the start point to the solution you just found:

\[
\text{problem.x0} = x;
\]

For example, Check Nearby Points (in the Optimization Toolbox documentation) shows the following:

\[
\text{options} = \text{optimset}('\text{Algorithm}', '\text{active-set}');
\]
Using GlobalSearch and MultiStart

ffun = @(x)(x(1)-(x(1)-x(2))^2);
problem = createOptimProblem('fmincon', ...
    'objective',ffun,'x0',[1/2 1/3], ... 
    'lb',[0 -1],'ub',[1 1],'options',options);
[x fval exitflag] = fmincon(problem)

x =
    1.0e-007 * 
    0    0.1614

fval =
    -2.6059e-016

exitflag =
    1

However, checking this purported solution with patternsearch shows that there is a better solution. Start patternsearch from the reported solution x:

% set the candidate solution x as the start point
problem.x0 = x;
problem.solver = 'patternsearch';
[xp fvalp exitflagp] = patternsearch(problem)

xp =
    1.0000   -1.0000

fvalp =
    -3.0000

exitflagp =
    1

Identify a Bounded Region That Contains a Global Solution

Suppose you have a smooth objective function in a bounded region. Given enough time and start points, MultiStart eventually locates a global solution.

Therefore, if you can bound the region where a global solution can exist, you can obtain some degree of assurance that MultiStart locates the global solution.
For example, consider the function

\[ f = x^6 + y^6 + \sin(x + y)(x^2 + y^2) - \cos\left(\frac{x^2}{1+y^2}\right)(2 + x^4 + x^2y^2 + y^4). \]

The initial summands \( x^6 + y^6 \) force the function to become large and positive for large values of \( |x| \) or \( |y| \). The components of the global minimum of the function must be within the bounds

\[ -10 \leq x, y \leq 10, \]

since \( 10^6 \) is much larger than all the multiples of \( 10^4 \) that occur in the other summands of the function.

You can identify smaller bounds for this problem; for example, the global minimum is between \(-2\) and \(2\). It is more important to identify reasonable bounds than it is to identify the best bounds.

**Use MultiStart with More Start Points**

To check whether there is a better solution to your problem, run MultiStart with additional start points. Use MultiStart instead of GlobalSearch for this task because GlobalSearch does not run the local solver from all start points.

For example, see “Example: Searching for a Better Solution” on page 3-60.

**Updating Unconstrained Problem from GlobalSearch.** If you use GlobalSearch on an unconstrained problem, change your problem structure before using MultiStart. You have two choices in updating a problem structure for an unconstrained problem using MultiStart:

- Change the solver field to ‘fminunc’:
  
  ```matlab
  problem.solver = 'fminunc';
  ```

  To avoid a warning if your objective function does not compute a gradient, change the local options structure to have LargeScale set to 'off':

  ```matlab
  problem.options.LargeScale = 'off';
  ```
• Add an artificial constraint, retaining `fmincon` as the local solver:

    `problem.lb = -Inf;`

To search a larger region than the default, see “Refining the Start Points” on page 3-56.

**Refining the Start Points**

If some components of your problem are unconstrained, `GlobalSearch` and `MultiStart` use artificial bounds to generate random start points uniformly in each component. However, if your problem has far-flung minima, you need widely dispersed start points to find these minima.

Use these methods to obtain widely dispersed start points:

• Give widely separated bounds in your `problem` structure.
• Use a `RandomStartPointSet` object with the `MultiStart` algorithm. Set a large value of the `ArtificialBound` property in the `RandomStartPointSet` object.
• Use a `CustomStartPointSet` object with the `MultiStart` algorithm. Use widely dispersed start points.

There are advantages and disadvantages of each method.

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Give bounds in <code>problem</code></td>
<td>Automatic point generation</td>
<td>Makes a more complex Hessian</td>
</tr>
<tr>
<td></td>
<td>Can use with <code>GlobalSearch</code></td>
<td>Unclear how large to set the bounds</td>
</tr>
<tr>
<td></td>
<td>Easy to do</td>
<td>Changes <code>problem</code></td>
</tr>
<tr>
<td></td>
<td>Bounds can be asymmetric</td>
<td>Only uniform points</td>
</tr>
<tr>
<td>Large <code>ArtificialBound</code> in</td>
<td>Automatic point generation</td>
<td><code>MultiStart</code> only</td>
</tr>
<tr>
<td><code>RandomStartPointSet</code></td>
<td>Does not change <code>problem</code></td>
<td>Only symmetric, uniform points</td>
</tr>
<tr>
<td></td>
<td>Easy to do</td>
<td>Unclear how large to set <code>ArtificialBound</code></td>
</tr>
</tbody>
</table>
### Methods of Generating Start Points

- “Uniform Grid” on page 3-57
- “Perturbed Grid” on page 3-58
- “Widely Dispersed Points for Unconstrained Components” on page 3-58

**Uniform Grid.** To generate a uniform grid of start points:

1. Generate multidimensional arrays with `ndgrid`. Give the lower bound, spacing, and upper bound for each component.

   For example, to generate a set of three-dimensional arrays with

   - First component from –2 through 0, spacing 0.5
   - Second component from 0 through 2, spacing 0.25
   - Third component from –10 through 5, spacing 1

   
   \[
   [X,Y,Z] = \text{ndgrid}(-2:.5:0,0:.25:2,-10:5);
   \]

2. Place the arrays into a single matrix, with each row representing one start point. For example:

   \[
   W = [X(:),Y(:),Z(:)];
   \]

   In this example, \( W \) is a 720-by-3 matrix.

3. Put the matrix into a `CustomStartPointSet` object. For example:

   \[
   \text{custpts} = \text{CustomStartPointSet}(W);
   \]

   Call `MultiStart` run with the `CustomStartPointSet` object as the third input. For example,
% Assume problem structure and ms MultiStart object exist
[x fval flag outpt manymins] = run(ms,problem,custpts);

**Perturbed Grid.** Integer start points can yield less robust solutions than slightly perturbed start points.

To obtain a perturbed set of start points:

1. Generate a matrix of start points as in steps 1–2 of “Uniform Grid” on page 3-57.
2. Perturb the start points by adding a random normal matrix with 0 mean and relatively small variance.

For the example in “Uniform Grid” on page 3-57, after making the W matrix, add a perturbation:

\[
\begin{align*}
[X,Y,Z] &= \text{ndgrid}(-2:.5:0,0:.25:2,-10:5); \\
W &= [X(:,),Y(:,),Z(,:)]; \\
W &= W + 0.01*\text{randn}(:,); \\
\end{align*}
\]

3. Put the matrix into a CustomStartPointSet object. For example:

\[
\text{custpts} = \text{CustomStartPointSet}(W);
\]

Call MultiStart run with the CustomStartPointSet object as the third input. For example,

% Assume problem structure and ms MultiStart object exist
[x fval flag outpt manymins] = run(ms,problem,custpts);

**Widely Dispersed Points for Unconstrained Components.** Some components of your problem can lack upper or lower bounds. For example:

- Although no explicit bounds exist, there are levels that the components cannot attain. For example, if one component represents the weight of a single diamond, there is an implicit upper bound of 1 kg (the Hope Diamond is under 10 g). In such a case, give the implicit bound as an upper bound.
- There truly is no upper bound. For example, the size of a computer file in bytes has no effective upper bound. The largest size can be in gigabytes or terabytes today, but in 10 years, who knows?
For truly unbounded components, you can use the following methods of sampling. To generate approximately $1/n$ points in each region $(\exp(n), \exp(n+1))$, use the following formula. If $u$ is random and uniformly distributed from 0 through 1, then $r = 2u - 1$ is uniformly distributed between $-1$ and 1. Take

$$y = \text{sgn}(r)(\exp(1/|r|) - e).$$

$y$ is symmetric and random. For a variable bounded below by $lb$, take

$$y = lb + (\exp(1/u) - e).$$

Similarly, for a variable bounded above by $ub$, take

$$y = ub - (\exp(1/u) - e).$$

For example, suppose you have a three-dimensional problem with

- $x(1) > 0$
- $x(2) < 100$
- $x(3)$ unconstrained

To make 150 start points satisfying these constraints:

```matlab
u = rand(150, 3);
r1 = 1./u(:,1);
r1 = exp(r1) - exp(1);
r2 = 1./u(:,2);
r2 = -exp(r2) + exp(1) + 100;
r3 = 1./(2*u(:,3)-1);
r3 = sign(r3).*(exp(abs(r3)) - exp(1));
custpts = CustomStartPointSet([r1,r2,r3]);
```

The following is a variant of this algorithm. Generate a number between 0 and infinity by the method for lower bounds. Use this number as the radius of a point. Generate the other components of the point by taking random numbers for each component and multiply by the radius. You can normalize the random numbers, before multiplying by the radius, so their norm is 1. For
Using GlobalSearch and MultiStart

a worked example of this method, see “MultiStart Without Bounds, Widely Dispersed Start Points” on page 3-88.

Example: Searching for a Better Solution

MultiStart fails to find the global minimum in “Multiple Local Minima Via MultiStart” on page 3-74. There are two simple ways to search for a better solution:

• Use more start points
• Give tighter bounds on the search space

Set up the problem structure and MultiStart object:

```matlab
problem = createOptimProblem('fminunc',
    'objective',@(x)sawtoothxy(x(1),x(2)),
    'x0',[100,-50],
    'options',
    optimset('LargeScale','off'));
ms = MultiStart;
```

**Use More Start Points.** Run MultiStart on the problem for 200 start points instead of 50:

```matlab
[x fval eflag output manymins] = run(ms,problem,200)
```

MultiStart completed some of the runs from the start points.

53 out of 200 local solver runs converged with a positive local solver exit flag.

```matlab
x =
    1.0e-005 *
     0.7460   -0.2550
fval =
    7.8236e-010
eflag =
    2
output =
```
funcCount: 32841
localSolverTotal: 200
localSolverSuccess: 53
localSolverIncomplete: 147
localSolverNoSolution: 0
message: [1x143 char]

manymins =
1x53 GlobalOptimSolution

Properties:
  X
  Fval
  Exitflag
  Output
  X0

This time MultiStart found the global minimum, and found 53 total local minima.

To see the range of local solutions, enter hist([manymins.Fval]).
**Tighter Bound on the Start Points.** Suppose you believe that the interesting local solutions have absolute values of all components less than 100. The default value of the bound on start points is 1000. To use a different value of the bound, generate a `RandomStartPointSet` with the `ArtificialBound` property set to 100:

```matlab
startpts = RandomStartPointSet('ArtificialBound',100,...
    'NumStartPoints',50);
[x fval eflag output manymins] = run(ms,problem,startpts)
```

MultiStart completed some of the runs from the start points.

29 out of 50 local solver runs converged with a positive local solver exit flag.

```matlab
x =
    1.0e-007 *
    0.1771  0.3198
fval =
    1.3125e-014
eflag =
    2
output =
    funcCount: 7557
    localSolverTotal: 50
    localSolverSuccess: 29
    localSolverIncomplete: 21
    localSolverNoSolution: 0
    message: [1x142 char]
manymins =
    1x25 GlobalOptimSolution

Properties:
    X
    Fval
    Exitflag
MultiStart found the global minimum, and found 25 distinct local solutions. To see the range of local solutions, enter `hist([manymins.Fval])`.

Compared to the minima found in “Use More Start Points” on page 3-60, this run found better (smaller) minima, and had a higher percentage of successful runs.

**Changing Options**

**How to Determine Which Options to Change**

After you run a global solver, you might want to change some global or local options. To determine which options to change, the guiding principle is:

- To affect the local solver, set local solver options.
- To affect the start points or solution set, change the problem structure, or set the global solver object properties.

For example, to obtain:

- More local minima — Set global solver object properties.
- Faster local solver iterations — Set local solver options.
- Different tolerances for considering local solutions identical (to obtain more or fewer local solutions) — Set global solver object properties.
• Different information displayed at the command line — Decide if you want iterative display from the local solver (set local solver options) or global information (set global solver object properties).

• Different bounds, to examine different regions — Set the bounds in the problem structure.

**Examples of Choosing Problem Options.**

• To start your local solver at points only satisfying inequality constraints, set the StartPointsToRun property in the global solver object to 'bounds-ineqs'. This setting can speed your solution, since local solvers do not have to attempt to find points satisfying these constraints. However, the setting can result in many fewer local solver runs, since the global solver can reject many start points. For an example, see “Example: Using Only Feasible Start Points” on page 3-77.

• To use the fmincon interior-point algorithm, set the local solver Algorithm option to 'interior-point'. For an example showing how to do this, see “Examples of Updating Problem Options” on page 3-65.

• For your local solver to have different bounds, set the bounds in the problem structure. Examine different regions by setting bounds.

• To see every solution that has positive local exit flag, set the TolX property in the global solver object to 0. For an example showing how to do this, see “Changing Global Options” on page 3-65.

**Changing Local Solver Options**

There are several ways to change values in a local options structure:

• Update the values using dot addressing and optimset. The syntax is

\[
\text{problem.options} = \text{optimset}(
\text{problem.options}, 'Parameter', \text{value}, ...);
\]

You can also replace the local options entirely:

\[
\text{problem.options} = \text{optimset}( 'Parameter', \text{value}, ...);
\]

• Use dot addressing on one local option. The syntax is
Improving Results

problem.options.Parameter = newvalue;

• Recreate the entire problem structure. For details, see “Create a Problem Structure” on page 3-4.

Examples of Updating Problem Options.

1 Create a problem structure:

   problem = createOptimProblem('fmincon','x0',[-1 2], ...
   'objective',@rosenboth);

2 Set the problem to use the sqp algorithm in fmincon:

   problem.options.Algorithm = 'sqp';

3 Update the problem to use the gradient in the objective function, have a
   TolFun value of 1e-8, and a TolX value of 1e-7:

   problem.options = optimset(problem.options,'GradObj','on', ... 
   'TolFun',1e-8,'TolX',1e-7);

Changing Global Options

There are several ways to change characteristics of a GlobalSearch or
MultiStart object:

• Use dot addressing. For example, suppose you have a default MultiStart
   object:

   ms = MultiStart

   Properties:
   UseParallel: 'never'
   StartPointsToRun: 'all'
   Display: 'final'
   TolFun: 1.0000e-006
   TolX: 1.0000e-006
   MaxTime: Inf

   To change ms to have its TolX value equal to 1e-3, update the TolX field:
ms.TolX = 1e-3

Properties:
    UseParallel: 'never'
    StartPointsToRun: 'all'
        Display: 'final'
            TolFun: 1.0000e-006
            TolX: 1.0000e-003
            MaxTime: Inf

- Reconstruct the object starting from the current settings. For example, to set the TolFun field in ms to 1e-3, retaining the nondefault value for TolX:

ms = MultiStart(ms,'TolFun',1e-3)

Properties:
    UseParallel: 'never'
    StartPointsToRun: 'all'
        Display: 'final'
            TolFun: 1.0000e-003
            TolX: 1.0000e-003
            MaxTime: Inf

- Convert a GlobalSearch object to a MultiStart object, or vice-versa. For example, with the ms object from the previous example, create a GlobalSearch object with the same values of TolX and TolFun:

gs = GlobalSearch(ms)

Properties:
            NumTrialPoints: 1000
            BasinRadiusFactor: 0.2000
            DistanceThresholdFactor: 0.7500
            MaxWaitCycle: 20
            NumStageOnePoints: 200
            PenaltyThresholdFactor: 0.2000
            StartPointsToRun: 'all'
                Display: 'final'
                    TolFun: 1.0000e-003 % from ms
                    TolX: 1.0000e-003 % from ms
                    MaxTime: Inf
Reproducing Results

GlobalSearch and MultiStart use pseudorandom numbers in choosing start points. Use the same pseudorandom number stream again to:

- Compare various algorithm settings.
- Have an example run repeatably.
- Extend a run, with known initial segment of a previous run.

Both GlobalSearch and MultiStart use the default random number stream.

Steps to Take in Reproducing Results

1 Before running your problem, store the current state of the default random number stream:

   stream = rng;

2 Run your GlobalSearch or MultiStart problem.

3 Restore the state of the random number stream:

   rng(stream)

4 If you run your problem again, you get the same result.

Example: Reproducing a GlobalSearch or MultiStart Result

This example shows how to obtain reproducible results for “Example: Finding Global or Multiple Local Minima” on page 3-70. The example follows the procedure in “Steps to Take in Reproducing Results” on page 3-67.

1 Store the current state of the default random number stream:

   stream = rng;

2 Create the sawtoothxy function file:

   function f = sawtoothxy(x,y)
   [t r] = cart2pol(x,y); % change to polar coordinates
   h = cos(2*t - 1/2)/2 + cos(t) + 2;
```matlab
g = (sin(r) - sin(2*r)/2 + sin(3*r)/3 - sin(4*r)/4 + 4) .* r.^2./(r+1);
f = g.*h;
end

3 Create the problem structure and GlobalSearch object:

```matlab
problem = createOptimProblem('fmincon', ...
    'objective', @(x)sawtoothxy(x(1),x(2)), ...
    'x0',[100,-50], 'options', ...
    optimset('Algorithm','sqp'));
gs = GlobalSearch('Display','iter');
```

4 Run the problem:

```matlab
[x fval] = run(gs,problem)
```

<table>
<thead>
<tr>
<th>Num Pts</th>
<th>Best F-count</th>
<th>f(x)</th>
<th>Penalty</th>
<th>Threshold</th>
<th>Local f(x)</th>
<th>Local exitflag</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>465</td>
<td>422.9</td>
<td></td>
<td></td>
<td>422.9</td>
<td>2</td>
<td>Initial Point</td>
</tr>
<tr>
<td>200</td>
<td>1730</td>
<td>1.547e-015</td>
<td></td>
<td></td>
<td>1.547e-015</td>
<td>1</td>
<td>Stage 1 Local</td>
</tr>
<tr>
<td>300</td>
<td>1830</td>
<td>1.547e-015</td>
<td>6.01e+004</td>
<td>1.074</td>
<td>1.547e-015</td>
<td>1</td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>400</td>
<td>1930</td>
<td>1.547e-015</td>
<td>1.47e+005</td>
<td>4.16</td>
<td>1.547e-015</td>
<td>1</td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>500</td>
<td>2030</td>
<td>1.547e-015</td>
<td>2.63e+004</td>
<td>11.84</td>
<td>2.63e+004</td>
<td>1</td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>600</td>
<td>2130</td>
<td>1.547e-015</td>
<td>1.341e+004</td>
<td>30.95</td>
<td>1.341e+004</td>
<td>1</td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>700</td>
<td>2230</td>
<td>1.547e-015</td>
<td>2.562e+004</td>
<td>65.25</td>
<td>2.562e+004</td>
<td>1</td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>800</td>
<td>2330</td>
<td>1.547e-015</td>
<td>5.217e+004</td>
<td>163.8</td>
<td>5.217e+004</td>
<td>1</td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>900</td>
<td>2430</td>
<td>1.547e-015</td>
<td>7.704e+004</td>
<td>409.2</td>
<td>7.704e+004</td>
<td>1</td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>981</td>
<td>2587</td>
<td>1.547e-015</td>
<td>42.24</td>
<td>516.6</td>
<td>42.24</td>
<td>7.573</td>
<td>1 Stage 2 Local</td>
</tr>
<tr>
<td>1000</td>
<td>2606</td>
<td>1.547e-015</td>
<td>3.299e+004</td>
<td>42.24</td>
<td>3.299e+004</td>
<td>1</td>
<td>Stage 2 Search</td>
</tr>
</tbody>
</table>

GlobalSearch stopped because it analyzed all the trial points.

All 3 local solver runs converged with a positive local solver exit flag.

```matlab
x =
    1.0e-007 *
    0.0414    0.1298

fval =
```

3-68
You might obtain a different result when running this problem, since the random stream was in an unknown state at the beginning of the run.

5 Restore the state of the random number stream:

```matlab
gn(stream)
```

6 Run the problem again. You get the same output.

```matlab
[x fval] = run(gs,problem)
```

<table>
<thead>
<tr>
<th>Num Pts</th>
<th>F-count</th>
<th>Best f(x)</th>
<th>Current f(x)</th>
<th>Threshold f(x)</th>
<th>Local f(x)</th>
<th>Local exitflag</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analyzed</td>
<td>0</td>
<td>465</td>
<td>422.9</td>
<td></td>
<td></td>
<td></td>
<td>Initial Point</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>1730</td>
<td>1.547e-015</td>
<td>422.9</td>
<td>2</td>
<td></td>
<td>Stage 1 Local</td>
</tr>
</tbody>
</table>

... Output deleted to save space ...

```
x =
   1.0e-007 *
   0.0414   0.1298
fval =
1.5467e-015
```

**Parallel Processing and Random Number Streams**

You obtain reproducible results from MultiStart when you run the algorithm in parallel the same way as you do for serial computation. Runs are reproducible because MultiStart generates pseudorandom start points locally, and then distributes the start points to parallel processors. Therefore, the parallel processors do not use random numbers.

To reproduce a parallel MultiStart run, use the procedure described in “Steps to Take in Reproducing Results” on page 3-67. For a description of how to run MultiStart in parallel, see “How to Use Parallel Processing” on page 8-11.
GlobalSearch and MultiStart Examples

In this section...

“Example: Finding Global or Multiple Local Minima” on page 3-70
“Example: Using Only Feasible Start Points” on page 3-77
“Example: Parallel MultiStart” on page 3-81
“Example: Isolated Global Minimum” on page 3-84

Example: Finding Global or Multiple Local Minima
This example illustrates how GlobalSearch finds a global minimum efficiently, and how MultiStart finds many more local minima.

The objective function for this example has many local minima and a unique global minimum. In polar coordinates, the function is

\[ f(r,t) = g(r)h(t), \]

where

\[ g(r) = \left( \sin(r) - \frac{\sin(2r)}{2} + \frac{\sin(3r)}{3} - \frac{\sin(4r)}{4} + 4 \right) \frac{r^2}{r + 1}, \]

\[ h(t) = 2 + \cos(t) + \frac{\cos\left(2t - \frac{1}{2}\right)}{2}. \]
The global minimum is at $r = 0$, with objective function 0. The function $g(r)$ grows approximately linearly in $r$, with a repeating sawtooth shape. The function $h(t)$ has two local minima, one of which is global.
Single Global Minimum Via GlobalSearch

1 Write a function file to compute the objective:

```matlab
function f = sawtoothxy(x,y)
    [t r] = cart2pol(x,y); % change to polar coordinates
    h = cos(2*t - 1/2)/2 + cos(t) + 2;
    g = (sin(r) - sin(2*r)/2 + sin(3*r)/3 - sin(4*r)/4 + 4) .* r.^2./(r+1);
    f = g.*h;
end
```
Create the problem structure. Use the 'sqp' algorithm for \texttt{fmincon}:

```matlab
problem = createOptimProblem('fmincon',... 'objective', @(x)sawtoothxy(x(1),x(2)),... 'x0',[100,-50], 'options',... optimset('Algorithm','sqp'));
```

The start point is \([100,-50]\) instead of \([0,0]\), so GlobalSearch does not start at the global solution.

Add an artificial bound, and validate the problem structure by running \texttt{fmincon}:

```matlab
problem.lb = -Inf;
[x fval] = fmincon(problem)
```

\[
x =
45.6965 \quad 45.6965 \quad
\]

\[
fval =
555.6941
\]

Create the \texttt{GlobalSearch} object, and set iterative display:

```matlab
gs = GlobalSearch('Display','iter');
```

Run the solver:

```matlab
[x fval] = run(gs,problem)
```

<table>
<thead>
<tr>
<th>Num Pts</th>
<th>F-count</th>
<th>Best f(x)</th>
<th>Current f(x)</th>
<th>Threshold Penalty</th>
<th>Local f(x)</th>
<th>Local exitflag</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>465</td>
<td>422.9</td>
<td></td>
<td></td>
<td>422.9</td>
<td>2</td>
<td>Initial Point</td>
</tr>
<tr>
<td>200</td>
<td>1730</td>
<td>1.547e-015</td>
<td></td>
<td></td>
<td>1.547e-015</td>
<td>1</td>
<td>Stage 1 Local</td>
</tr>
<tr>
<td>300</td>
<td>1830</td>
<td>1.547e-015</td>
<td>5.709e+004</td>
<td>1.074</td>
<td></td>
<td></td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>400</td>
<td>1930</td>
<td>1.547e-015</td>
<td>1.646e+005</td>
<td>4.16</td>
<td></td>
<td></td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>500</td>
<td>2030</td>
<td>1.547e-015</td>
<td>2.262e+004</td>
<td>11.84</td>
<td></td>
<td></td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>600</td>
<td>2130</td>
<td>1.547e-015</td>
<td>1680</td>
<td>30.95</td>
<td></td>
<td></td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>700</td>
<td>2230</td>
<td>1.547e-015</td>
<td>1.138e+004</td>
<td>65.25</td>
<td></td>
<td></td>
<td>Stage 2 Search</td>
</tr>
<tr>
<td>800</td>
<td>2330</td>
<td>1.547e-015</td>
<td>1.573e+005</td>
<td>163.8</td>
<td></td>
<td></td>
<td>Stage 2 Search</td>
</tr>
</tbody>
</table>
Using GlobalSearch and MultiStart

GlobalSearch stopped because it analyzed all the trial points.

All 3 local solver runs converged with a positive local solver exit flag.

\[ x = \begin{bmatrix} 1.0e-007 \times \\ 0.0414 \\ 0.1298 \end{bmatrix} \]

\[ fval = 1.5467e-015 \]

You can get different results, since GlobalSearch is stochastic.

The solver found three local minima, and it found the global minimum near [0,0].

**Multiple Local Minima Via MultiStart**

1. Write a function file to compute the objective:

   ```matlab
   function f = sawtoothxy(x,y)
   [t r] = cart2pol(x,y); % change to polar coordinates
   h = cos(2*t - 1/2)/2 + cos(t) + 2;
   g = (sin(r) - sin(2*r)/2 + sin(3*r)/3 - sin(4*r)/4 + 4) ... 
       .*r.^2./(r+1);
   f = g.*h;
   end
   ```

2. Create the problem structure. Use the fminunc algorithm with the LargeScale option set to 'off'. The reasons for these choices are:

   - The problem is unconstrained. Therefore, fminunc is the appropriate solver; see “Optimization Decision Table” in the Optimization Toolbox documentation.
3 Validate the problem structure by running it:

```matlab
[x fval] = fminunc(problem)
```

```matlab
x =
   8.4420  -110.2602

fval =
   435.2573
```

4 Create a default MultiStart object:

```matlab
ms = MultiStart;
```

5 Run the solver for 50 iterations, recording the local minima:

```matlab
[x fval eflag output manymins] = run(ms,problem,50)
```

MultiStart completed some of the runs from the start points.

16 out of 50 local solver runs converged with a positive local solver exit flag.

```matlab
x =
   -379.3434 559.6154

fval =
   1.7590e+003

eflag =
   2
```
output =
    funcCount: 7803
    localSolverTotal: 50
    localSolverSuccess: 16
    localSolverIncomplete: 34
    localSolverNoSolution: 0
    message: [1x142 char]

manymins =
    1x16 GlobalOptimSolution

Properties:
    X
    Fval
    Exitflag
    Output
    X0

You can get different results, since MultiStart is stochastic.

The solver did not find the global minimum near \([0,0]\). It found 16 distinct local minima.

6 Plot the function values at the local minima:

    hist([manymins.Fval])

![Histogram of function values at local minima](image)
Plot the function values at the three best points:

```matlab
bestf = [manymins.Fval];
hist(bestf(1:3))
```

MultiStart started `fminunc` from start points with components uniformly distributed between –1000 and 1000. `fminunc` often got stuck in one of the many local minima. `fminunc` exceeded its iteration limit or function evaluation limit 34 times.

**Example: Using Only Feasible Start Points**

You can set the `StartPointsToRun` option so that MultiStart and GlobalSearch use only start points that satisfy inequality constraints. This option can speed your optimization, since the local solver does not have to search for a feasible region. However, the option can cause the solvers to miss some basins of attraction.

There are three settings for the `StartPointsToRun` option:

- `all` — Accepts all start points
- `bounds` — Rejects start points that do not satisfy bounds
- `bounds-ineqs` — Rejects start points that do not satisfy bounds or inequality constraints
For example, suppose your objective function is

```matlab
function y = tiltcircle(x)
    vx = x(:)-[4;4]; % ensure vx is in column form
    y = vx'*[1;1] + sqrt(16 - vx'*vx); % complex if norm(x-[4;4])>4
end
```

`tiltcircle` returns complex values for `norm(x - [4 4]) > 4`.

Write a constraint function that is positive on the set where `norm(x - [4 4]) > 4`

```matlab
function [c ceq] = myconstraint(x)
    ceq = [];
end
```
cx = x(:) - [4;4]; % ensure x is a column vector
c = cx'*cx - 16; % negative where tiltcircle(x) is real

Set GlobalSearch to use only start points satisfying inequality constraints:

    gs = GlobalSearch('StartPointsToRun','bounds-ineqs');

To complete the example, create a problem structure and run the solver:

    opts = optimset('Algorithm','interior-point');
    problem = createOptimProblem('fmincon',
        'x0',[4 4], 'objective',@tiltcircle,
        'nonlcon',@myconstraint, 'lb', [-10 -10],
        'ub', [10 10], 'options', opts);
    [x,fval,exitflag,output,solutionset] = run(gs,problem)

GlobalSearch stopped because it analyzed all the trial points.

All 5 local solver runs converged with a positive local solver exit flag.

x =
    1.1716   1.1716

fval =
   -5.6530

exitflag =
    1

output =
    funcCount: 3302
    localSolverTotal: 5
    localSolverSuccess: 5
    localSolverIncomplete: 0
    localSolverNoSolution: 0
    message: [1x137 char]

solutionset =
    1x4 GlobalOptimSolution

Properties:
Why Does GlobalSearch Find Several Local Minima?

The `tiltcircle` function has just one local minimum. Yet `GlobalSearch` (`fmincon`) stops at several points. Does this mean `fmincon` makes an error?
The reason that `fmincon` stops at several boundary points is subtle. The `tiltcircle` function has an infinite gradient on the boundary, as you can see from a one-dimensional calculation:

\[
\frac{d}{dx} \sqrt{16 - x^2} = \frac{-x}{\sqrt{16 - x^2}} = \pm \infty \text{ at } |x| = 4.
\]

So there is a huge gradient normal to the boundary. This gradient overwhelms the small additional tilt from the linear term. As far as `fmincon` can tell, boundary points are stationary points for the constrained problem.

This behavior can arise whenever you have a function that has a square root.

**Example: Parallel MultiStart**

If you have a multicore processor or access to a processor network, you can use Parallel Computing Toolbox™ functions with `MultiStart`. This example shows how to find multiple minima in parallel for a problem, using a processor with two cores. The problem is the same as in “Multiple Local Minima Via MultiStart” on page 3-74.

1 Write a function file to compute the objective:

```matlab
function f = sawtoothxy(x,y)
    [t r] = cart2pol(x,y); % change to polar coordinates
    h = cos(2*t - 1/2)/2 + cos(t) + 2;
    g = (sin(r) - sin(2*r)/2 + sin(3*r)/3 - sin(4*r)/4 + 4) ... 
        .*r.^2./(r+1);
    f = g.*h;
end
```

2 Create the problem structure:

```matlab
problem = createOptimProblem('fminunc',...
    'objective',@(x)sawtoothxy(x(1),x(2)),...
    'x0',[100,-50],
    'options',...
    optimset('LargeScale','off'));
```

3 Validate the problem structure by running it:
Using GlobalSearch and MultiStart

\[ [x \ fval] = \text{fminunc}(\text{problem}) \]

\[ x = \]
\[ 8.4420 \ -110.2602 \]

\[ fval = \]
\[ 435.2573 \]

4. Create a MultiStart object, and set the object to use parallel processing and iterative display:

\[ \text{ms} = \text{MultiStart}('\text{UseParallel}', 'always', 'Display', 'iter'); \]

5. Set up parallel processing:

\[ \text{matlabpool open 2} \]

Starting matlabpool using the 'local' configuration
... connected to 2 labs.

6. Run the problem on 50 start points:

\[ [x \ fval \ eflag \ output \ manymins] = \text{run}(\text{ms}, \text{problem}, 50); \]

Running the local solvers in parallel.

<table>
<thead>
<tr>
<th>Run Index</th>
<th>Local exitflag</th>
<th>Local ( f(x) )</th>
<th>Local # iter</th>
<th>Local F-count</th>
<th>First-order optimality</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>2</td>
<td>3953</td>
<td>4</td>
<td>21</td>
<td>0.1626</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>1331</td>
<td>45</td>
<td>201</td>
<td>65.02</td>
</tr>
<tr>
<td>34</td>
<td>0</td>
<td>7271</td>
<td>54</td>
<td>201</td>
<td>520.9</td>
</tr>
<tr>
<td>33</td>
<td>2</td>
<td>8249</td>
<td>4</td>
<td>18</td>
<td>2.968</td>
</tr>
</tbody>
</table>

... Many iterations omitted ...

<table>
<thead>
<tr>
<th>Run Index</th>
<th>Local exitflag</th>
<th>Local ( f(x) )</th>
<th>Local # iter</th>
<th>Local F-count</th>
<th>First-order optimality</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>2</td>
<td>2740</td>
<td>5</td>
<td>21</td>
<td>0.0422</td>
</tr>
<tr>
<td>35</td>
<td>0</td>
<td>8501</td>
<td>48</td>
<td>201</td>
<td>424.8</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
<td>1225</td>
<td>40</td>
<td>201</td>
<td>21.89</td>
</tr>
</tbody>
</table>

MultiStart completed some of the runs from the start points.

17 out of 50 local solver runs converged with a positive local solver exit flag.
Notice that the run indexes look random. Parallel `MultiStart` runs its start points in an unpredictable order.

Notice that `MultiStart` confirms parallel processing in the first line of output, which states: “Running the local solvers in parallel.”

7 When finished, shut down the parallel environment:

```matlab
matlabpool close
Sending a stop signal to all the labs ... stopped.
```

For an example of how to obtain better solutions to this problem, see “Example: Searching for a Better Solution” on page 3-60. You can use parallel processing along with the techniques described in that example.

**Speedup with Parallel Computing**

The results of `MultiStart` runs are stochastic. The timing of runs is stochastic, too. Nevertheless, some clear trends are apparent in the following table. The data for the table came from one run at each number of start points, on a machine with two cores.

<table>
<thead>
<tr>
<th>Start Points</th>
<th>Parallel Seconds</th>
<th>Serial Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>3.6</td>
<td>3.4</td>
</tr>
<tr>
<td>100</td>
<td>4.9</td>
<td>5.7</td>
</tr>
<tr>
<td>200</td>
<td>8.3</td>
<td>10</td>
</tr>
<tr>
<td>500</td>
<td>16</td>
<td>23</td>
</tr>
<tr>
<td>1000</td>
<td>31</td>
<td>46</td>
</tr>
</tbody>
</table>

Parallel computing can be slower than serial when you use only a few start points. As the number of start points increases, parallel computing becomes increasingly more efficient than serial.

There are many factors that affect speedup (or slowdown) with parallel processing. For more information, see “Improving Performance with Parallel Computing” in the Optimization Toolbox documentation.
Example: Isolated Global Minimum

Finding a start point in the basin of attraction of the global minimum can be difficult when the basin is small or when you are unsure of the location of the minimum. To solve this type of problem you can:

- Add sensible bounds
- Take a huge number of random start points
- Make a methodical grid of start points
- For an unconstrained problem, take widely dispersed random start points

This example shows these methods and some variants.

The function $-\text{sech}(x)$ is nearly 0 for all $|x| > 5$, and $-\text{sech}(0) = -1$. The example is a two-dimensional version of the sech function, with one minimum at $[1, 1]$, the other at $[1\text{e}5, -1\text{e}5]$:

$$f(x,y) = -10\text{sech}(|x - (1,1)|) - 20\text{sech}(0.0003(|x - (1\text{e}5,-1\text{e}5)|)) - 1.$$

$f$ has a global minimum of $-21$ at $(1\text{e}5,-1\text{e}5)$, and a local minimum of $-11$ at $(1,1)$. 
The minimum at (1e5,–1e5) shows as a narrow spike. The minimum at (1,1) does not show since it is too narrow.

The following sections show various methods of searching for the global minimum. Some of the methods are not successful on this problem. Nevertheless, you might find each method useful for different problems.

- “Default Settings Cannot Find the Global Minimum — Add Bounds” on page 3-86
- “GlobalSearch with Bounds and More Start Points” on page 3-86
- “MultiStart with Bounds and Many Start Points” on page 3-87
Default Settings Cannot Find the Global Minimum — Add Bounds

GlobalSearch and MultiStart cannot find the global minimum using default global options, since the default start point components are in the range (–9999,10001) for GlobalSearch and (–1000,1000) for MultiStart.

With additional bounds of –1e6 and 1e6 in problem, GlobalSearch usually does not find the global minimum:

```matlab
x1 = [1;1]; x2 = [1e5;-1e5];
f = @(x)-10*sech(norm(x(:)-x1)) -20*sech((norm(x(:)-x2))*3e-4) -1;
problem = createOptimProblem('fmincon','x0',[0,0],'objective',f,...
       'lb',[-1e6;-1e6],'ub',[1e6;1e6]);
gs = GlobalSearch;
[xfinal fval] = run(gs,problem)

GlobalSearch stopped because it analyzed all the trial points.

All 57 local solver runs converged with a positive local solver exit flag.

xfinal =
    1.0000
    1.0000

fval =
    -11.0000
```

GlobalSearch with Bounds and More Start Points

To find the global minimum, you can search more points. This example uses 1e5 start points, and a MaxTime of 300 s:

```matlab
gs.NumTrialPoints = 1e5;
gs.MaxTime = 300;
```
GlobalSearch and MultiStart Examples

[xg fvalg] = run(gs,problem)

GlobalSearch stopped because maximum time is exceeded.

GlobalSearch called the local solver 2186 times before exceeding the clock time limit (MaxTime = 300 seconds).
1943 local solver runs converged with a positive local solver exit flag.

xg =
    1.0e+005 *
    1.0000
    -1.0000

fvalg =
    -21.0000

In this case, GlobalSearch found the global minimum.

MultiStart with Bounds and Many Start Points

Alternatively, you can search using MultiStart with many start points. This example uses 1e5 start points, and a MaxTime of 300 s:

ms = MultiStart(gs);
[xm fvalm] = run(ms,problem,1e5)

MultiStart stopped because maximum time was exceeded.

MultiStart called the local solver 17266 times before exceeding the clock time limit (MaxTime = 300 seconds).
17266 local solver runs converged with a positive local solver exit flag.

xm =
    1.0000
    1.0000

fvalm =
    -11.0000
In this case, MultiStart failed to find the global minimum.

**MultiStart Without Bounds, Widely Dispersed Start Points**

You can also use MultiStart to search an unbounded region to find the global minimum. Again, you need many start points to have a good chance of finding the global minimum.

The first five lines of code generate 10,000 widely dispersed random start points using the method described in “Widely Dispersed Points for Unconstrained Components” on page 3-58. newprob is a problem structure using the fminunc local solver and no bounds:

```matlab
u = rand(1e4,1);
u = 1./u;
u = exp(u) - exp(1);
s = rand(1e4,1)*2*pi;
stpts = [u.*cos(s),u.*sin(s)];
startpts = CustomStartPointSet(stpts);
newprob = createOptimProblem('fminunc','x0',[0;0], 'objective',f);
[xcust fcust] = run(ms,newprob,startpts)
```

MultiStart completed the runs from all start points.

All 10000 local solver runs converged with a positive local solver exit flag.

```
xcust =
   1.0e+004 *
       10.0000
      -10.0000

fcust =
   -21.0000
```

In this case, MultiStart found the global minimum.
**MultiStart with a Regular Grid of Start Points**

You can also use a grid of start points instead of random start points. To learn how to construct a regular grid for more dimensions, or one that has small perturbations, see “Uniform Grid” on page 3-57 or “Perturbed Grid” on page 3-58.

```matlab
xx = -1e6:1e4:1e6;
[xxx yyy] = meshgrid(xx,xx);
z = [xxx(:),yyy(:)];
bigstart = CustomStartPointSet(z);
[xgrid fgrid] = run(ms,newprob,bigstart)
```

MultiStart completed the runs from all start points.

All 10000 local solver runs converged with a positive local solver exit flag.

```matlab
xgrid =
1.0e+004 *
10.0000
-10.0000
fgrid =
-21.0000
```

In this case, MultiStart found the global minimum.

**MultiStart with Regular Grid and Promising Start Points**

Making a regular grid of start points, especially in high dimensions, can use an inordinate amount of memory or time. You can filter the start points to run only those with small objective function value.

To perform this filtering most efficiently, write your objective function in a vectorized fashion. For information, see “Example: Writing a Vectorized Function” on page 2-3 or “Vectorizing the Objective and Constraint Functions” on page 4-83. The following function handle computes a vector of objectives based on an input matrix whose rows represent start points:

```matlab
x1 = [1;1];x2 = [1e5;-1e5];
```
g = @(x) -10*sech(sqrt((x(:,1)-x1(1)).^2 + (x(:,2)-x1(2)).^2)) ... -20*sech(sqrt((x(:,1)-x2(1)).^2 + (x(:,2)-x2(2)).^2))-1;

Suppose you want to run the local solver only for points where the value is less than –2. Start with a denser grid than in “MultiStart with a Regular Grid of Start Points” on page 3-89, then filter out all the points with high function value:

\[
\begin{align*}
&xx = -1e6:1e3:1e6; \\
&[xxx ~ yyy] = \text{meshgrid}(xx,xx); \\
&z = [xxx(:), yyy(:)]; \\
&idx = \text{g}(z) < -2; \quad \% \text{index of promising start points} \\
&zz = z(idx,:); \\
&\text{smallstartset} = \text{CustomStartPointSet}(zz); \\
&\text{opts} = \text{optimset}('LargeScale','off','Display','off'); \\
&\text{newprobg} = \text{createOptimProblem}('fminunc','x0',[0,0],... \\
&\quad \text{objective'},g,'options',opts); \\
&\quad \% \text{row vector } x0 \text{ since } g \text{ expects rows} \\
&[xfew ffew] = \text{run(ms,newprobg,smallstartset)}
\end{align*}
\]

MultiStart completed the runs from all start points.

All 2 local solver runs converged with a positive local solver exit flag.

\[
\begin{align*}
&xfew = \\
&\quad 100000 \quad -100000 \\
&ffew = \\
&\quad -21
\end{align*}
\]

In this case, MultiStart found the global minimum. There are only two start points in smallstartset, one of which is the global minimum.
Using Direct Search

- “What Is Direct Search?” on page 4-2
- “Performing a Pattern Search” on page 4-3
- “Example: Finding the Minimum of a Function Using the GPS Algorithm” on page 4-7
- “Pattern Search Terminology” on page 4-11
- “How Pattern Search Polling Works” on page 4-14
- “Searching and Polling” on page 4-24
- “Description of the Nonlinear Constraint Solver” on page 4-30
- “Performing a Pattern Search Using the Optimization Tool GUI” on page 4-33
- “Performing a Pattern Search from the Command Line” on page 4-43
- “Pattern Search Examples: Setting Options” on page 4-49
What Is Direct Search?

Direct search is a method for solving optimization problems that does not require any information about the gradient of the objective function. Unlike more traditional optimization methods that use information about the gradient or higher derivatives to search for an optimal point, a direct search algorithm searches a set of points around the current point, looking for one where the value of the objective function is lower than the value at the current point. You can use direct search to solve problems for which the objective function is not differentiable, or is not even continuous.

Global Optimization Toolbox functions include three direct search algorithms called the generalized pattern search (GPS) algorithm, the generating set search (GSS) algorithm, and the mesh adaptive search (MADS) algorithm. All are pattern search algorithms that compute a sequence of points that approach an optimal point. At each step, the algorithm searches a set of points, called a mesh, around the current point—the point computed at the previous step of the algorithm. The mesh is formed by adding the current point to a scalar multiple of a set of vectors called a pattern. If the pattern search algorithm finds a point in the mesh that improves the objective function at the current point, the new point becomes the current point at the next step of the algorithm.

The GPS algorithm uses fixed direction vectors. The GSS algorithm is identical to the GPS algorithm, except when there are linear constraints, and when the current point is near a linear constraint boundary. The MADS algorithm uses a random selection of vectors to define the mesh. For details, see “Patterns” on page 4-11.
Performing a Pattern Search

In this section...

“Calling patternsearch at the Command Line” on page 4-3
“Using the Optimization Tool for Pattern Search” on page 4-3

Calling patternsearch at the Command Line
To perform a pattern search on an unconstrained problem at the command line, call the function `patternsearch` with the syntax

\[
[x \ fval] = \text{patternsearch}(\@\text{objfun}, \ x0)
\]

where

• `@objfun` is a handle to the objective function.
• `x0` is the starting point for the pattern search.

The results are:

• `x` — Point at which the final value is attained
• `fval` — Final value of the objective function

“Performing a Pattern Search from the Command Line” on page 4-43 explains in detail how to use the `patternsearch` function.

Using the Optimization Tool for Pattern Search
To open the Optimization Tool, enter

\[
\text{optimtool('patternsearch')}
\]

at the command line, or enter `optimtool` and then choose `patternsearch` from the Solver menu.
You can also start the tool from the MATLAB **Start** menu as pictured:
To use the Optimization Tool, first enter the following information:

- **Objective function** — The objective function you want to minimize. Enter the objective function in the form @objfun, where objfun.m is a file that computes the objective function. The @ sign creates a function handle to objfun.

- **Start point** — The initial point at which the algorithm starts the optimization.
In the **Constraints** pane, enter linear constraints, bounds, or a nonlinear constraint function as a function handle for the problem. If the problem is unconstrained, leave these fields blank.

Then, click **Start**. The tool displays the results of the optimization in the **Run solver and view results** pane.

In the **Options** pane, set the options for the pattern search. To view the options in a category, click the + sign next to it.

“Finding the Minimum of the Function” on page 4-8 gives an example of using the Optimization Tool.

The “Graphical Optimization Tool” chapter in the Optimization Toolbox documentation provides a detailed description of the Optimization Tool.
Example: Finding the Minimum of a Function Using the GPS Algorithm

**Objective Function**
This example uses the objective function, `ps_example`, which is included with Global Optimization Toolbox software. View the code for the function by entering

```
type ps_example
```

The following figure shows a plot of the function.

![3D Plot of Objective Function](image)
**Finding the Minimum of the Function**

To find the minimum of `ps_example`, perform the following steps:

1. Enter
   ```matlab
   optimtool
   ```
   and then choose the `patternsearch` solver.

2. In the **Objective function** field of the Optimization Tool, enter `@ps_example`.

3. In the **Start point** field, type `[2.1 1.7]`.
   ![Image of Optimization Tool](image)

   Leave the fields in the **Constraints** pane blank because the problem is unconstrained.

4. Click **Start** to run the pattern search.

The **Run solver and view results** pane displays the results of the pattern search.
The reason the optimization terminated is that the mesh size became smaller than the acceptable tolerance value for the mesh size, defined by the Mesh tolerance parameter in the Stopping criteria pane. The minimum function value is approximately –2. The Final point pane displays the point at which the minimum occurs.

**Plotting the Objective Function Values and Mesh Sizes**

To see the performance of the pattern search, display plots of the best function value and mesh size at each iteration. First, select the following check boxes in the Plot functions pane:

- **Best function value**
- **Mesh size**
Then click **Start** to run the pattern search. This displays the following plots.

The upper plot shows the objective function value of the best point at each iteration. Typically, the objective function values improve rapidly at the early iterations and then level off as they approach the optimal value.

The lower plot shows the mesh size at each iteration. The mesh size increases after each successful iteration and decreases after each unsuccessful one, explained in “How Pattern Search Polling Works” on page 4-14.
Pattern Search Terminology

In this section...

“Patterns” on page 4-11
“Meshes” on page 4-12
“Polling” on page 4-13
“Expanding and Contracting” on page 4-13

Patterns

A pattern is a set of vectors \( \{v_i\} \) that the pattern search algorithm uses to determine which points to search at each iteration. The set \( \{v_i\} \) is defined by the number of independent variables in the objective function, \( N \), and the positive basis set. Two commonly used positive basis sets in pattern search algorithms are the maximal basis, with \( 2^N \) vectors, and the minimal basis, with \( N+1 \) vectors.

With GPS, the collection of vectors that form the pattern are fixed-direction vectors. For example, if there are three independent variables in the optimization problem, the default for a \( 2^N \) positive basis consists of the following pattern vectors:

\[
\begin{align*}
v_1 &= [1 \ 0 \ 0] \\
v_2 &= [0 \ 1 \ 0] \\
v_3 &= [0 \ 0 \ 1] \\
v_4 &= [-1 \ 0 \ 0] \\
v_5 &= [0 \ -1 \ 0] \\
v_6 &= [0 \ 0 \ -1]
\end{align*}
\]

An \( N+1 \) positive basis consists of the following default pattern vectors.

\[
\begin{align*}
v_1 &= [1 \ 0 \ 0] \\
v_2 &= [0 \ 1 \ 0] \\
v_3 &= [0 \ 0 \ 1] \\
v_4 &= [-1 \ -1 \ -1]
\end{align*}
\]

With GSS, the pattern is identical to the GPS pattern, except when there are linear constraints and the current point is near a constraint boundary. For a description of the way in which GSS forms a pattern with linear constraints, see Kolda, Lewis, and Torczon [1]. The GSS algorithm is more efficient than the GPS algorithm when you have linear constraints. For an example showing the efficiency gain, see “Example: Comparing the Efficiency of Poll Options” on page 4-55.
With MADS, the collection of vectors that form the pattern are randomly selected by the algorithm. Depending on the poll method choice, the number of vectors selected will be $2N$ or $N+1$. As in GPS, $2N$ vectors consist of $N$ vectors and their $N$ negatives, while $N+1$ vectors consist of $N$ vectors and one that is the negative of the sum of the others.


**Meshes**

At each step, `patternsearch` searches a set of points, called a *mesh*, for a point that improves the objective function. `patternsearch` forms the mesh by

1. Generating a set of vectors $\{d_i\}$ by multiplying each pattern vector $v_i$ by a scalar $\Delta^m$. $\Delta^m$ is called the *mesh size*.

2. Adding the $\{d_i\}$ to the *current point*—the point with the best objective function value found at the previous step.

For example, using the GPS algorithm. suppose that:

- The current point is $[1.6\ 3.4]$.
- The pattern consists of the vectors
  
  \[
  v_1 = [1\ 0] \\
  v_2 = [0\ 1] \\
  v_3 = [-1\ 0] \\
  v_4 = [0\ -1]
  \]

- The current mesh size $\Delta^m$ is 4.

The algorithm multiplies the pattern vectors by 4 and adds them to the current point to obtain the following mesh.

\[
[1.6\ 3.4] + 4*[1\ 0] = [5.6\ 3.4]
\]
\[
\begin{align*}
[1.6, 3.4] + 4*[0, 1] &= [1.6, 7.4] \\
[1.6, 3.4] + 4*[-1, 0] &= [-2.4, 3.4] \\
[1.6, 3.4] + 4*[0, -1] &= [1.6, -0.6]
\end{align*}
\]

The pattern vector that produces a mesh point is called its *direction*.

**Polling**

At each step, the algorithm polls the points in the current mesh by computing their objective function values. When the **Complete poll** option has the (default) setting *On*, the algorithm stops polling the mesh points as soon as it finds a point whose objective function value is less than that of the current point. If this occurs, the poll is called *successful* and the point it finds becomes the current point at the next iteration.

The algorithm only computes the mesh points and their objective function values up to the point at which it stops the poll. If the algorithm fails to find a point that improves the objective function, the poll is called *unsuccessful* and the current point stays the same at the next iteration.

When the **Complete poll** option has the setting *On*, the algorithm computes the objective function values at all mesh points. The algorithm then compares the mesh point with the smallest objective function value to the current point. If that mesh point has a smaller value than the current point, the poll is successful.

**Expanding and Contracting**

After polling, the algorithm changes the value of the mesh size $\Delta^m$. The default is to multiply $\Delta^m$ by 2 after a successful poll, and by 0.5 after an unsuccessful poll.
How Pattern Search Polling Works

In this section...

“Context” on page 4-14
“Successful Polls” on page 4-15
“An Unsuccessful Poll” on page 4-18
“Displaying the Results at Each Iteration” on page 4-19
“More Iterations” on page 4-20
“Stopping Conditions for the Pattern Search” on page 4-21

Context

patternsearch finds a sequence of points, \( x_0, x_1, x_2, \ldots \), that approach an optimal point. The value of the objective function either decreases or remains the same from each point in the sequence to the next. This section explains how pattern search works for the function described in “Example: Finding the Minimum of a Function Using the GPS Algorithm” on page 4-7.

To simplify the explanation, this section describes how the generalized pattern search (GPS) works using a maximal positive basis of \( 2N \), with Scale set to Off in Mesh options.
How Pattern Search Polling Works

The problem setup:

**Successful Polls**
The pattern search begins at the initial point \( x_0 \) that you provide. In this example, \( x_0 = [2.1 \ 1.7] \).

**Iteration 1**
At the first iteration, the mesh size is 1 and the GPS algorithm adds the pattern vectors to the initial point \( x_0 = [2.1 \ 1.7] \) to compute the following mesh points:

\[
\begin{align*}
[1 \ 0] + x_0 &= [3.1 \ 1.7] \\
[0 \ 1] + x_0 &= [2.1 \ 2.7] \\
[-1 \ 0] + x_0 &= [1.1 \ 1.7] \\
[0 \ -1] + x_0 &= [2.1 \ 0.7]
\end{align*}
\]
The algorithm computes the objective function at the mesh points in the order shown above. The following figure shows the value of `ps_example` at the initial point and mesh points.

The algorithm polls the mesh points by computing their objective function values until it finds one whose value is smaller than 4.6347, the value at x_0. In this case, the first such point it finds is [1.1 1.7], at which the value of the objective function is 4.5146, so the poll at iteration 1 is successful. The algorithm sets the next point in the sequence equal to

\[ x_1 = [1.1 \ 1.7] \]
Note By default, the GPS pattern search algorithm stops the current iteration as soon as it finds a mesh point whose fitness value is smaller than that of the current point. Consequently, the algorithm might not poll all the mesh points. You can make the algorithm poll all the mesh points by setting **Complete poll** to **On**.

**Iteration 2**
After a successful poll, the algorithm multiplies the current mesh size by 2, the default value of **Expansion factor** in the **Mesh** options pane. Because the initial mesh size is 1, at the second iteration the mesh size is 2. The mesh at iteration 2 contains the following points:

\[
\begin{align*}
2 \times [1, 0] + x_1 &= [3.1, 1.7] \\
2 \times [0, 1] + x_1 &= [1.1, 3.7] \\
2 \times [-1, 0] + x_1 &= [-0.9, 1.7] \\
2 \times [0, -1] + x_1 &= [1.1, -0.3]
\end{align*}
\]

The following figure shows the point \( x_1 \) and the mesh points, together with the corresponding values of \( ps\_example \).
Using Direct Search

The algorithm polls the mesh points until it finds one whose value is smaller than 4.5146, the value at \( x_1 \). The first such point it finds is \([-0.9, 1.7]\), at which the value of the objective function is 3.25, so the poll at iteration 2 is again successful. The algorithm sets the second point in the sequence equal to

\[
x_2 = [-0.9, 1.7]
\]

Because the poll is successful, the algorithm multiplies the current mesh size by 2 to get a mesh size of 4 at the third iteration.

**An Unsuccessful Poll**

By the fourth iteration, the current point is

\[
x_3 = [-4.9, 1.7]
\]

and the mesh size is 8, so the mesh consists of the points

\[
8*[1 \ 0] + x_3 = [3.1 \ 1.7] \\
8*[0 \ 1] + x_3 = [-4.9 \ 9.7] \\
8*[-1 \ 0] + x_3 = [-12.9 \ 1.7]
\]
8*[0 -1] + x3 = [-4.9 -1.3]

The following figure shows the mesh points and their objective function values.

At this iteration, none of the mesh points has a smaller objective function value than the value at x3, so the poll is unsuccessful. In this case, the algorithm does not change the current point at the next iteration. That is,

\[ x4 = x3; \]

At the next iteration, the algorithm multiplies the current mesh size by 0.5, the default value of \textbf{Contraction factor} in the \textbf{Mesh} options pane, so that the mesh size at the next iteration is 4. The algorithm then polls with a smaller mesh size.

\textbf{Displaying the Results at Each Iteration}

You can display the results of the pattern search at each iteration by setting \textbf{Level of display} to \textit{Iterative} in the \textbf{Display to command window} options. This enables you to evaluate the progress of the pattern search and to make changes to options if necessary.
Using Direct Search

With this setting, the pattern search displays information about each iteration at the command line. The first four iterations are

<table>
<thead>
<tr>
<th>Iter</th>
<th>f-count</th>
<th>f(x)</th>
<th>MeshSize</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>4.63474</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>4.51464</td>
<td>2</td>
<td>Successful Poll</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>3.25</td>
<td>4</td>
<td>Successful Poll</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>-0.264905</td>
<td>8</td>
<td>Successful Poll</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>-0.264905</td>
<td>4</td>
<td>Refine Mesh</td>
</tr>
</tbody>
</table>

The entry Successful Poll below Method indicates that the current iteration was successful. For example, the poll at iteration 2 is successful. As a result, the objective function value of the point computed at iteration 2, displayed below f(x), is less than the value at iteration 1.

At iteration 4, the entry Refine Mesh tells you that the poll is unsuccessful. As a result, the function value at iteration 4 remains unchanged from iteration 3.

By default, the pattern search doubles the mesh size after each successful poll and halves it after each unsuccessful poll.

**More Iterations**

The pattern search performs 60 iterations before stopping. The following plot shows the points in the sequence computed in the first 13 iterations of the pattern search.
How Pattern Search Polling Works

The numbers below the points indicate the first iteration at which the algorithm finds the point. The plot only shows iteration numbers corresponding to successful polls, because the best point doesn’t change after an unsuccessful poll. For example, the best point at iterations 4 and 5 is the same as at iteration 3.

Stopping Conditions for the Pattern Search

The criteria for stopping the pattern search algorithm are listed in the Stopping criteria section of the Optimization Tool:
The algorithm stops when any of the following conditions occurs:

- The mesh size is less than **Mesh tolerance**.
- The number of iterations performed by the algorithm reaches the value of **Max iteration**.
- The total number of objective function evaluations performed by the algorithm reaches the value of **Max function evaluations**.
- The time, in seconds, the algorithm runs until it reaches the value of **Time limit**.
- The distance between the point found in two consecutive iterations and the mesh size are both less than **X tolerance**.
The change in the objective function in two consecutive iterations and the mesh size are both less than **Function tolerance**.

**Nonlinear constraint tolerance** is not used as stopping criterion. It determines the feasibility with respect to nonlinear constraints.

The MADS algorithm uses an additional parameter called the poll parameter, $\Delta_p$, in the mesh size stopping criterion:

$$
\Delta_p = \begin{cases} 
N \sqrt{\Delta_m} & \text{for positive basis } N+1 \text{ poll} \\
\sqrt{\Delta_m} & \text{for positive basis } 2N \text{ poll}, 
\end{cases}
$$

where $\Delta_m$ is the mesh size. The MADS stopping criterion is:

$$
\Delta_p \leq \text{Mesh tolerance}.
$$
Searching and Polling

In this section...

“Definition of Search” on page 4-24
“How To Use a Search Method” on page 4-26
“Search Types” on page 4-27
“When to Use Search” on page 4-27
“Example: Search and Poll” on page 4-28

Definition of Search

In patternsearch, a search is an algorithm that runs before a poll. The search attempts to locate a better point than the current point. (Better means one with lower objective function value.) If the search finds a better point, the better point becomes the current point, and no polling is done at that iteration. If the search does not find a better point, patternsearch performs a poll.

By default, patternsearch does not use search. To search, see “How To Use a Search Method” on page 4-26.

The figure patternsearch With a Search Method on page 4-25 contains a flow chart of direct search including using a search method.
patternsearch With a Search Method
Iteration limit applies to all built-in search methods except those that are poll methods. If you select an iteration limit for the search method, the search is enabled until the iteration limit is reached. Afterwards, `patternsearch` stops searching and only polls.

**How To Use a Search Method**

To use search in `patternsearch`:

- In Optimization Tool, choose a **Search method** in the **Search** pane.

![Search Method Selection](image)

- At the command line, create an options structure with a search method using `psoptimset`. For example, to use Latin hypercube search:

  ```matlab
  opts = psoptimset('SearchMethod', @searchlhs);
  ```

  For more information, including a list of all built-in search methods, consult the `psoptimset` function reference page, and the “Search Options” on page 9-14 section of the options reference.

You can write your own search method. Use the syntax described in “Structure of the Search Function” on page 9-17. To use your search method in a pattern search, give its function handle as the **Custom Function (SearchMethod)** option.
Search Types

- Poll methods — You can use any poll method as a search algorithm. `patternsearch` conducts one poll step as a search. For this type of search to be beneficial, your search type should be different from your poll type. (`patternsearch` does not search if the selected search method is the same as the poll type.) Therefore, use a MADS search with a GSS or GPS poll, or use a GSS or GPS search with a MADS poll.

- `fminsearch`, also called Nelder-Mead — `fminsearch` is for unconstrained problems only. `fminsearch` runs to its natural stopping criteria; it does not take just one step. Therefore, use `fminsearch` for just one iteration. This is the default setting. To change settings, see “Search Options” on page 9-14.

- `ga` — `ga` runs to its natural stopping criteria; it does not take just one step. Therefore, use `ga` for just one iteration. This is the default setting. To change settings, see “Search Options” on page 9-14.

- Latin hypercube search — Described in “Search Options” on page 9-14. By default, searches $15n$ points, where $n$ is the number of variables, and only searches during the first iteration. To change settings, see “Search Options” on page 9-14.

When to Use Search

There are two main reasons to use a search method:

- To speed an optimization (see “Search Methods for Increased Speed” on page 4-27)
- To obtain a better local solution, or to obtain a global solution (see “Search Methods for Better Solutions” on page 4-28)

Search Methods for Increased Speed

Generally, you do not know beforehand whether a search method speeds an optimization or not. So try a search method when:

- You are performing repeated optimizations on similar problems, or on the same problem with different parameters.
- You can experiment with different search methods to find a lower solution time.
Search does not always speed an optimization. For one example where it does, see “Example: Search and Poll” on page 4-28.

**Search Methods for Better Solutions**

Since search methods run before poll methods, using search can be equivalent to choosing a different starting point for your optimization. This comment holds for the Nelder-Mead, ga, and Latin hypercube search methods, all of which, by default, run once at the beginning of an optimization. ga and Latin hypercube searches are stochastic, and can search through several basins of attraction.

**Example: Search and Poll**

patternsearch takes a long time to minimize Rosenbrock’s function. The function is \( f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \).

Rosenbrock’s function is described and plotted in “Example: Nonlinear Constrained Minimization” in the Optimization Toolbox documentation. The dejong2fcn.m file, which in included in the toolbox, calculates this function.

1. Set patternsearch options to MaxFunEvals = 5000 and MaxIter = 2000:
   
   ```
   opts = psoptimset('MaxFunEvals',5000,'MaxIter',2000);
   ```

2. Run patternsearch starting from \([-1.9, 2]\):
   
   ```
   [x feval eflag output] = patternsearch(@dejong2fcn,...
   [-1.9,2],[],[],[],[],[],[],[],[],opts);
   ```

   Maximum number of function evaluations exceeded: increase options.MaxFunEvals.

   feval

   feval =

   0.8560

   The optimization did not complete, and the result is not very close to the optimal value of 0.
3 Set the options to use `fminsearch` as the search method:

```matlab
topts = psoptimset(topts,'SearchMethod',@searchneldermead);
```

4 Rerun the optimization, the results are much better:

```matlab
[x2 feval2 eflag2 output2] = patternsearch(@dejong2fcn,...
    [-1.9,2],[[],[],[],[],[],[],[],[],opts);
```

Optimization terminated: mesh size less than options.TolMesh.

```matlab
feval2
```

```matlab
feval2 =
    4.0686e-010
```

`fminsearch` is not as closely tied to coordinate directions as the default GPS
`patternsearch` poll method. Therefore, `fminsearch` is more efficient at
getting close to the minimum of Rosenbrock's function. Adding the search
method in this case is effective.
Description of the Nonlinear Constraint Solver

The pattern search algorithm uses the Augmented Lagrangian Pattern Search (ALPS) algorithm to solve nonlinear constraint problems. The optimization problem solved by the ALPS algorithm is

$$\min_{x} f(x)$$

such that

$$c_i(x) \leq 0, \ i = 1 \ldots m$$
$$ceq_i(x) = 0, \ i = m + 1 \ldots mt$$
$$A \cdot x \leq b$$
$$A_{eq} \cdot x = beq$$
$$lb \leq x \leq ub,$$

where $c(x)$ represents the nonlinear inequality constraints, $ceq(x)$ represents the equality constraints, $m$ is the number of nonlinear inequality constraints, and $mt$ is the total number of nonlinear constraints.

The ALPS algorithm attempts to solve a nonlinear optimization problem with nonlinear constraints, linear constraints, and bounds. In this approach, bounds and linear constraints are handled separately from nonlinear constraints. A subproblem is formulated by combining the objective function and nonlinear constraint function using the Lagrangian and the penalty parameters. A sequence of such optimization problems are approximately minimized using a pattern search algorithm such that the linear constraints and bounds are satisfied.

Each subproblem solution represents one iteration. The number of function evaluations per iteration is therefore much higher when using nonlinear constraints than otherwise.

A subproblem formulation is defined as
\[ \Theta(x, \lambda, s, \rho) = f(x) - \sum_{i=1}^{m} \lambda_i s_i \log(s_i - c_i(x)) + \sum_{i=m+1}^{mt} \lambda_i c_{eq_i}(x) + \frac{\rho}{2} \sum_{i=m+1}^{mt} c_{eq_i}(x)^2, \]

where

- The components \( \lambda_i \) of the vector \( \lambda \) are nonnegative and are known as Lagrange multiplier estimates.
- The elements \( s_i \) of the vector \( s \) are nonnegative shifts.
- \( \rho \) is the positive penalty parameter.

The algorithm begins by using an initial value for the penalty parameter (InitialPenalty).

The pattern search minimizes a sequence of subproblems, each of which is an approximation of the original problem. Each subproblem has a fixed value of \( \lambda, s, \) and \( \rho \). When the subproblem is minimized to a required accuracy and satisfies feasibility conditions, the Lagrangian estimates are updated. Otherwise, the penalty parameter is increased by a penalty factor (PenaltyFactor). This results in a new subproblem formulation and minimization problem. These steps are repeated until the stopping criteria are met.

Each subproblem solution represents one iteration. The number of function evaluations per iteration is therefore much higher when using nonlinear constraints than otherwise.

For a complete description of the algorithm, see the following references:


Performing a Pattern Search Using the Optimization Tool GUI

In this section...

“Example: A Linearly Constrained Problem” on page 4-33
“Displaying Plots” on page 4-36
“Example: Working with a Custom Plot Function” on page 4-37

Example: A Linearly Constrained Problem

This section presents an example of performing a pattern search on a constrained minimization problem. The example minimizes the function

\[ F(x) = \frac{1}{2} x^T H x + f^T x, \]

where

\[
H = \begin{bmatrix}
36 & 17 & 19 & 12 & 8 & 15 \\
17 & 33 & 18 & 11 & 7 & 14 \\
19 & 18 & 43 & 13 & 8 & 16 \\
12 & 11 & 13 & 18 & 6 & 11 \\
8 & 7 & 8 & 6 & 9 & 8 \\
15 & 14 & 16 & 11 & 8 & 29
\end{bmatrix},
\]

\[
f = [20 \ 15 \ 21 \ 18 \ 29 \ 24],
\]

subject to the constraints

\[ A \cdot x \leq b, \]
\[ Aeq \cdot x = beq, \]

where
Performing a Pattern Search on the Example

To perform a pattern search on the example, first enter

```matlab
optimtool('patternsearch')
```

to open the Optimization Tool, or enter `optimtool` and then choose `patternsearch` from the **Solver** menu. Then type the following function in the **Objective function** field:

```matlab
@lincontest7
```

*lincontest7* is a file included in Global Optimization Toolbox software that computes the objective function for the example. Because the matrices and vectors defining the starting point and constraints are large, it is more convenient to set their values as variables in the MATLAB workspace first and then enter the variable names in the Optimization Tool. To do so, enter

```matlab
x0 = [2 1 0 9 1 0];
Aineq = [-8 7 3 -4 9 0];
bineq = 7;
Aeq = [7 1 8 3 3 3; 5 0 -5 1 -5 8; -2 -6 7 1 1 9; 1 -1 2 -2 3 -3];
beq = [84 62 65 1];
```

Then, enter the following in the Optimization Tool:

- Set **Start point** to `x0`.
- Set the following **Linear inequalities**:
  - Set `A` to `Aineq`.
  - Set `b` to `bineq`.
- Set $A_{eq}$ to $A_{eq}$.
- Set $b_{eq}$ to $b_{eq}$.

The following figure shows these settings in the Optimization Tool.

Since this is a linearly constrained problem, set the Poll method to GSS Positive basis 2N. For more information about the efficiency of the GSS search methods for linearly constrained problems, see “Example: Comparing the Efficiency of Poll Options” on page 4-55.

Then click Start to run the pattern search. When the search is finished, the results are displayed in Run solver and view results pane, as shown in the following figure.
Displaying Plots

The **Plot functions** pane, shown in the following figure, enables you to display various plots of the results of a pattern search.

Select the check boxes next to the plots you want to display. For example, if you select **Best function value** and **Mesh size**, and run the example described in “Example: Finding the Minimum of a Function Using the GPS Algorithm” on page 4-7, the tool displays the plots shown in the following figure.
Performing a Pattern Search Using the Optimization Tool GUI

The upper plot displays the objective function value at each iteration. The lower plot displays the mesh size at each iteration.

**Note** When you display more than one plot, you can open a larger version of a plot in a separate window. Right-click (Ctrl-click for Mac) on a blank area in a plot while patternsearch is running, or after it has stopped, and choose the sole menu item.

“Plot Options” on page 9-32 describes the types of plots you can create.

**Example: Working with a Custom Plot Function**

To use a plot function other than those included with the software, you can write your own custom plot function that is called at each iteration of the pattern search to create the plot. This example shows how to create a plot.
function that displays the logarithmic change in the best objective function value from the previous iteration to the current iteration.

This section covers the following topics:

- “Creating the Custom Plot Function” on page 4-38
- “Setting Up the Problem” on page 4-39
- “Using the Custom Plot Function” on page 4-40
- “How the Plot Function Works” on page 4-41

Creating the Custom Plot Function

To create the plot function for this example, copy and paste the following code into a new function file in the MATLAB Editor:

```matlab
function stop = psplotchange(optimvalues, flag)
    % PSPLTCHANGE Plots the change in the best objective function value from the previous iteration.

    % Best objective function value in the previous iteration
    persistent last_best
    stop = false;

    if(strcmp(flag,'init'))
        set(gca,'Yscale','log'); % Set up the plot
        hold on;
        xlabel('Iteration');
        ylabel('Log Change in Values');
        title(['Change in Best Function Value']);
    end

    % Best objective function value in the current iteration
    best = min(optimvalues.fval);

    % Set last_best to best
    if optimvalues.iteration == 0
        last_best = best;
    else
```
Performing a Pattern Search Using the Optimization Tool GUI

%Change in objective function value
change = last_best - best;
plot(optimvalues.iteration, change, '.r');
end

Then save the file as psplotchange.m in a folder on the MATLAB path.

**Setting Up the Problem**
The problem is the same as “Example: A Linearly Constrained Problem” on page 4-33. To set up the problem:

1 Enter the following at the MATLAB command line:

   ```matlab
   x0 = [2 1 0 9 1 0];
   Aineq = [-8 7 3 -4 9 0];
  bineq = 7;
   Aeq = [7 1 8 3 3 3; 5 0 -5 1 -5 8; -2 -6 7 1 1 9; 1 -1 2 -2 3 -3];
   beq = [84 62 65 1];
   ```

2 Enter `optimtool` to open the Optimization Tool.

3 Choose the `patternsearch` solver.

4 Set up the problem to match the following figure.
5 Since this is a linearly constrained problem, set the Poll method to GSS Positive basis 2N.

![Poll method selection](image)

**Using the Custom Plot Function**

To use the custom plot function, select Custom function in the Plot functions pane and enter @psplotchange in the field to the right. To compare the custom plot with the best function value plot, also select Best function value.

![Plot functions pane](image)

Now, when you run the example, the pattern search tool displays the plots shown in the following figure.
Performing a Pattern Search Using the Optimization Tool GUI

Note that because the scale of the y-axis in the lower custom plot is logarithmic, the plot will only show changes that are greater than 0.

**How the Plot Function Works**
The plot function uses information contained in the following structures, which the Optimization Tool passes to the function as input arguments:

- `optimvalues` — Structure containing the current state of the solver
- `flag` — String indicating the current status of the algorithm

The most important statements of the custom plot function, `psplotchange.m`, are summarized in the following table.
# Custom Plot Function Statements

<table>
<thead>
<tr>
<th>Statement</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>persistent last_best</code></td>
<td>Creates the persistent variable <code>last_best</code>, the best objective function value in the previous generation. Persistent variables are preserved over multiple calls to the plot function.</td>
</tr>
<tr>
<td><code>set(gca,'Yscale','log')</code></td>
<td>Sets up the plot before the algorithm starts.</td>
</tr>
<tr>
<td><code>best = min(optimvalues.fval)</code></td>
<td>Sets <code>best</code> equal to the minimum objective function value. The field <code>optimvalues.fval</code> contains the objective function value in the current iteration. The variable <code>best</code> is the minimum objective function value. For a complete description of the fields of the structure <code>optimvalues</code>, see “Structure of the Plot Functions” on page 9-11.</td>
</tr>
<tr>
<td><code>change = last_best - best</code></td>
<td>Sets the variable <code>change</code> to the best objective function value at the previous iteration minus the best objective function value in the current iteration.</td>
</tr>
<tr>
<td><code>plot(optimvalues.iteration, change, '.r')</code></td>
<td>Plots the variable <code>change</code> at the current objective function value, for the current iteration contained in <code>optimvalues.iteration</code>.</td>
</tr>
</tbody>
</table>
Performing a Pattern Search from the Command Line

In this section...

“Calling patternsearch with the Default Options” on page 4-43
“Setting Options for patternsearch at the Command Line” on page 4-45
“Using Options and Problems from the Optimization Tool” on page 4-47

Calling patternsearch with the Default Options
This section describes how to perform a pattern search with the default options.

Pattern Search on Unconstrained Problems
For an unconstrained problem, call patternsearch with the syntax

\[ [x \ fval] = \text{patternsearch}(@\text{objectfun}, \ x0) \]

The output arguments are

- \( x \) — The final point
- \( fval \) — The value of the objective function at \( x \)

The required input arguments are

- \( @\text{objectfun} \) — A function handle to the objective function \( \text{objectfun} \), which you can write as a function file. See “Computing Objective Functions” on page 2-2 to learn how to do this.
- \( x0 \) — The initial point for the pattern search algorithm.

As an example, you can run the example described in “Example: Finding the Minimum of a Function Using the GPS Algorithm” on page 4-7 from the command line by entering

\[ [x \ fval] = \text{patternsearch}(@\text{ps_example}, \ [2.1 \ 1.7]) \]

This returns
Optimization terminated: mesh size less than options.TolMesh.

\[ x = \begin{bmatrix} -4.7124 \\ 0.0000 \end{bmatrix} \]

\[ \text{fval} = -2.0000 \]

**Pattern Search on Constrained Problems**

If your problem has constraints, use the syntax

\[
\begin{bmatrix} x \ fval \end{bmatrix} = \text{patternsearch}(\text{@objfun}, x_0, A, b, Aeq, beq, lb, ub, \text{nonlcon})
\]

where

- \( A \) is a matrix and \( b \) is vector that represent inequality constraints of the form \( A \cdot x \leq b \).
- \( Aeq \) is a matrix and \( beq \) is a vector that represent equality constraints of the form \( Aeq \cdot x = beq \).
- \( lb \) and \( ub \) are vectors representing bound constraints of the form \( lb \leq x \) and \( x \leq ub \), respectively.
- \( \text{nonlcon} \) is a function that returns the nonlinear equality and inequality vectors, \( c \) and \( ceq \), respectively. The function is minimized such that \( c(x) \leq 0 \) and \( ceq(x) = 0 \).

You only need to pass in the constraints that are part of the problem. For example, if there are no bound constraints or a nonlinear constraint function, use the syntax

\[
\begin{bmatrix} x \ fval \end{bmatrix} = \text{patternsearch}(\text{@objfun}, x_0, A, b, Aeq, beq)
\]

Use empty brackets [ ] for constraint arguments that are not needed for the problem. For example, if there are no inequality constraints or a nonlinear constraint function, use the syntax

\[
\begin{bmatrix} x \ fval \end{bmatrix} = \text{patternsearch}(\text{@objfun}, x_0, [], [], Aeq, beq, lb, ub)
\]
Additional Output Arguments
To get more information about the performance of the pattern search, you can call `patternsearch` with the syntax

\[
[x\ fval\ exitflag\ output] = \text{patternsearch}(@objfun,x0)
\]

Besides \(x\) and \(fval\), this returns the following additional output arguments:

- `exitflag` — Integer indicating whether the algorithm was successful
- `output` — Structure containing information about the performance of the solver

For more information about these arguments, see the `patternsearch` reference page.

Setting Options for `patternsearch` at the Command Line
You can specify any available `patternsearch` options by passing an `options` structure as an input argument to `patternsearch` using the syntax

\[
[x\ fval] = \text{patternsearch}(@fitnessfun,nvars, ... \nonlcon,options)
\]

Pass in empty brackets `[]` for any constraints that do not appear in the problem.

You create the `options` structure using the function `psoptimset`.

\[
\text{options} = \text{psoptimset}(@\text{patternsearch})
\]

This returns the `options` structure with the default values for its fields.

\[
\text{options} =
\begin{align*}
\text{TolMesh} & : 1.0000e-006 \\
\text{TolCon} & : 1.0000e-006 \\
\text{TolX} & : 1.0000e-006 \\
\text{TolFun} & : 1.0000e-006 \\
\text{TolBind} & : 1.0000e-003 \\
\text{MaxIter} & : '100*\text{numberofvariables}'
\end{align*}
\]
Using Direct Search

MaxFunEvals: '2000*numberofvariables'
TimeLimit: Inf
MeshContraction: 0.5000
MeshExpansion: 2
MeshAccelerator: 'off'
MeshRotate: 'on'
InitialMeshSize: 1
ScaleMesh: 'on'
MaxMeshSize: Inf
InitialPenalty: 10
PenaltyFactor: 100
PollMethod: 'gpspositivebasis2n'
CompletePoll: 'off'
PollingOrder: 'consecutive'
SearchMethod: []
CompleteSearch: 'off'
Display: 'final'
OutputFcns: []
PlotFcns: []
PlotInterval: 1
Cache: 'off'
CacheSize: 10000
CacheTol: 2.2204e-016
Vectorized: 'off'
UseParallel: 'never'

The patternsearch function uses these default values if you do not pass
in options as an input argument.

The value of each option is stored in a field of the options structure, such as
options.MeshExpansion. You can display any of these values by entering
options followed by the name of the field. For example, to display the mesh
expansion factor for the pattern search, enter

options.MeshExpansion

ans =
  2
To create an options structure with a field value that is different from the default, use \texttt{psoptimset}. For example, to change the mesh expansion factor to 3 instead of its default value 2, enter

\begin{verbatim}
options = psoptimset('MeshExpansion',3);
\end{verbatim}

This creates the options structure with all values set to empty except for \texttt{MeshExpansion}, which is set to 3. (An empty field causes \texttt{patternsearch} to use the default value.)

If you now call \texttt{patternsearch} with the argument \texttt{options}, the pattern search uses a mesh expansion factor of 3.

If you subsequently decide to change another field in the options structure, such as setting \texttt{PlotFcns} to \texttt{@psplotmeshsize}, which plots the mesh size at each iteration, call \texttt{psoptimset} with the syntax

\begin{verbatim}
options = psoptimset(options, 'PlotFcns', @psplotmeshsize)
\end{verbatim}

This preserves the current values of all fields of options except for \texttt{PlotFcns}, which is changed to \texttt{@plotmeshsize}. Note that if you omit the options input argument, \texttt{psoptimset} resets \texttt{MeshExpansion} to its default value, which is 2.

You can also set both \texttt{MeshExpansion} and \texttt{PlotFcns} with the single command

\begin{verbatim}
options = psoptimset('MeshExpansion',3,'PlotFcns',@plotmeshsize)
\end{verbatim}

### Using Options and Problems from the Optimization Tool

As an alternative to creating the options structure using \texttt{psoptimset}, you can set the values of options in the Optimization Tool and then export the options to a structure in the MATLAB workspace, as described in the “Importing and Exporting Your Work” section of the Optimization Toolbox documentation. If you export the default options in the Optimization Tool, the resulting options structure has the same settings as the default structure returned by the command

\begin{verbatim}
options = psoptimset
\end{verbatim}

except for the default value of 'Display', which is 'final' when created by \texttt{psoptimset}, but 'none' when created in the Optimization Tool.
You can also export an entire problem from the Optimization Tool and run it from the command line. Enter

```
patternsearch(problem)
```

where `problem` is the name of the exported problem.
Pattern Search Examples: Setting Options

In this section...

“Poll Method” on page 4-49
“Complete Poll” on page 4-51
“Example: Comparing the Efficiency of Poll Options” on page 4-55
“Using a Search Method” on page 4-62
“Mesh Expansion and Contraction” on page 4-66
“Mesh Accelerator” on page 4-72
“Using Cache” on page 4-75
“Setting Tolerances for the Solver” on page 4-79
“Constrained Minimization Using patternsearch” on page 4-80
“Vectorizing the Objective and Constraint Functions” on page 4-83

Note All examples use the generalized pattern search (GPS) algorithm, but can be applied to the other algorithms as well.

Poll Method

At each iteration, the pattern search polls the points in the current mesh—that is, it computes the objective function at the mesh points to see if there is one whose function value is less than the function value at the current point. “How Pattern Search Polling Works” on page 4-14 provides an example of polling. You can specify the pattern that defines the mesh by the Poll method option. The default pattern, GPS Positive basis 2N, consists of the following 2N directions, where N is the number of independent variables for the objective function.

\[
\begin{bmatrix}
1 & 0 & 0 & \ldots & 0 \\
0 & 1 & 0 & \ldots & 0 \\
& & & & \\
0 & 0 & 0 & \ldots & 1 \\
-1 & 0 & 0 & \ldots & 0 \\
\end{bmatrix}
\]
[0 -1 0...0]
[0 0 0...-1].

For example, if the objective function has three independent variables, the GPS Positive basis 2N, consists of the following six vectors.

[1 0 0]
[0 1 0]
[0 0 1]
[-1 0 0]
[0 -1 0]
[0 0 -1].

Alternatively, you can set Poll method to GPS Positive basis NP1, the pattern consisting of the following $N + 1$ directions.

[1 0 0...0]
[0 1 0...0]
...
[0 0 0...1]
[-1 -1 -1...-1].

For example, if objective function has three independent variables, the GPS Positive basis Np1, consists of the following four vectors.

[1 0 0]
[0 1 0]
[0 0 1]
[-1 -1 -1].

A pattern search will sometimes run faster using GPS Positive basis Np1 rather than the GPS Positive basis 2N as the Poll method, because the algorithm searches fewer points at each iteration. Although not being addressed in this example, the same is true when using the MADS Positive basis Np1 over the MADS Positive basis 2N, and similarly for GSS. For example, if you run a pattern search on the example described in “Example: A Linearly Constrained Problem” on page 4-33, the algorithm performs 1588 function evaluations with GPS Positive basis 2N, the default Poll method, but only 877 function evaluations using GPS Positive basis Np1. For more detail, see “Example: Comparing the Efficiency of Poll Options” on page 4-55.
However, if the objective function has many local minima, using GPS Positive basis 2N as the Poll method might avoid finding a local minimum that is not the global minimum, because the search explores more points around the current point at each iteration.

**Complete Poll**

By default, if the pattern search finds a mesh point that improves the value of the objective function, it stops the poll and sets that point as the current point for the next iteration. When this occurs, some mesh points might not get polled. Some of these unpolled points might have an objective function value that is even lower than the first one the pattern search finds.

For problems in which there are several local minima, it is sometimes preferable to make the pattern search poll all the mesh points at each iteration and choose the one with the best objective function value. This enables the pattern search to explore more points at each iteration and thereby potentially avoid a local minimum that is not the global minimum.

In the Optimization Tool you can make the pattern search poll the entire mesh setting Complete poll to On in Poll options. At the command line, use psoptimset to set the CompletePoll option to 'on'.

**Example: Using a Complete Poll in a Generalized Pattern Search**

As an example, consider the following function.

\[
    f(x_1, x_2) = \begin{cases} 
    x_1^2 + x_2^2 - 25 & \text{for } x_1^2 + x_2^2 \leq 25 \\
    x_1^2 + (x_2 - 9)^2 - 16 & \text{for } x_1^2 + (x_2 - 9)^2 \leq 16 \\
    0 & \text{otherwise.}
    \end{cases}
\]

The following figure shows a plot of the function.
The global minimum of the function occurs at (0, 0), where its value is -25. However, the function also has a local minimum at (0, 9), where its value is -16.

To create a file that computes the function, copy and paste the following code into a new file in the MATLAB Editor.

```matlab
function z = poll_example(x)
if x(1)^2 + x(2)^2 <= 25
    z = x(1)^2 + x(2)^2 - 25;
elseif x(1)^2 + (x(2) - 9)^2 <= 16
    z = x(1)^2 + (x(2) - 9)^2 - 16;
else
    z = 0;
end
```

Then save the file as `poll_example.m` in a folder on the MATLAB path.
To run a pattern search on the function, enter the following in the Optimization Tool:

- Set **Solver** to `patternsearch`.
- Set **Objective function** to `@poll_example`.
- Set **Start point** to `[0 5]`.
- Set **Level of display** to **Iterative** in the Display to command window options.

Click **Start** to run the pattern search with **Complete poll** set to **Off**, its default value. The Optimization Tool displays the results in the **Run solver and view results** pane, as shown in the following figure.

The pattern search returns the local minimum at (0, 9). At the initial point, (0, 5), the objective function value is 0. At the first iteration, the search polls the following mesh points.

\[
f((0, 5) + (1, 0)) = f(1, 5) = 0
\]

\[
f((0, 5) + (0, 1)) = f(0, 6) = -7
\]

As soon as the search polls the mesh point (0, 6), at which the objective function value is less than at the initial point, it stops polling the current mesh and sets the current point at the next iteration to (0, 6). Consequently,
the search moves toward the local minimum at \((0, 9)\) at the first iteration. You see this by looking at the first two lines of the command line display.

<table>
<thead>
<tr>
<th>Iter</th>
<th>f-count</th>
<th>f(x)</th>
<th>MeshSize</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>-7</td>
<td>2</td>
<td>Successful Poll</td>
</tr>
</tbody>
</table>

Note that the pattern search performs only two evaluations of the objective function at the first iteration, increasing the total function count from 1 to 3.

Next, set **Complete poll** to **On** and click **Start**. The **Run solver and view results** pane displays the following results.

![Run solver and view results](image)

This time, the pattern search finds the global minimum at \((0, 0)\). The difference between this run and the previous one is that with **Complete poll** set to **On**, at the first iteration the pattern search polls all four mesh points.

\[
\begin{align*}
  f((0, 5) + (1, 0)) &= f(1, 5) = 0 \\
  f((0, 5) + (0, 1)) &= f(0, 6) = -6 \\
  f((0, 5) + (-1, 0)) &= f(-1, 5) = 0 \\
  f((0, 5) + (0, -1)) &= f(0, 4) = -9
\end{align*}
\]
Because the last mesh point has the lowest objective function value, the pattern search selects it as the current point at the next iteration. The first two lines of the command-line display show this.

<table>
<thead>
<tr>
<th>Iter</th>
<th>f-count</th>
<th>f(x)</th>
<th>MeshSize</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>-9</td>
<td>2</td>
<td>Successful Poll</td>
</tr>
</tbody>
</table>

In this case, the objective function is evaluated four times at the first iteration. As a result, the pattern search moves toward the global minimum at (0, 0).

The following figure compares the sequence of points returned when Complete poll is set to Off with the sequence when Complete poll is On.

---

**Example: Comparing the Efficiency of Poll Options**

This example shows how several poll options interact in terms of iterations and total function evaluations. The main results are:
• GSS is more efficient than GPS or MADS for linearly constrained problems.

• Whether setting CompletePoll to 'on' increases efficiency or decreases efficiency is unclear, although it affects the number of iterations.

• Similarly, whether having a $2N$ poll is more or less efficient than having an $Np1$ poll is also unclear. The most efficient poll is GSS Positive Basis $Np1$ with Complete poll set to on. The least efficient is MADS Positive Basis $Np1$ with Complete poll set to on.

**Note** The efficiency of an algorithm depends on the problem. GSS is efficient for linearly constrained problems. However, predicting the efficiency implications of the other poll options is difficult, as is knowing which poll type works best with other constraints.

**Problem setup**
The problem is the same as in “Performing a Pattern Search on the Example” on page 4-34. This linearly constrained problem uses the lincontest7 file that comes with the toolbox:

1 Enter the following into your MATLAB workspace:

   ```matlab
   x0 = [2 1 0 9 1 0];
   Aineq = [-8 7 3 -4 9 0];
   bineq = 7;
   Aeq = [7 1 8 3 3 3; 5 0 -5 -1 -5 8; -2 -6 7 1 1 9; 1 -1 2 -2 -3 -3];
   beq = [84 62 65 1];
   ```

2 Open the Optimization Tool by entering `optimtool` at the command line.

3 Choose the `patternsearch` solver.

4 Enter the problem and constraints as pictured.
5 Ensure that the Poll method is GPS Positive basis 2N.

Generate the Results

1 Run the optimization.

2 Choose File > Export to Workspace.
3 Export the results to a structure named gps2noff.

4 Set Options > Poll > Complete poll to on.
5 Run the optimization.

6 Export the result to a structure named gps2non.

7 Set **Options > Poll > Poll method** to **GPS Positive basis Np1** and set **Complete poll** to off.

8 Run the optimization.

9 Export the result to a structure namedgpsnp1off.

10 Set **Complete poll** to on.

11 Run the optimization.

12 Export the result to a structure named gpsnp1on.

13 Continue in a like manner to create solution structures for the other poll methods with **Complete poll** set on and off: **gss2noff, gss2non, gssnp1off, gssnp1on, mads2noff, mads2non, madsnp1off,** and **madsnp1on**.

**Examine the Results**

You have the results of 12 optimization runs. The following table shows the efficiency of the runs, measured in total function counts and in iterations. Your MADS results could differ, since MADS is a stochastic algorithm.
<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Function Count</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS2N, complete poll off</td>
<td>1206</td>
<td>106</td>
</tr>
<tr>
<td>GPS2N, complete poll on</td>
<td>1362</td>
<td>94</td>
</tr>
<tr>
<td>GPSNp1, complete poll off</td>
<td>914</td>
<td>122</td>
</tr>
<tr>
<td>GPSNp1, complete poll on</td>
<td>991</td>
<td>106</td>
</tr>
<tr>
<td>GSS2N, complete poll off</td>
<td>683</td>
<td>74</td>
</tr>
<tr>
<td>GSS2N, complete poll on</td>
<td>889</td>
<td>74</td>
</tr>
<tr>
<td>GSSNp1, complete poll off</td>
<td>529</td>
<td>94</td>
</tr>
<tr>
<td>GSSNp1, complete poll on</td>
<td>519</td>
<td>74</td>
</tr>
<tr>
<td>MADS2N, complete poll off</td>
<td>907</td>
<td>159</td>
</tr>
<tr>
<td>MADS2N, complete poll on</td>
<td>1292</td>
<td>160</td>
</tr>
<tr>
<td>MADSNp1, complete poll off</td>
<td>1215</td>
<td>208</td>
</tr>
<tr>
<td>MADSNp1, complete poll on</td>
<td>1714</td>
<td>207</td>
</tr>
</tbody>
</table>

To obtain, say, the first row in the table, enter `gps2noff.output.funccount` and `gps2noff.output.iterations`. You can also examine a structure in the Variable Editor by double-clicking the structure in the Workspace Browser, and then double-clicking the output structure.
The main results gleaned from the table are:

- Setting **Complete poll** to on generally lowers the number of iterations for GPS and GSS, but the change in number of function evaluations is unpredictable.
- Setting **Complete poll** to on does not change the number of iterations for MADS, but substantially increases the number of function evaluations.
- The most efficient algorithm/options settings, with efficiency meaning lowest function count:
Using Direct Search

1. GSS Positive basis Np1 with Complete poll set to on (function count 519)

2. GSS Positive basis Np1 with Complete poll set to off (function count 529)

3. GSS Positive basis 2N with Complete poll set to off (function count 683)

4. GSS Positive basis 2N with Complete poll set to on (function count 889)

The other poll methods had function counts exceeding 900.

- For this problem, the most efficient poll is GSS Positive Basis Np1 with Complete poll set to on, although the Complete poll setting makes only a small difference. The least efficient poll is MADS Positive Basis Np1 with Complete poll set to on. In this case, the Complete poll setting makes a substantial difference.

Using a Search Method

In addition to polling the mesh points, the pattern search algorithm can perform an optional step at every iteration, called search. At each iteration, the search step applies another optimization method to the current point. If this search does not improve the current point, the poll step is performed.

The following example illustrates the use of a search method on the problem described in “Example: A Linearly Constrained Problem” on page 4-33. To set up the example, enter the following commands at the MATLAB prompt to define the initial point and constraints.

```matlab
x0 = [2 1 0 9 1 0];
Aineq = [-8 7 3 -4 9 0];
bineq = 7;
Aeq = [7 1 8 3 3 3; 5 0 -5 1 -5 8; -2 -6 7 1 1 9; 1 -1 2 -2 3 -3];
beq = [84 62 65 1];
```

Then enter the settings shown in the following figures in the Optimization Tool.
For comparison, click **Start** to run the example without a search method. This displays the plots shown in the following figure.
To see the effect of using a search method, select MADS Positive Basis 2N in the Search method field in Search options.
This sets the search method to be a pattern search using the pattern for MADS Positive Basis 2N. Then click Start to run the pattern search. This displays the following plots.

Note that using the search method reduces the total function evaluations—from 1206 to 1159—and reduces the number of iterations from 106 to 97.
Mesh Expansion and Contraction

The Expansion factor and Contraction factor options, in Mesh options, control how much the mesh size is expanded or contracted at each iteration. With the default Expansion factor value of 2, the pattern search multiplies the mesh size by 2 after each successful poll. With the default Contraction factor value of 0.5, the pattern search multiplies the mesh size by 0.5 after each unsuccessful poll.

You can view the expansion and contraction of the mesh size during the pattern search by selecting Mesh size in the Plot functions pane. To also display the values of the mesh size and objective function at the command line, set Level of display to Iterative in the Display to command window options.

For example, set up the problem described in “Example: A Linearly Constrained Problem” on page 4-33 as follows:

1 Enter the following at the command line:

   ```
   x0 = [2 1 0 9 1 0];
   Aineq = [-8 7 3 -4 9 0];
   bineq = 7;
   Aeq = [7 1 8 3 3 3; 5 0 -5 1 -5 8; -2 -6 7 1 1 9; 1 -1 2 -2 3 -3];
   beq = [84 62 65 1];
   ```

2 Set up your problem in the Optimization Tool to match the following figures.
3 Run the optimization.
Using Direct Search

The Optimization Tool displays the following plot.

To see the changes in mesh size more clearly, change the y-axis to logarithmic scaling as follows:

1 Select **Axes Properties** from the **Edit** menu in the plot window.
2 In the Properties Editor, select the **Y Axis** tab.
3 Set **Scale** to **Log**.

Updating these settings in the MATLAB Property Editor will show the plot in the following figure.
The first 5 iterations result in successful polls, so the mesh sizes increase steadily during this time. You can see that the first unsuccessful poll occurs at iteration 6 by looking at the command-line display:

<table>
<thead>
<tr>
<th>Iter</th>
<th>f-count</th>
<th>f(x)</th>
<th>MeshSize</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2434.15</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2419.37</td>
<td>2</td>
<td>Successful Poll</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2392.12</td>
<td>4</td>
<td>Successful Poll</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2346.85</td>
<td>8</td>
<td>Successful Poll</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>2293.23</td>
<td>16</td>
<td>Successful Poll</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>1921.94</td>
<td>32</td>
<td>Successful Poll</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>1921.94</td>
<td>16</td>
<td>Refine Mesh</td>
</tr>
</tbody>
</table>

Note that at iteration 5, which is successful, the mesh size doubles for the next iteration. But at iteration 6, which is unsuccessful, the mesh size is multiplied by 0.5.

To see how Expansion factor and Contraction factor affect the pattern search, make the following changes:
• Set **Expansion factor** to 3.0.
• Set **Contraction factor** to 0.75.

Then click **Start**. The **Run solver and view results** pane shows that the final point is approximately the same as with the default settings of **Expansion factor** and **Contraction factor**, but that the pattern search takes longer to reach that point.
When you change the scaling of the y-axis to logarithmic, the mesh size plot appears as shown in the following figure.
4 Using Direct Search

Note that the mesh size increases faster with Expansion factor set to 3.0, as compared with the default value of 2.0, and decreases more slowly with Contraction factor set to 0.75, as compared with the default value of 0.5.

Mesh Accelerator

The mesh accelerator can make a pattern search converge faster to an optimal point by reducing the number of iterations required to reach the mesh tolerance. When the mesh size is below a certain value, the pattern search contracts the mesh size by a factor smaller than the Contraction factor factor. Mesh accelerator applies only to the GPS and GSS algorithms.

Note For best results, use the mesh accelerator for problems in which the objective function is not too steep near the optimal point, or you might lose some accuracy. For differentiable problems, this means that the absolute value of the derivative is not too large near the solution.
To use the mesh accelerator, set **Accelerator** to **on** in the **Mesh** options. Or, at the command line, set the `MeshAccelerator` option to `on`.

For example, set up the problem described in “Example: A Linearly Constrained Problem” on page 4-33 as follows:

1. Enter the following at the command line:

   ```matlab
   x0 = [2 1 0 9 1 0];
   Aineq = [-8 7 3 -4 9 0];
   bineq = 7;
   Aeq = [7 1 8 3 3 3; 5 0 -5 1 -5 8; -2 -6 7 1 1 9; 1 -1 2 -2 3 3 -3];
   beq = [84 62 65 1];
   ```

2. Set up your problem in the Optimization Tool to match the following figures.

![Optimization Tool](image-url)
3 Run the optimization.

The number of iterations required to reach the mesh tolerance is 61, as compared with 74 when Accelerator is set to Off.

You can see the effect of the mesh accelerator by setting Level of display to Iterative in Display to command window. Run the example with Accelerator set to On, and then run it again with Accelerator set to Off. The mesh sizes are the same until iteration 56, but differ at iteration 57. The MATLAB Command Window displays the following lines for iterations 56 and 57 with Accelerator set to Off.

<table>
<thead>
<tr>
<th>Iter</th>
<th>f-count</th>
<th>f(x)</th>
<th>MeshSize</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>56</td>
<td>509</td>
<td>1919.54</td>
<td>6.104e-005</td>
<td>Refine Mesh</td>
</tr>
<tr>
<td>57</td>
<td>521</td>
<td>1919.54</td>
<td>3.052e-005</td>
<td>Refine Mesh</td>
</tr>
</tbody>
</table>

Note that the mesh size is multiplied by 0.5, the default value of Contraction factor.

For comparison, the Command Window displays the following lines for the same iteration numbers with Accelerator set to On.
Pattern Search Examples: Setting Options

<table>
<thead>
<tr>
<th>Iter</th>
<th>f-count</th>
<th>f(x)</th>
<th>MeshSize</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>56</td>
<td>509</td>
<td>1919.54</td>
<td>6.104e-005</td>
<td>Refine Mesh</td>
</tr>
<tr>
<td>57</td>
<td>521</td>
<td>1919.54</td>
<td>1.526e-005</td>
<td>Refine Mesh</td>
</tr>
</tbody>
</table>

In this case the mesh size is multiplied by 0.25.

**Using Cache**

Typically, at any given iteration of a pattern search, some of the mesh points might coincide with mesh points at previous iterations. By default, the pattern search recomputes the objective function at these mesh points even though it has already computed their values and found that they are not optimal. If computing the objective function takes a long time—say, several minutes—this can make the pattern search run significantly longer.

You can eliminate these redundant computations by using a cache, that is, by storing a history of the points that the pattern search has already visited. To do so, set `Cache` to `On` in `Cache` options. At each poll, the pattern search checks to see whether the current mesh point is within a specified tolerance, `Tolerance`, of a point in the cache. If so, the search does not compute the objective function for that point, but uses the cached function value and moves on to the next point.

**Note** When `Cache` is set to `On`, the pattern search might fail to identify a point in the current mesh that improves the objective function because it is within the specified tolerance of a point in the cache. As a result, the pattern search might run for more iterations with `Cache` set to `On` than with `Cache` set to `Off`. It is generally a good idea to keep the value of `Tolerance` very small, especially for highly nonlinear objective functions.

For example, set up the problem described in “Example: A Linearly Constrained Problem” on page 4-33 as follows:

1. Enter the following at the command line:

   ```matlab
   x0 = [2 1 0 9 1 0];
   Aineq = [-8 7 3 -4 9 0];
   bineq = 7;
   ```
Set up your problem in the Optimization Tool to match the following figures.
3 Run the optimization.

After the pattern search finishes, the plots appear as shown in the following figure.
Using Direct Search

Note that the total function count is 683.

Now, set Cache to On and run the example again. This time, the plots appear as shown in the following figure.
This time, the total function count is reduced to 664.

**Setting Tolerances for the Solver**

Tolerance refers to how small a parameter, such as a mesh size, can become before the search is halted or changed in some way. You can specify the value of the following tolerances:

- **Mesh tolerance** — When the current mesh size is less than the value of **Mesh tolerance**, the algorithm halts.
• **X tolerance** — After a successful poll, if the distance from the previous best point to the current best point is less than the value of **X tolerance**, the algorithm halts.

• **Function tolerance** — After a successful poll, if the difference between the function value at the previous best point and function value at the current best point is less than the value of **Function tolerance**, the algorithm halts.

• **Nonlinear constraint tolerance** — The algorithm treats a point to be feasible if constraint violation is less than **TolCon**.

• **Bind tolerance** — Bind tolerance applies to linearly constrained problems. It specifies how close a point must get to the boundary of the feasible region before a linear constraint is considered to be active. When a linear constraint is active, the pattern search polls points in directions parallel to the linear constraint boundary as well as the mesh points.

Usually, you should set **Bind tolerance** to be at least as large as the maximum of **Mesh tolerance**, **X tolerance**, and **Function tolerance**.

**Constrained Minimization Using patternsearch**

Suppose you want to minimize the simple objective function of two variables $x_1$ and $x_2$,

$$\min_x f(x) = \left(4 - 2.1x_1^2 - x_1^{4/3}\right)x_1^2 + x_1x_2 + \left(-4 + 4x_2^2\right)x_2^2$$

subject to the following nonlinear inequality constraints and bounds

$$x_1x_2 + x_1 - x_2 + 1.5 \leq 0 \quad \text{(nonlinear constraint)}$$

$$10 - x_1x_2 \leq 0 \quad \text{(nonlinear constraint)}$$

$$0 \leq x_1 \leq 1 \quad \text{(bound)}$$

$$0 \leq x_2 \leq 13 \quad \text{(bound)}$$

Begin by creating the objective and constraint functions. First, create a file named **simple_objective.m** as follows:

```matlab
function y = simple_objective(x)
    y = (4 - 2.1*x(1)^2 - x(1)^{4/3})*x(1)^2 + x(1)*x(2) + (-4 + 4*x(2)^2)*x(2)^2;
```
The pattern search solver assumes the objective function will take one input \( x \) where \( x \) has as many elements as number of variables in the problem. The objective function computes the value of the function and returns that scalar value in its one return argument \( y \).

Then create a file named `simple_constraint.m` containing the constraints:

```matlab
function \[ c, ceq \] = simple_constraint(x)
c = [1.5 + x(1)*x(2) + x(1) - x(2);
    -x(1)*x(2) + 10];
ceq = [];
```

The pattern search solver assumes the constraint function will take one input \( x \), where \( x \) has as many elements as the number of variables in the problem. The constraint function computes the values of all the inequality and equality constraints and returns two vectors, \( c \) and \( ceq \), respectively.

Next, to minimize the objective function using the `patternsearch` function, you need to pass in a function handle to the objective function as well as specifying a start point as the second argument. Lower and upper bounds are provided as \( LB \) and \( UB \) respectively. In addition, you also need to pass a function handle to the nonlinear constraint function.

```matlab
ObjectiveFunction = @simple_objective;
X0 = [0 0]; % Starting point
LB = [0 0]; % Lower bound
UB = [1 13]; % Upper bound
ConstraintFunction = @simple_constraint;
[x,fval] = patternsearch(ObjectiveFunction,X0,[],[],[],...
    LB,UB,ConstraintFunction)
```

Optimization terminated: mesh size less than options.TolMesh and constraint violation is less than options.TolCon.

\[
\begin{align*}
x &= \begin{bmatrix} 0.8122 & 12.3122 \end{bmatrix} \\
\text{fval} &= 9.1324e+004
\end{align*}
\]
Next, plot the results. Create an options structure using `psoptimset` that selects two plot functions. The first plot function `psplotbestf` plots the best objective function value at every iteration. The second plot function `psplotmaxconstr` plots the maximum constraint violation at every iteration.

**Note** You can also visualize the progress of the algorithm by displaying information to the Command Window using the 'Display' option.

```plaintext
options = psoptimset('PlotFcns',{@psplotbestf,@psplotmaxconstr},'Display','iter');
[x,fval] = patternsearch(ObjectiveFunction,X0,[],[],[],[],LB,UB,ConstraintFunction,options)
```

max
Iter f-count f(x) constraint MeshSize Method
0 1 0 10 0.8919
1 28 113580 0 0.001 Increase penalty
2 105 91324 1.782e-007 1e-005 Increase penalty
3 192 91324 1.188e-011 1e-007 Increase penalty

Optimization terminated: mesh size less than options.TolMesh
and constraint violation is less than options.TolCon.

x =
0.8122 12.3122

fval =
9.1324e+004
Vectorizing the Objective and Constraint Functions

Direct search often runs faster if you vectorize the objective and nonlinear constraint functions. This means your functions evaluate all the points in a poll or search pattern at once, with one function call, without having to loop through the points one at a time. Therefore, the option `Vectorize = 'on'` works only when `CompletePoll` or `CompleteSearch` are also set to 'on'.

If there are nonlinear constraints, the objective function and the nonlinear constraints all need to be vectorized in order for the algorithm to compute in a vectorized manner.
Note  Write your vectorized objective function or nonlinear constraint function to accept a matrix with an arbitrary number of points. \texttt{patternsearch} sometimes evaluates a single point even during a vectorized calculation.

Vectorized Objective Function

A vectorized objective function accepts a matrix as input and generates a vector of function values, where each function value corresponds to one row or column of the input matrix. \texttt{patternsearch} resolves the ambiguity in whether the rows or columns of the matrix represent the points of a pattern as follows. Suppose the input matrix has $m$ rows and $n$ columns:

- If the initial point $x_0$ is a column vector of size $m$, the objective function takes each column of the matrix as a point in the pattern and returns a vector of size $n$.
- If the initial point $x_0$ is a row vector of size $n$, the objective function takes each row of the matrix as a point in the pattern and returns a vector of size $m$.
- If the initial point $x_0$ is a scalar, the matrix has one row ($m = 1$, the matrix is a vector), and each entry of the matrix represents one point to evaluate.

Pictorially, the matrix and calculation are represented by the following figure.
**Structure of Vectorized Functions**

For example, suppose the objective function is

\[
f(x) = x_1^4 + x_2^4 - 4x_1^2 - 2x_2^2 + 3x_1 - x_2 / 2.
\]

If the initial vector \(x_0\) is a column vector, such as \([0;0]\), a function for vectorized evaluation is

```matlab
function f = vectorizedc(x)
    f = x(1,:).^4+x(2,:).^4-4*x(1,:).^2-2*x(2,:).^2 ... 
    +3*x(1,:)-.5*x(2,:);
```

If \(x_0\) is a row vector, each row represents one search/poll point. Similarly, if \(x_0\) is a column vector, each column represents one search/poll point.
If the initial vector \( x_0 \) is a row vector, such as \([0,0]\), a function for vectorized evaluation is

```
function f = vectorizedr(x)

    f = x(:,1).^4+x(:,2).^4-4*x(:,1).^2-2*x(:,2).^2 ...
        +3*x(:,1)-.5*x(:,2);
```

If you want to use the same objective (fitness) function for both pattern search and genetic algorithm, write your function to have the points represented by row vectors, and write \( x_0 \) as a row vector. The genetic algorithm always takes individuals as the rows of a matrix. This was a design decision—the genetic algorithm does not require a user-supplied population, so needs to have a default format.

To minimize \( \text{vectorizedc} \), enter the following commands:

```
options=psoptimset('Vectorized','on','CompletePoll','on');
x0=[0;0];
[x fval]=patternsearch(@vectorizedc,x0,...
    [],[],[],[],[],[],[],options)
```

MATLAB returns the following output:

```
Optimization terminated: mesh size less than options.TolMesh.

x =
    -1.5737
    1.0575

fval =
    -10.0088
```

**Vectorized Constraint Functions**

Only nonlinear constraints need to be vectorized; bounds and linear constraints are handled automatically. If there are nonlinear constraints, the objective function and the nonlinear constraints all need to be vectorized in order for the algorithm to compute in a vectorized manner.
The same considerations hold for constraint functions as for objective functions: the initial point \( x_0 \) determines the type of points (row or column vectors) in the poll or search. If the initial point is a row vector of size \( k \), the matrix \( x \) passed to the constraint function has \( k \) columns. Similarly, if the initial point is a column vector of size \( k \), the matrix of poll or search points has \( k \) rows. The figure Structure of Vectorized Functions on page 4-85 may make this clear.

Your nonlinear constraint function returns two matrices, one for inequality constraints, and one for equality constraints. Suppose there are \( n_c \) nonlinear inequality constraints and \( n_{ceq} \) nonlinear equality constraints. For row vector \( x_0 \), the constraint matrices have \( n_c \) and \( n_{ceq} \) columns respectively, and the number of rows is the same as in the input matrix. Similarly, for a column vector \( x_0 \), the constraint matrices have \( n_c \) and \( n_{ceq} \) rows respectively, and the number of columns is the same as in the input matrix. In figure Structure of Vectorized Functions on page 4-85, “Results” includes both \( n_c \) and \( n_{ceq} \).

**Example of Vectorized Objective and Constraints**

Suppose that the nonlinear constraints are

\[
\frac{x_1^2}{9} + \frac{x_2^2}{4} \leq 1 \text{ (the interior of an ellipse)},
\]

\[
x_2 \geq \cosh(x_1) - 1.
\]

Write a function for these constraints for row-form \( x_0 \) as follows:

```matlab
function [c, ceq] = ellipsecosh(x)
    c(:,1)=x(:,1).^2/9+x(:,2).^2/4-1;
    c(:,2)=cosh(x(:,1))-x(:,2)-1;
    ceq=[];
```

Minimize \texttt{vectorizedr} (defined in “Vectorized Objective Function” on page 4-84) subject to the constraints \texttt{ellipsecosh}:

```matlab
x0=[0,0];
options=psoptimset('Vectorized','on','CompletePoll','on');
[x fval]=patternsearch(@vectorizedr,x0,...
    [],[],[],[],[],@ellipsecosh,options)
```
MATLAB returns the following output:

Optimization terminated: mesh size less than options.TolMesh
and constraint violation is less than options.TolCon.

\[
x = \\
-1.3516 \quad 1.0612
\]

\[
fval = \\
-9.5394
\]
Using the Genetic Algorithm

- “What Is the Genetic Algorithm?” on page 5-2
- “Performing a Genetic Algorithm Optimization” on page 5-3
- “Example: Rastrigin’s Function” on page 5-8
- “Some Genetic Algorithm Terminology” on page 5-18
- “How the Genetic Algorithm Works” on page 5-21
- “Mixed Integer Optimization” on page 5-28
- “Description of the Nonlinear Constraint Solver” on page 5-38
- “Genetic Algorithm Optimizations Using the Optimization Tool GUI” on page 5-40
- “Using the Genetic Algorithm from the Command Line” on page 5-50
- “Genetic Algorithm Examples” on page 5-60
What Is the Genetic Algorithm?

The genetic algorithm is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The genetic algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm selects individuals at random from the current population to be parents and uses them to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution. You can apply the genetic algorithm to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, nondifferentiable, stochastic, or highly nonlinear. The genetic algorithm can address problems of mixed integer programming, where some components are restricted to be integer-valued.

The genetic algorithm uses three main types of rules at each step to create the next generation from the current population:

- **Selection rules** select the individuals, called parents, that contribute to the population at the next generation.
- **Crossover rules** combine two parents to form children for the next generation.
- **Mutation rules** apply random changes to individual parents to form children.

The genetic algorithm differs from a classical, derivative-based, optimization algorithm in two main ways, as summarized in the following table.

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<th>Genetic Algorithm</th>
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<td>Generates a single point at each iteration. The sequence of points approaches an optimal solution.</td>
<td>Generates a population of points at each iteration. The best point in the population approaches an optimal solution.</td>
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<tr>
<td>Selects the next point in the sequence by a deterministic computation.</td>
<td>Selects the next population by computation which uses random number generators.</td>
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Performing a Genetic Algorithm Optimization

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Calling the Function ga at the Command Line

To use the genetic algorithm at the command line, call the genetic algorithm function ga with the syntax

\[
[x \text{ } \text{fval}] = \text{ga}(\text{@fitnessfun}, \text{nvars}, \text{options})
\]

where

- @fitnessfun is a handle to the fitness function.
- nvars is the number of independent variables for the fitness function.
- options is a structure containing options for the genetic algorithm. If you do not pass in this argument, ga uses its default options.

The results are given by

- \(x\) — Point at which the final value is attained
- \(fval\) — Final value of the fitness function

Using the function ga is convenient if you want to

- Return results directly to the MATLAB workspace
- Run the genetic algorithm multiple times with different options, by calling ga from a file

“Using the Genetic Algorithm from the Command Line” on page 5-50 provides a detailed description of using the function ga and creating the options structure.
Using the Optimization Tool

To open the Optimization Tool, enter

\[ \text{optimtool('ga')} \]

at the command line, or enter \texttt{optimtool} and then choose \texttt{ga} from the \texttt{Solver} menu.
Performing a Genetic Algorithm Optimization

You can also start the tool from the MATLAB **Start** menu as pictured:
To use the Optimization Tool, you must first enter the following information:

- **Fitness function** — The objective function you want to minimize. Enter the fitness function in the form `@fitnessfun`, where `fitnessfun.m` is a file that computes the fitness function. “Computing Objective Functions” on page 2-2 explains how write this file. The `@` sign creates a function handle to `fitnessfun`. 
Performing a Genetic Algorithm Optimization

- **Number of variables** — The length of the input vector to the fitness function. For the function `my_fun` described in “Computing Objective Functions” on page 2-2, you would enter 2.

You can enter constraints or a nonlinear constraint function for the problem in the **Constraints** pane. If the problem is unconstrained, leave these fields blank.

To run the genetic algorithm, click the **Start** button. The tool displays the results of the optimization in the **Run solver and view results** pane.

You can change the options for the genetic algorithm in the **Options** pane. To view the options in one of the categories listed in the pane, click the + sign next to it.

For more information,

- See the “Graphical Optimization Tool” chapter in the Optimization Toolbox documentation.
- See “Example: Rastrigin’s Function” on page 5-8 for an example of using the tool.
Example: Rastrigin’s Function

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Rastrigin’s Function

This section presents an example that shows how to find the minimum of Rastrigin’s function, a function that is often used to test the genetic algorithm.

For two independent variables, Rastrigin’s function is defined as

\[ Ras(x) = 20 + x_1^2 + x_2^2 - 10(\cos 2\pi x_1 + \cos 2\pi x_2). \]

Global Optimization Toolbox software contains the `rastriginsfcn.m` file, which computes the values of Rastrigin’s function. The following figure shows a plot of Rastrigin’s function.
As the plot shows, Rastrigin’s function has many local minima—the “valleys” in the plot. However, the function has just one global minimum, which occurs at the point [0 0] in the x-y plane, as indicated by the vertical line in the plot, where the value of the function is 0. At any local minimum other than [0 0], the value of Rastrigin’s function is greater than 0. The farther the local minimum is from the origin, the larger the value of the function is at that point.

Rastrigin’s function is often used to test the genetic algorithm, because its many local minima make it difficult for standard, gradient-based methods to find the global minimum.

The following contour plot of Rastrigin’s function shows the alternating maxima and minima.
Finding the Minimum of Rastrigin’s Function

This section explains how to find the minimum of Rastrigin’s function using the genetic algorithm.

**Note** Because the genetic algorithm uses random number generators, the algorithm returns slightly different results each time you run it.

To find the minimum, do the following steps:

1. Enter `optimtool('ga')` at the command line to open the Optimization Tool.
2. Enter the following in the Optimization Tool:
In the **Fitness function** field, enter `@rastriginsfcn`.

In the **Number of variables** field, enter 2, the number of independent variables for Rastrigin’s function.

The **Fitness function** and **Number of variables** fields should appear as shown in the following figure.

![Fitness function and Number of variables fields](image)

3. Click the **Start** button in the **Run solver and view results** pane, as shown in the following figure.

![Run solver and view results pane](image)

While the algorithm is running, the **Current iteration** field displays the number of the current generation. You can temporarily pause the algorithm by clicking the **Pause** button. When you do so, the button name changes to **Resume**. To resume the algorithm from the point at which you paused it, click **Resume**.

When the algorithm is finished, the **Run solver and view results** pane appears as shown in the following figure. Your numerical results might differ from those in the figure, since `ga` is stochastic.
The display shows:

- The final value of the fitness function when the algorithm terminated:

  \[
  \text{Objective function value: } 0.05531602101322264
  \]

  Note that the value shown is very close to the actual minimum value of Rastrigin's function, which is \( 0 \). “Genetic Algorithm Examples” on page 5-60 describes some ways to get a result that is closer to the actual minimum.

- The reason the algorithm terminated.

  \[
  \text{Optimization terminated: average change in the fitness value less than options.TolFun.}
  \]

- The final point, which in this example is \([ 0.001 \ -0.017 ]\).

**Finding the Minimum from the Command Line**

To find the minimum of Rastrigin's function from the command line, enter

\[
[x \ fval \ exitflag] = \text{ga}(\text{rastriginsfcn}, \ 2)
\]

This returns

\[
\text{Optimization terminated: average change in the fitness value less than options.TolFun.}
\]
Example: Rastrigin’s Function

\[ x = \\
0.0229 \quad 0.0106 \]

\[ fval = \\
0.1258 \]

\[ exitflag = \\
1 \]

• \( x \) is the final point returned by the algorithm.
• \( fval \) is the fitness function value at the final point.
• \( exitflag \) is integer value corresponding to the reason that the algorithm terminated.

**Note** Because the genetic algorithm uses random number generators, the algorithm returns slightly different results each time you run it.

**Displaying Plots**

The Optimization Tool **Plot functions** pane enables you to display various plots that provide information about the genetic algorithm while it is running. This information can help you change options to improve the performance of the algorithm. For example, to plot the best and mean values of the fitness function at each generation, select the box next to **Best fitness**, as shown in the following figure.
When you click **Start**, the Optimization Tool displays a plot of the best and mean values of the fitness function at each generation.

Try this on “Example: Rastrigin’s Function” on page 5-8:

When the algorithm stops, the plot appears as shown in the following figure.
The points at the bottom of the plot denote the best fitness values, while the points above them denote the averages of the fitness values in each generation. The plot also displays the best and mean values in the current generation numerically at the top.

To get a better picture of how much the best fitness values are decreasing, you can change the scaling of the y-axis in the plot to logarithmic scaling. To do so,

1. Select **Axes Properties** from the **Edit** menu in the plot window to open the Property Editor attached to your figure window as shown below.
2 Click the **Y Axis** tab.

3 In the **Y Scale** pane, select **Log**.

   The plot now appears as shown in the following figure.
Typically, the best fitness value improves rapidly in the early generations, when the individuals are farther from the optimum. The best fitness value improves more slowly in later generations, whose populations are closer to the optimal point.
Some Genetic Algorithm Terminology

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**Fitness Functions**

The *fitness function* is the function you want to optimize. For standard optimization algorithms, this is known as the objective function. The toolbox software tries to find the minimum of the fitness function.

Write the fitness function as a file or anonymous function, and pass it as a function handle input argument to the main genetic algorithm function.

**Individuals**

An *individual* is any point to which you can apply the fitness function. The value of the fitness function for an individual is its score. For example, if the fitness function is

\[ f(x_1, x_2, x_3) = (2x_1 + 1)^2 + (3x_2 + 4)^2 + (x_3 - 2)^2, \]

the vector (2, -3, 1), whose length is the number of variables in the problem, is an individual. The score of the individual (2, -3, 1) is \( f(2, -3, 1) = 51 \).

An individual is sometimes referred to as a *genome* and the vector entries of an individual as *genes*. 
**Populations and Generations**

A *population* is an array of individuals. For example, if the size of the population is 100 and the number of variables in the fitness function is 3, you represent the population by a 100-by-3 matrix. The same individual can appear more than once in the population. For example, the individual (2, -3, 1) can appear in more than one row of the array.

At each iteration, the genetic algorithm performs a series of computations on the current population to produce a new population. Each successive population is called a new *generation*.

**Diversity**

*Diversity* refers to the average distance between individuals in a population. A population has high diversity if the average distance is large; otherwise it has low diversity. In the following figure, the population on the left has high diversity, while the population on the right has low diversity.

Diversity is essential to the genetic algorithm because it enables the algorithm to search a larger region of the space.
Fitness Values and Best Fitness Values
The fitness value of an individual is the value of the fitness function for that individual. Because the toolbox software finds the minimum of the fitness function, the best fitness value for a population is the smallest fitness value for any individual in the population.

Parents and Children
To create the next generation, the genetic algorithm selects certain individuals in the current population, called parents, and uses them to create individuals in the next generation, called children. Typically, the algorithm is more likely to select parents that have better fitness values.
Outline of the Algorithm

The following outline summarizes how the genetic algorithm works:

1 The algorithm begins by creating a random initial population.

2 The algorithm then creates a sequence of new populations. At each step, the algorithm uses the individuals in the current generation to create the next population. To create the new population, the algorithm performs the following steps:
   a Scores each member of the current population by computing its fitness value.
   b Scales the raw fitness scores to convert them into a more usable range of values.
   c Selects members, called parents, based on their fitness.
   d Some of the individuals in the current population that have lower fitness are chosen as *elite*. These elite individuals are passed to the next population.
   e Produces children from the parents. Children are produced either by making random changes to a single parent—*mutation*—or by combining the vector entries of a pair of parents—*crossover*.
   f Replaces the current population with the children to form the next generation.
3 The algorithm stops when one of the stopping criteria is met. See “Stopping Conditions for the Algorithm” on page 5-25.

**Initial Population**

The algorithm begins by creating a random initial population, as shown in the following figure.

In this example, the initial population contains 20 individuals, which is the default value of **Population size** in the **Population** options. Note that all the individuals in the initial population lie in the upper-right quadrant of the picture, that is, their coordinates lie between 0 and 1, because the default value of **Initial range** in the **Population** options is $[0;1]$. If you know approximately where the minimal point for a function lies, you should set **Initial range** so that the point lies near the middle of that range. For example, if you believe that the minimal point for Rastrigin’s function is near the point $[0\ 0]$, you could set **Initial range** to be $[-1;1]$. However, as this example shows, the genetic algorithm can find the minimum even with a less than optimal choice for **Initial range**.
**Creating the Next Generation**

At each step, the genetic algorithm uses the current population to create the children that make up the next generation. The algorithm selects a group of individuals in the current population, called *parents*, who contribute their *genes*—the entries of their vectors—to their children. The algorithm usually selects individuals that have better fitness values as parents. You can specify the function that the algorithm uses to select the parents in the `Selection function` field in the *Selection* options.

The genetic algorithm creates three types of children for the next generation:

- *Elite children* are the individuals in the current generation with the best fitness values. These individuals automatically survive to the next generation.
- *Crossover children* are created by combining the vectors of a pair of parents.
- *Mutation children* are created by introducing random changes, or mutations, to a single parent.

The following schematic diagram illustrates the three types of children.

```
| Elite child | Crossover child | Mutation child |
```

![Diagram of children creation](image-url)
“Mutation and Crossover” on page 5-74 explains how to specify the number of children of each type that the algorithm generates and the functions it uses to perform crossover and mutation.

The following sections explain how the algorithm creates crossover and mutation children.

### Crossover Children
The algorithm creates crossover children by combining pairs of parents in the current population. At each coordinate of the child vector, the default crossover function randomly selects an entry, or gene, at the same coordinate from one of the two parents and assigns it to the child.

### Mutation Children
The algorithm creates mutation children by randomly changing the genes of individual parents. By default, the algorithm adds a random vector from a Gaussian distribution to the parent.

The following figure shows the children of the initial population, that is, the population at the second generation, and indicates whether they are elite, crossover, or mutation children.
**Plots of Later Generations**

The following figure shows the populations at iterations 60, 80, 95, and 100.

As the number of generations increases, the individuals in the population get closer together and approach the minimum point \([0\ 0]\).

**Stopping Conditions for the Algorithm**

The genetic algorithm uses the following conditions to determine when to stop:

- **Generations** — The algorithm stops when the number of generations reaches the value of Generations.
• **Time limit** — The algorithm stops after running for an amount of time in seconds equal to **Time limit**.

• **Fitness limit** — The algorithm stops when the value of the fitness function for the best point in the current population is less than or equal to **Fitness limit**.

• **Stall generations** — The algorithm stops when the weighted average change in the fitness function value over **Stall generations** is less than **Function tolerance**.

• **Stall time limit** — The algorithm stops if there is no improvement in the objective function during an interval of time in seconds equal to **Stall time limit**.

• **Function Tolerance** — The algorithm runs until the weighted average change in the fitness function value over **Stall generations** is less than **Function tolerance**.

• **Nonlinear constraint tolerance** — The **Nonlinear constraint tolerance** is not used as stopping criterion. It is used to determine the feasibility with respect to nonlinear constraints.

The algorithm stops as soon as any one of these conditions is met. You can specify the values of these criteria in the **Stopping criteria** pane in the Optimization Tool. The default values are shown in the pane.
When you run the genetic algorithm, the Run solver and view results panel displays the criterion that caused the algorithm to stop.

The options Stall time limit and Time limit prevent the algorithm from running too long. If the algorithm stops due to one of these conditions, you might improve your results by increasing the values of Stall time limit and Time limit.
Mixed Integer Optimization

In this section...

“Solving Mixed Integer Optimization Problems” on page 5-28
“Characteristics of the Integer ga Solver” on page 5-30
“Integer ga Algorithm” on page 5-36

Solving Mixed Integer Optimization Problems

ga can solve problems when certain variables are integer-valued. Give IntCon, a vector of the x components that are integers:

\[
[x, fval, exitflag] = \text{ga}(fitnessfcn, nvars, A, b, [], [], ... \\
\text{lb, ub, nonlcon, IntCon, options})
\]

IntCon is a vector of positive integers that contains the x components that are integer-valued. For example, if you want to restrict x(2) and x(10) to be integers, set IntCon to [2, 10].

Note  Restrictions exist on the types of problems that ga can solve with integer variables. In particular, ga does not accept any equality constraints when there are integer variables. For details, see “Characteristics of the Integer ga Solver” on page 5-30.

Tip  ga solves integer problems best when you provide lower and upper bounds for every x component.

Example: Mixed Integer Optimization of Rastrigin’s Function

Suppose you want to find the minimum of Rastrigin’s function as in “Example: Rastrigin’s Function” on page 5-8, but you want to restrict the first component of x to be an integer. Suppose further that you want to look only in the region \(5\pi \leq x(1) \leq 20\pi, -20\pi \leq x(2) \leq -4\pi\).

1 Set up the bounds for your problem:
lb = [5*pi,-20*pi];
ub = [20*pi,-4*pi];

2 Set a plot function so you can view the progress of ga:

opts = gaoptimset('PlotFcns',@gaplotbestf);

3 Call the ga solver where x(1) has integer values:

IntCon = 1;
[x,fval,exitflag] = ga(@rastriginsfcn,2,[],[],[],[],[],lb,ub,[],IntCon,opts)

Optimization terminated: average change in the
penalty fitness value less than options.TolFun
and constraint violation is less than options.TolCon.

x =
    16.0000   -12.9325

fval =
    424.1355

exitflag =
    1
ga converges quickly to the solution.

**Characteristics of the Integer ga Solver**

There are some restrictions on the types of problems that ga can solve when you include integer constraints:

- No linear equality constraints. You must have $A_{eq} = []$ and $beq = []$. For a possible workaround, see “No Equality Constraints” on page 5-31.

- No nonlinear equality constraints. Any nonlinear constraint function must return [] for the nonlinear equality constraint. For a possible workaround, see “Example: Integer Programming with a Nonlinear Equality Constraint” on page 5-32.

- Only doubleVector population type.

- No custom creation function (CreationFcn option), crossover function (CrossoverFcn option), mutation function (MutationFcn option), or initial
scores (*InitialScores* option). If you supply any of these, *ga* overrides their settings.

- *ga* uses only the binary tournament selection function (*SelectionFcn* option), and overrides any other setting.
- No hybrid function. *ga* overrides any setting of the *HybridFcn* option.
- *ga* ignores the *ParetoFraction*, *DistanceMeasureFcn*, *InitialPenalty*, and *PenaltyFactor* options.

The listed restrictions are mainly natural, not arbitrary. For example:

- There are no hybrid functions that support integer constraints. So *ga* does not use hybrid functions when there are integer constraints.
- To obtain integer variables, *ga* uses special creation, crossover, and mutation functions.

**No Equality Constraints**

You cannot use equality constraints and integer constraints in the same problem. You can try to work around this restriction by including two inequality constraints for each linear equality constraint. For example, to try to include the constraint

\[ 3x_1 - 2x_2 = 5, \]

create two inequality constraints:

\[ 3x_1 - 2x_2 \leq 5 \]
\[ 3x_1 - 2x_2 \geq 5. \]

To write these constraints in the form \( Ax \leq b \), multiply the second inequality by -1:

\[ -3x_1 + 2x_2 \leq -5. \]

You can try to include the equality constraint using \( A = [3, -2; -3, 2] \) and \( b = [5; -5] \).
Be aware that this procedure can fail; ga has difficulty with simultaneous integer and equality constraints.

**Example: Integer Programming with a Nonlinear Equality Constraint.**

This example attempts to locate the minimum of the Ackley function in five dimensions with these constraints:

- $x(1)$, $x(3)$, and $x(5)$ are integers.
- $\|\mathbf{x}\| = 4$.

The Ackley function, described briefly in “Example: Resuming the Genetic Algorithm from the Final Population” on page 5-45, is difficult to minimize. Adding integer and equality constraints increases the difficulty.

To include the nonlinear equality constraint, give a small tolerance $\texttt{tol}$ that allows the norm of $\mathbf{x}$ to be within $\texttt{tol}$ of 4. Without a tolerance, the nonlinear equality constraint is never satisfied, and the solver does not realize when it has a feasible solution.

1. Write the expression $\|\mathbf{x}\| = 4$ as two “less than zero” inequalities:

   \[
   \begin{align*}
   \|\mathbf{x}\| - 4 & \leq 0 \\
   -(\|\mathbf{x}\| - 4) & \leq 0.
   \end{align*}
   \]

2. Allow a small tolerance in the inequalities:

   \[
   \begin{align*}
   \|\mathbf{x}\| - 4 - \texttt{tol} & \leq 0 \\
   -(\|\mathbf{x}\| - 4) - \texttt{tol} & \leq 0.
   \end{align*}
   \]

3. Write a nonlinear inequality constraint function that implements these inequalities:

   ```matlab
   function [c, ceq] = eqCon(x)
   
   ceq = [];
   rad = 4;
   tol = 1e-3;
   confcnval = norm(x) - rad;
   c = [confcnval - tol; -confcnval - tol];
   ```
4 Set options:

- StallGenLimit = 50 — Allow the solver to try for a while.
- TolFun = 1e-10 — Specify a stricter stopping criterion than usual.
- Generations = 300 — Allow more generations than default.
- PlotFcns = @gaplotbestfun — Observe the optimization.

```matlab
opts = gaoptimset('StallGenLimit',50,'TolFun',1e-10,...
                 'Generations',300,'PlotFcns',@gaplotbestfun);
```

5 Set lower and upper bounds to help the solver:

```matlab
nVar = 5;
lb = -5*ones(1,nVar);
ub = 5*ones(1,nVar);
```

6 Solve the problem:

```matlab
rng(2,'twister') % for reproducibility
[x,fval,exitflag] = ga(@ackleyfcn,nVar,[],[],[],[],...
        lb,ub,@eqCon,[1 3 5],opts);
```

Optimization terminated: average change in the penalty fitness
value less than options.TolFun and constraint violation is
less than options.TolCon.
Using the Genetic Algorithm

Examine the solution:

\[ x, fval, \text{exitflag}, \text{norm}(x) \]

\[ x = \begin{bmatrix} 0 & 0.1943 & -2.0000 & -1.7203 & 3.0000 \end{bmatrix} \]

\[ fval = 6.0542 \]

\[ \text{exitflag} = 1 \]

\[ \text{ans} = 3.9996 \]
The odd $x$ components are integers, as specified. The norm of $x$ is 4, to within the given tolerance of 1e-3.

8 Despite the exit flag of 1, the solution is not the global optimum. Run the problem again and examine the solution:

```matlab
opts = gaoptimset(opts,'Display','off');
[x2,fval2,exitflag2] = ga(@ackleyfcn,nVar,[],[],[],[],...,lb,ub,@eqCon,[1 3 5],opts);
```

Examine the second solution:

```matlab
x2,fval2,exitflag2,norm(x2)
```

```matlab
x2 =  
    -1.0000   2.9991   2.0000   0.9987  -1.0000
```
fval2 =
  4.2361

exitflag2 =
  1

ans =
  3.9990

The second run gives a better solution (lower fitness function value). Again, the odd $x$ components are integers, and the norm of $x_2$ is 4, to within the given tolerance of $1e-3$.

Be aware that this procedure can fail; `ga` has difficulty with simultaneous integer and equality constraints.

**Integer `ga` Algorithm**

Integer programming with `ga` involves several modifications of the basic algorithm (see “How the Genetic Algorithm Works” on page 5-21). For integer programming:

- Special creation, crossover, and mutation functions enforce variables to be integers. For details, see Deep et al. [2].

- The genetic algorithm attempts to minimize a penalty function, not the fitness function. The penalty function includes a term for infeasibility. This penalty function is combined with binary tournament selection to select individuals for subsequent generations. The penalty function value of a member of a population is:
  - If the member is feasible, the penalty function is the fitness function.
  - If the member is infeasible, the penalty function is the maximum fitness function among feasible members of the population, plus a sum of the constraint violations of the (infeasible) point.

For details of the penalty function, see Deb [1].

Description of the Nonlinear Constraint Solver

The genetic algorithm uses the Augmented Lagrangian Genetic Algorithm (ALGA) to solve nonlinear constraint problems without integer constraints. The optimization problem solved by the ALGA algorithm is

$$\min_{x} f(x)$$

such that

$$c_i(x) \leq 0, \ i = 1 \ldots m$$
$$ceq_i(x) = 0, \ i = m + 1 \ldots mt$$
$$A \cdot x \leq b$$
$$Aeq \cdot x = beq$$
$$lb \leq x \leq ub,$$

where \(c(x)\) represents the nonlinear inequality constraints, \(ceq(x)\) represents the equality constraints, \(m\) is the number of nonlinear inequality constraints, and \(mt\) is the total number of nonlinear constraints.

The Augmented Lagrangian Genetic Algorithm (ALGA) attempts to solve a nonlinear optimization problem with nonlinear constraints, linear constraints, and bounds. In this approach, bounds and linear constraints are handled separately from nonlinear constraints. A subproblem is formulated by combining the fitness function and nonlinear constraint function using the Lagrangian and the penalty parameters. A sequence of such optimization problems are approximately minimized using the genetic algorithm such that the linear constraints and bounds are satisfied.

A subproblem formulation is defined as

$$\Theta(x, \lambda, s, \rho) = f(x) - \sum_{i=1}^{m} \lambda_i s_i \log(s_i - c_i(x)) + \sum_{i=m+1}^{mt} \lambda_i ceq_i(x) + \frac{\rho}{2} \sum_{i=m+1}^{mt} ceq_i(x)^2,$$

where
• The components \( \lambda_i \) of the vector \( \lambda \) are nonnegative and are known as Lagrange multiplier estimates

• The elements \( s_i \) of the vector \( s \) are nonnegative shifts

• \( \rho \) is the positive penalty parameter.

The algorithm begins by using an initial value for the penalty parameter (InitialPenalty).

The genetic algorithm minimizes a sequence of subproblems, each of which is an approximation of the original problem. Each subproblem has a fixed value of \( \lambda \), \( s \), and \( \rho \). When the subproblem is minimized to a required accuracy and satisfies feasibility conditions, the Lagrangian estimates are updated. Otherwise, the penalty parameter is increased by a penalty factor (PenaltyFactor). This results in a new subproblem formulation and minimization problem. These steps are repeated until the stopping criteria are met.

Each subproblem solution represents one generation. The number of function evaluations per generation is therefore much higher when using nonlinear constraints than otherwise.

For a complete description of the algorithm, see the following references:


Genetic Algorithm Optimizations Using the Optimization Tool GUI

In this section...

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Introduction

The Optimization Tool GUI is described in the chapter Optimization Tool in the Optimization Toolbox documentation. This section describes some places where there are some differences between the use of the genetic algorithm in the Optimization Tool and the use of other solvers.

Displaying Plots

The Plot functions pane, shown in the following figure, enables you to display various plots of the results of the genetic algorithm.

Select the check boxes next to the plots you want to display.
For example, to set up the example described in “Example: Rastrigin’s Function” on page 5-8, set the Optimization Tool as pictured.

When you run the problem, you see plots similar to those shown in the following figure.

The upper plot displays the best and mean fitness values in each generation. The lower plot displays the coordinates of the point with the best fitness value in the current generation.

**Note** When you display more than one plot, you can open a larger version of a plot in a separate window. Right-click (Ctrl-click for Mac) on a blank area in a plot while ga is running, or after it has stopped, and choose the sole menu item.
“Plot Options” on page 9-32 describes the types of plots you can create.

**Example: Creating a Custom Plot Function**

If none of the plot functions that come with the software is suitable for the output you want to plot, you can write your own custom plot function, which the genetic algorithm calls at each generation to create the plot. This example shows how to create a plot function that displays the change in the best fitness value from the previous generation to the current generation.

This section covers the following topics:

- “Creating the Custom Plot Function” on page 5-42
- “Using the Plot Function” on page 5-43
- “How the Plot Function Works” on page 5-44

**Creating the Custom Plot Function**

To create the plot function for this example, copy and paste the following code into a new file in the MATLAB Editor:

```matlab
function state = gaplotchange(options, state, flag)
% GAPLOTCHANGE Plots the logarithmic change in the best score from the
% previous generation.
% persistent last_best % Best score in the previous generation

if(strcmp(flag,'init')) % Set up the plot
    set(gca,'xlim',[1,options.Generations],'Yscale','log');
    hold on;
    xlabel 'Generation'
    title('Change in Best Fitness Value')
end

best = min(state.Score); % Best score in the current generation
if state.Generation == 0 % Set last_best to best.
    last_best = best;
else
    change = last_best - best; % Change in best score
```

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last_best=best;
plot(state.Generation, change, '.r');
title(['Change in Best Fitness Value'])
end

Then save the file as gaplotchange.m in a folder on the MATLAB path.

**Using the Plot Function**

To use the custom plot function, select **Custom** in the **Plot functions** pane and enter `@gaplotchange` in the field to the right. To compare the custom plot with the best fitness value plot, also select **Best fitness**. Now, if you run the example described in “Example: Rastrigin’s Function” on page 5-8, the tool displays plots similar to those shown in the following figure.

![Plot Examples](image)

Best: 0.0020445 Mean: 0.033263

Note that because the scale of the y-axis in the lower custom plot is logarithmic, the plot only shows changes that are greater than 0. The logarithmic scale enables you to see small changes in the fitness function that the upper plot does not reveal.
How the Plot Function Works

The plot function uses information contained in the following structures, which the genetic algorithm passes to the function as input arguments:

- **options** — The current options settings
- **state** — Information about the current generation
- **flag** — String indicating the current status of the algorithm

The most important lines of the plot function are the following:

- **persistent last_best**
  
  Creates the persistent variable `last_best`—the best score in the previous generation. Persistent variables are preserved over multiple calls to the plot function.

- **set(gca,'xlim',[1,options.Generations],'Yscale','log');**
  
  Sets up the plot before the algorithm starts. `options.Generations` is the maximum number of generations.

- **best = min(state.Score)**
  
  The field `state.Score` contains the scores of all individuals in the current population. The variable `best` is the minimum score. For a complete description of the fields of the structure `state`, see “Structure of the Plot Functions” on page 9-34.

- **change = last_best - best**
  
  The variable `change` is the best score at the previous generation minus the best score in the current generation.

- **plot(state.Generation, change, '.r')**
  
  Plots the change at the current generation, whose number is contained in `state.Generation`.

The code for `gaplotchange` contains many of the same elements as the code for `gaplotbestf`, the function that creates the best fitness plot.
Reproducing Your Results
To reproduce the results of the last run of the genetic algorithm, select the **Use random states from previous run** check box. This resets the states of the random number generators used by the algorithm to their previous values. If you do not change any other settings in the Optimization Tool, the next time you run the genetic algorithm, it returns the same results as the previous run.

Normally, you should leave **Use random states from previous run** unselected to get the benefit of randomness in the genetic algorithm. Select the **Use random states from previous run** check box if you want to analyze the results of that particular run or show the exact results to others. After the algorithm has run, you can clear your results using the **Clear Status** button in the **Run solver** settings.

**Note** If you select **Include information needed to resume this run**, then selecting **Use random states from previous run** has no effect on the initial population created when you import the problem and run the genetic algorithm on it. The latter option is only intended to reproduce results from the beginning of a new run, not from a resumed run.

Example: Resuming the Genetic Algorithm from the Final Population
The following example shows how export a problem so that when you import it and click **Start**, the genetic algorithm resumes from the final population saved with the exported problem. To run the example, enter the following in the Optimization Tool:

1 Set **Fitness function** to @ackleyfcn, which computes Ackley’s function, a test function provided with the software.

2 Set **Number of variables** to 10.

3 Select **Best fitness** in the **Plot functions** pane.

4 Click **Start**.

This displays the following plot.
Using the Genetic Algorithm

Suppose you want to experiment by running the genetic algorithm with other options settings, and then later restart this run from its final population with its current options settings. You can do this using the following steps:

1. Click **Export to Workspace**.

2. In the dialog box that appears,
   - Select **Export problem and options to a MATLAB structure named**.
   - Enter a name for the problem and options, such as `ackley_uniform`, in the text field.
   - Select **Include information needed to resume this run**.

The dialog box should now appear as in the following figure.
3 Click OK.

This exports the problem and options to a structure in the MATLAB workspace. You can view the structure in the MATLAB Command Window by entering

```matlab
ackley_uniform
```

```matlab
ackley_uniform =
    fitnessfcn: @ackleyfcn
    nvars: 10
    Aineq: []
    bineq: []
    Aeq: []
    beq: []
    lb: []
    ub: []
    nonlcon: []
    rngstate: []
    solver: 'ga'
    options: [1x1 struct]
```

After running the genetic algorithm with different options settings or even a different fitness function, you can restore the problem as follows:
1 Select **Import Problem** from the **File** menu. This opens the dialog box shown in the following figure.

![Import Optimization Problem dialog box](image)

2 Select **ackley_uniform**.

3 Click **Import**.

This sets the **Initial population** and **Initial scores** fields in the **Population** panel to the final population of the run before you exported the problem. All other options are restored to their setting during that run. When you click **Start**, the genetic algorithm resumes from the saved final population. The following figure shows the best fitness plots from the original run and the restarted run.
**Note** If, after running the genetic algorithm with the imported problem, you want to restore the genetic algorithm’s default behavior of generating a random initial population, delete the population in the **Initial population** field.

The version of Ackley’s function in the toolbox differs from the published version of Ackley’s function in Ackley [1]. The toolbox version has another exponential applied, leading to flatter regions, so a more difficult optimization problem.

Using the Genetic Algorithm from the Command Line

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Running ga with the Default Options

To run the genetic algorithm with the default options, call ga with the syntax

\[
[x \ fval] = \text{ga}(\text{@fitnessfun, nvars})
\]

The input arguments to ga are

- \text{@fitnessfun} — A function handle to the file that computes the fitness function. “Computing Objective Functions” on page 2-2 explains how to write this file.
- \text{nvars} — The number of independent variables for the fitness function.

The output arguments are

- \text{x} — The final point
- \text{fval} — The value of the fitness function at x

For a description of additional input and output arguments, see the reference page for ga.

You can run the example described in “Example: Rastrigin’s Function” on page 5-8 from the command line by entering

\[
[x \ fval] = \text{ga}(\text{@rastriginsfcn, 2})
\]
Using the Genetic Algorithm from the Command Line

This returns

\[ x = \\
\begin{align*}
0.0027 & \quad -0.0052 \\
\end{align*}
\]

\[ fval = \\
0.0068 \\
\]

**Additional Output Arguments**
To get more information about the performance of the genetic algorithm, you can call `ga` with the syntax

\[
[x \ fval \ exitflag \ output \ population \ scores] = \text{ga}(@fitnessfcn, nvars)
\]

Besides `x` and `fval`, this function returns the following additional output arguments:

- **exitflag** — Integer value corresponding to the reason the algorithm terminated
- **output** — Structure containing information about the performance of the algorithm at each generation
- **population** — Final population
- **scores** — Final scores

See the `ga` reference page for more information about these arguments.

**Setting Options for ga at the Command Line**
You can specify any of the options that are available for `ga` by passing an options structure as an input argument to `ga` using the syntax

\[
[x \ fval] = \text{ga}(@fitnessfun, nvars, [],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[],[], [],
\]

This syntax does not specify any linear equality, linear inequality, or nonlinear constraints.

You create the options structure using the function `gaoptimset`. 
options = gaoptimset(@ga)

This returns the structure options with the default values for its fields.

options =
    PopulationType: 'doubleVector'
    PopInitRange: [2x1 double]
    PopulationSize: 20
    EliteCount: 2
    CrossoverFraction: 0.8000
    ParetoFraction: []
    MigrationDirection: 'forward'
    MigrationInterval: 20
    MigrationFraction: 0.2000
    Generations: 100
    TimeLimit: Inf
    FitnessLimit: -Inf
    StallGenLimit: 50
    StallTimeLimit: Inf
    TolFun: 1.00000e-006
    TolCon: 1.00000e-006
    InitialPopulation: []
    InitialScores: []
    InitialPenalty: 10
    PenaltyFactor: 100
    PlotInterval: 1
    CreationFcn: @gacreationuniform
    FitnessScalingFcn: @fitscalingrank
    SelectionFcn: @selectionstochunif
    CrossoverFcn: @crossoverscattered
    MutationFcn: {[1x1 function_handle] [1] [1]}
    DistanceMeasureFcn: []
    HybridFcn: []
    Display: 'final'
    PlotFcns: []
    OutputFcns: []
    Vectorized: 'off'
    UseParallel: 'never'
The function `ga` uses these default values if you do not pass in options as an input argument.

The value of each option is stored in a field of the options structure, such as `options.PopulationSize`. You can display any of these values by entering `options. followed by the name of the field. For example, to display the size of the population for the genetic algorithm, enter

```matlab
options.PopulationSize
```

```matlab
ans =
20
```

To create an options structure with a field value that is different from the default — for example to set `PopulationSize` to 100 instead of its default value 20 — enter

```matlab
options = gaoptimset('PopulationSize', 100)
```

This creates the options structure with all values set to their defaults except for `PopulationSize`, which is set to 100.

If you now enter,

```matlab
ga(@fitnessfun,nvars,[],[],[],[],[],[],options)
```

ga runs the genetic algorithm with a population size of 100.

If you subsequently decide to change another field in the options structure, such as setting `PlotFcns` to `@gaplotbestf`, which plots the best fitness function value at each generation, call `gaoptimset` with the syntax

```matlab
options = gaoptimset(options, 'PlotFcns', @plotbestf)
```

This preserves the current values of all fields of `options` except for `PlotFcns`, which is changed to `@plotbestf`. Note that if you omit the input argument `options`, `gaoptimset` resets `PopulationSize` to its default value 20.

You can also set both `PopulationSize` and `PlotFcns` with the single command

```matlab
options = gaoptimset('PopulationSize',100,'PlotFcns',@plotbestf)
```
Using Options and Problems from the Optimization Tool

As an alternative to creating an options structure using `gaoptimset`, you can set the values of options in the Optimization Tool and then export the options to a structure in the MATLAB workspace, as described in the “Importing and Exporting Your Work” section of the Optimization Toolbox documentation. If you export the default options in the Optimization Tool, the resulting structure `options` has the same settings as the default structure returned by the command

```
options = gaoptimset(@ga)
```

except that the option 'Display' defaults to 'off' in an exported structure, and is 'final' in the default at the command line.

If you export a problem structure, `ga_problem`, from the Optimization Tool, you can apply `ga` to it using the syntax

```
[x fval] = ga(ga_problem)
```

The problem structure contains the following fields:

- `fitnessfcn` — Fitness function
- `nvars` — Number of variables for the problem
- `Aineq` — Matrix for inequality constraints
- `Bineq` — Vector for inequality constraints
- `Aeq` — Matrix for equality constraints
- `Beq` — Vector for equality constraints
- `LB` — Lower bound on `x`
- `UB` — Upper bound on `x`
- `nonlcon` — Nonlinear constraint function
- `options` — Options structure
Reproducing Your Results

Because the genetic algorithm is stochastic—that is, it makes random choices—you get slightly different results each time you run the genetic algorithm. The algorithm uses the default MATLAB pseudorandom number stream. For more information about random number streams, see RandStream. Each time `ga` calls the stream, its state changes. So that the next time `ga` calls the stream, it returns a different random number. This is why the output of `ga` differs each time you run it.

If you need to reproduce your results exactly, you can call `ga` with an output argument that contains the current state of the default stream, and then reset the state to this value before running `ga` again. For example, to reproduce the output of `ga` applied to Rastrigin's function, call `ga` with the syntax

```matlab
[x fval exitflag output] = ga(@rastriginsfcn, 2);
```

Suppose the results are

```matlab
x =
  0.0027   -0.0052
fval =
  0.0068
```

The state of the stream is stored in `output.rngstate`:

```matlab
output =
    problemtype: 'unconstrained'
    rngstate: [1x1 struct]
    generations: 68
    funccount: 1380
    message: 'Optimization terminated: average change in the fitness value less than options.TolFun.'
```

To reset the state, enter

```matlab
stream = RandStream.getGlobalStream;
stream.State = output.rngstate.state;
```

If you now run `ga` a second time, you get the same results.
You can reproduce your run in the Optimization Tool by checking the box **Use random states from previous run** in the Run solver and view results section.

**Note** If you do not need to reproduce your results, it is better not to set the state of the stream, so that you get the benefit of the randomness in the genetic algorithm.

### Resuming ga from the Final Population of a Previous Run

By default, `ga` creates a new initial population each time you run it. However, you might get better results by using the final population from a previous run as the initial population for a new run. To do so, you must have saved the final population from the previous run by calling `ga` with the syntax

```matlab
[x,fval,exitflag,output,final_pop] = ga(@fitnessfcn, nvars);
```

The last output argument is the final population. To run `ga` using `final_pop` as the initial population, enter

```matlab
options = gaoptimset('InitialPop', final_pop);
[x,fval,exitflag,output,final_pop2] = ...
    ga(@fitnessfcn,nvars,[],[],[],[],[],[],options);
```

You can then use `final_pop2`, the final population from the second run, as the initial population for a third run.

In Optimization Tool, you can choose to export a problem in a way that lets you resume the run. Simply check the box **Include information needed to resume this run** when exporting the problem.
This saves the final population, which becomes the initial population when imported.

If you want to run a problem that was saved with the final population, but would rather not use the initial population, simply delete or otherwise change the initial population in the **Options > Population** pane.

### Running ga From a File

The command-line interface enables you to run the genetic algorithm many times, with different options settings, using a file. For example, you can run the genetic algorithm with different settings for **Crossover fraction** to see which one gives the best results. The following code runs the function *ga* 21 times, varying options.*CrossoverFraction* from 0 to 1 in increments of 0.05, and records the results.

```matlab
options = gaoptimset('Generations',300,'Display','none');
rng('default') % for reproducibility
record=[];
for n=0:.05:1
    options = gaoptimset(options,'CrossoverFraction',n);
    [x fval]=ga(@rastriginsfcn,2,[],[],[],[],[],[],[],options);
    record = [record; fval];
end
```

You can plot the values of `fval` against the crossover fraction with the following commands:
Using the Genetic Algorithm

plot(0:.05:1, record);
xlabel('Crossover Fraction');
ylabel('fval')

The following plot appears.

The plot suggests that you get the best results by setting 
options.CrossoverFraction to a value somewhere between 0.4 and 0.8.

You can get a smoother plot of fval as a function of the crossover fraction by 
running ga 20 times and averaging the values of fval for each crossover 
fraction. The following figure shows the resulting plot.
This plot also suggests the range of best choices for options.CrossoverFraction is 0.4 to 0.8.
Genetic Algorithm Examples

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Improving Your Results

To get the best results from the genetic algorithm, you usually need to experiment with different options. Selecting the best options for a problem involves start and error. This section describes some ways you can change options to improve results. For a complete description of the available options, see “Genetic Algorithm Options” on page 9-31.

Population Diversity

One of the most important factors that determines the performance of the genetic algorithm performs is the diversity of the population. If the average distance between individuals is large, the diversity is high; if the average distance is small, the diversity is low. Getting the right amount of diversity is
a matter of start and error. If the diversity is too high or too low, the genetic algorithm might not perform well.

This section explains how to control diversity by setting the Initial range of the population. “Setting the Amount of Mutation” on page 5-75 describes how the amount of mutation affects diversity.

This section also explains how to set the population size.

**Example: Setting the Initial Range**

By default, `ga` creates a random initial population using a creation function. You can specify the range of the vectors in the initial population in the Initial range field in Population options.

**Note** The initial range restricts the range of the points in the initial population by specifying the lower and upper bounds. Subsequent generations can contain points whose entries do not lie in the initial range. Set upper and lower bounds for all generations in the Bounds fields in the Constraints panel.

If you know approximately where the solution to a problem lies, specify the initial range so that it contains your guess for the solution. However, the genetic algorithm can find the solution even if it does not lie in the initial range, if the population has enough diversity.

The following example shows how the initial range affects the performance of the genetic algorithm. The example uses Rastrigin’s function, described in “Example: Rastrigin’s Function” on page 5-8. The minimum value of the function is 0, which occurs at the origin.

To run the example, open the ga solver in the Optimization Tool by entering `optimtool('ga')` at the command line. Set the following:

- Set Fitness function to @rastriginsfcn.
- Set Number of variables to 2.
- Select Best fitness in the Plot functions pane of the Options pane.
- Select **Distance** in the **Plot functions** pane.
- Set **Initial range** in the **Population** pane of the **Options** pane to \([1; 1.1]\).

Click **Start** in **Run solver and view results**. Although the results of genetic algorithm computations are random, your results are similar to the following figure, with a best fitness function value of approximately 2.

The upper plot, which displays the best fitness at each generation, shows little progress in lowering the fitness value. The lower plot shows the average distance between individuals at each generation, which is a good measure of the diversity of a population. For this setting of initial range, there is too little diversity for the algorithm to make progress.
Next, try setting Initial range to [1;100] and running the algorithm. This time the results are more variable. You might obtain a plot with a best fitness value of 35, as in the following plot. You might obtain different results.

![Fitness vs Generation](image1)

![Average Distance vs Generation](image2)

This time, the genetic algorithm makes progress, but because the average distance between individuals is so large, the best individuals are far from the optimal solution.

Finally, set Initial range to [1;2] and run the genetic algorithm. Again, there is variability in the result, but you might obtain a result similar to the following figure. Run the optimization several times, and you eventually obtain a final point near [0;0], with a fitness function value near 0.
Using the Genetic Algorithm

The diversity in this case is better suited to the problem, so `ga` usually returns a better result than in the previous two cases.

**Example: Linearly Constrained Population and Custom Plot Function**

This example shows how the default creation function for linearly constrained problems, `gacreationlinearfeasible`, creates a well-dispersed population that satisfies linear constraints and bounds. It also contains an example of a custom plot function.

The problem uses the objective function in `lincontest6.m`, a quadratic:
\[ f(x) = \frac{x_1^2}{2} + x_2^2 - x_1 x_2 - 2x_1 - 6x_2. \]

To see code for the function, enter `type lincontest6` at the command line.
The constraints are three linear inequalities:

\[
\begin{align*}
x_1 + x_2 & \leq 2, \\
-x_1 + 2x_2 & \leq 2, \\
2x_1 + x_2 & \leq 3.
\end{align*}
\]

Also, the variables \( x_i \) are restricted to be positive.

1. Create a custom plot function file by cutting and pasting the following code into a new function file in the MATLAB Editor:

```matlab
function state = gaplotshowpopulation2(unused,state,flag,fcn)
% This plot function works in 2-d only
if size(state.Population,2) > 2
    return;
end
if nargin < 4 % check to see if fitness function exists
    fcn = [];
end
% Dimensions to plot
dimensionsToPlot = [1 2];

switch flag
% Plot initialization
    case 'init'
        pop = state.Population(:,dimensionsToPlot);
        plotHandle = plot(pop(:,1),pop(:,2),'*');
        set(plotHandle,'Tag','gaplotshowpopulation2')
        title('Population plot in two dimension',...
            'interp','none')
        xlabelStr = sprintf('%s %s','Variable ',...
            num2str(dimensionsToPlot(1)))
        ylabelStr = sprintf('%s %s','Variable ',...
            num2str(dimensionsToPlot(2)))
        xlabel(xxlabelStr,'interp','none');
        ylabel(ylabelStr,'interp','none');
        hold on;
```
% plot the inequalities
plot([0 1.5],[2 0.5], 'm-.') % x1 + x2 <= 2
plot([0 1.5],[1 3.5/2], 'm-.'); % -x1 + 2*x2 <= 2
plot([0 1.5],[3 0], 'm-.'); % 2*x1 + x2 <= 3
% plot lower bounds
plot([0 0], [0 2], 'm-.'); % lb = [ 0 0];
plot([0 1.5], [0 0], 'm-.'); % lb = [ 0 0];
set(gca,'xlim',[-0.7,2.2])
set(gca,'ylim',[-0.7,2.7])

% Contour plot the objective function
if ~isempty(fcn) % if there is a fitness function
    range = [-0.5,2; -0.5,2];
    pts = 100;
    span = diff(range')/(pts - 1);
    x = range(1,1): span(1) : range(1,2);
    y = range(2,1): span(2) : range(2,2);

    pop = zeros(pts * pts,2);
    values = zeros(pts,1);
    k = 1;
    for i = 1:pts
        for j = 1:pts
            pop(k,:) = [x(i),y(j)];
            values(k) = fcn(pop(k,:));
            k = k + 1;
        end
    end
    values = reshape(values,pts,pts);
    contour(x,y,values);
    colorbar
end
% Pause for three seconds to view the initial plot
pause(3);
case 'iter'
    pop = state.Population(:,dimensionsToPlot);
    plotHandle = findobj(get(gca,'Children'),'Tag','gaplotshowpopulation2');
    set(plotHandle,'Xdata',pop(:,1),'Ydata',pop(:,2));
The custom plot function plots the lines representing the linear inequalities and bound constraints, plots level curves of the fitness function, and plots the population as it evolves. This plot function expects to have not only the usual inputs (options, state, flag), but also a function handle to the fitness function, @lincontest6 in this example. To generate level curves, the custom plot function needs the fitness function.

2 At the command line, enter the constraints as a matrix and vectors:

\[
A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}; \quad b = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}; \quad lb = \text{zeros}(2,1);
\]

3 Set options to use gaplotshowpopulation2, and pass in @lincontest6 as the fitness function handle:

\[
\text{options} = \text{gaoptimset}('\text{PlotFcn}',... \{[@\text{gaplotshowpopulation2,}@\text{lincontest6}]\});
\]

4 Run the optimization using options:

\[
[x,fval] = \text{ga}(@\text{lincontest6}, 2, A, b, [], [], lb, [], [], \text{options});
\]

A plot window appears showing the linear constraints, bounds, level curves of the objective function, and initial distribution of the population:
You can see that the initial population is biased to lie on the constraints.

The population eventually concentrates around the minimum point:
Setting the Population Size

The Population size field in Population options determines the size of the population at each generation. Increasing the population size enables the genetic algorithm to search more points and thereby obtain a better result. However, the larger the population size, the longer the genetic algorithm takes to compute each generation.

**Note** You should set Population size to be at least the value of Number of variables, so that the individuals in each population span the space being searched.

You can experiment with different settings for Population size that return good results without taking a prohibitive amount of time to run.
Fitness Scaling

Fitness scaling converts the raw fitness scores that are returned by the fitness function to values in a range that is suitable for the selection function. The selection function uses the scaled fitness values to select the parents of the next generation. The selection function assigns a higher probability of selection to individuals with higher scaled values.

The range of the scaled values affects the performance of the genetic algorithm. If the scaled values vary too widely, the individuals with the highest scaled values reproduce too rapidly, taking over the population gene pool too quickly, and preventing the genetic algorithm from searching other areas of the solution space. On the other hand, if the scaled values vary only a little, all individuals have approximately the same chance of reproduction and the search will progress very slowly.

The default fitness scaling option, **Rank**, scales the raw scores based on the rank of each individual instead of its score. The rank of an individual is its position in the sorted scores: the rank of the most fit individual is 1, the next most fit is 2, and so on. The rank scaling function assigns scaled values so that

- The scaled value of an individual with rank $n$ is proportional to $1/\sqrt{n}$.
- The sum of the scaled values over the entire population equals the number of parents needed to create the next generation.

Rank fitness scaling removes the effect of the spread of the raw scores.

The following plot shows the raw scores of a typical population of 20 individuals, sorted in increasing order.
The following plot shows the scaled values of the raw scores using rank scaling.

Because the algorithm minimizes the fitness function, lower raw scores have higher scaled values. Also, because rank scaling assigns values that depend
only on an individual’s rank, the scaled values shown would be the same for any population of size 20 and number of parents equal to 32.

**Comparing Rank and Top Scaling**

To see the effect of scaling, you can compare the results of the genetic algorithm using rank scaling with one of the other scaling options, such as Top. By default, top scaling assigns 40 percent of the fittest individuals to the same scaled value and assigns the rest of the individuals to value 0. Using the default selection function, only 40 percent of the fittest individuals can be selected as parents.

The following figure compares the scaled values of a population of size 20 with number of parents equal to 32 using rank and top scaling.

![Comparison of Rank and Top Scaling](image)

Because top scaling restricts parents to the fittest individuals, it creates less diverse populations than rank scaling. The following plot compares the
variances of distances between individuals at each generation using rank and top scaling.

### Selection

The selection function chooses parents for the next generation based on their scaled values from the fitness scaling function. An individual can be selected more than once as a parent, in which case it contributes its genes to more than one child. The default selection option, **Stochastic uniform**, lays out a line in which each parent corresponds to a section of the line of length proportional to its scaled value. The algorithm moves along the line in steps of equal size. At each step, the algorithm allocates a parent from the section it lands on.

A more deterministic selection option is **Remainder**, which performs two steps:

- In the first step, the function selects parents deterministically according to the integer part of the scaled value for each individual. For example, if an individual’s scaled value is 2.3, the function selects that individual twice as a parent.
• In the second step, the selection function selects additional parents using the fractional parts of the scaled values, as in stochastic uniform selection. The function lays out a line in sections, whose lengths are proportional to the fractional part of the scaled value of the individuals, and moves along the line in equal steps to select the parents.

Note that if the fractional parts of the scaled values all equal 0, as can occur using Top scaling, the selection is entirely deterministic.

**Reproduction Options**

Reproduction options control how the genetic algorithm creates the next generation. The options are

• **Elite count** — The number of individuals with the best fitness values in the current generation that are guaranteed to survive to the next generation. These individuals are called *elite children*. The default value of **Elite count** is 2.

  When **Elite count** is at least 1, the best fitness value can only decrease from one generation to the next. This is what you want to happen, since the genetic algorithm minimizes the fitness function. Setting **Elite count** to a high value causes the fittest individuals to dominate the population, which can make the search less effective.

• **Crossover fraction** — The fraction of individuals in the next generation, other than elite children, that are created by crossover. “Setting the Crossover Fraction” on page 5-77 describes how the value of **Crossover fraction** affects the performance of the genetic algorithm.

**Mutation and Crossover**

The genetic algorithm uses the individuals in the current generation to create the children that make up the next generation. Besides elite children, which correspond to the individuals in the current generation with the best fitness values, the algorithm creates

• Crossover children by selecting vector entries, or genes, from a pair of individuals in the current generation and combines them to form a child

• Mutation children by applying random changes to a single individual in the current generation to create a child
Both processes are essential to the genetic algorithm. Crossover enables the algorithm to extract the best genes from different individuals and recombine them into potentially superior children. Mutation adds to the diversity of a population and thereby increases the likelihood that the algorithm will generate individuals with better fitness values.

See “Creating the Next Generation” on page 5-23 for an example of how the genetic algorithm applies mutation and crossover.

You can specify how many of each type of children the algorithm creates as follows:

- **Elite count**, in Reproduction options, specifies the number of elite children.
- **Crossover fraction**, in Reproduction options, specifies the fraction of the population, other than elite children, that are crossover children.

For example, if the Population size is 20, the Elite count is 2, and the Crossover fraction is 0.8, the numbers of each type of children in the next generation are as follows:

- There are two elite children.
- There are 18 individuals other than elite children, so the algorithm rounds 0.8*18 = 14.4 to 14 to get the number of crossover children.
- The remaining four individuals, other than elite children, are mutation children.

**Setting the Amount of Mutation**

The genetic algorithm applies mutations using the option that you specify on the Mutation function pane. The default mutation option, Gaussian, adds a random number, or mutation, chosen from a Gaussian distribution, to each entry of the parent vector. Typically, the amount of mutation, which is proportional to the standard deviation of the distribution, decreases at each new generation. You can control the average amount of mutation that the algorithm applies to a parent in each generation through the Scale and Shrink options:
- **Scale** controls the standard deviation of the mutation at the first generation, which is **Scale** multiplied by the range of the initial population, which you specify by the **Initial range** option.

- **Shrink** controls the rate at which the average amount of mutation decreases. The standard deviation decreases linearly so that its final value equals $1 - \text{Shrink}$ times its initial value at the first generation. For example, if **Shrink** has the default value of 1, then the amount of mutation decreases to 0 at the final step.

You can see the effect of mutation by selecting the plot options **Distance** and **Range**, and then running the genetic algorithm on a problem such as the one described in “Example: Rastrigin’s Function” on page 5-8. The following figure shows the plot.

![Plot](image)

The upper plot displays the average distance between points in each generation. As the amount of mutation decreases, so does the average distance between individuals, which is approximately 0 at the final generation. The lower plot displays a vertical line at each generation, showing the range from the smallest to the largest fitness value, as well as mean fitness value. As the amount of mutation decreases, so does the range. These plots show
that reducing the amount of mutation decreases the diversity of subsequent generations.

For comparison, the following figure shows the plots for Distance and Range when you set Shrink to 0.5.

With Shrink set to 0.5, the average amount of mutation decreases by a factor of 1/2 by the final generation. As a result, the average distance between individuals decreases by approximately the same factor.

**Setting the Crossover Fraction**

The Crossover fraction field, in the Reproduction options, specifies the fraction of each population, other than elite children, that are made up of crossover children. A crossover fraction of 1 means that all children other than elite individuals are crossover children, while a crossover fraction of 0 means that all children are mutation children. The following example show that neither of these extremes is an effective strategy for optimizing a function.
The example uses the fitness function whose value at a point is the sum of the absolute values of the coordinates at the points. That is,

\[ f(x_1, x_2, ..., x_n) = |x_1| + |x_2| + \cdots + |x_n|. \]

You can define this function as an anonymous function by setting **Fitness function** to

\[ @(x) \text{sum}(\text{abs}(x)) \]

To run the example,

* Set **Fitness function** to @(x) sum(abs(x)).
* Set **Number of variables** to 10.
* Set **Initial range** to [-1; 1].
* Select **Best fitness** and **Distance** in the **Plot functions** pane.

Run the example with the default value of 0.8 for **Crossover fraction**, in the **Options > Reproduction** pane. This returns the best fitness value of approximately 0.2 and displays the following plots.
Crossover Without Mutation

To see how the genetic algorithm performs when there is no mutation, set Crossover fraction to 1.0 and click Start. This returns the best fitness value of approximately 1.3 and displays the following plots.
In this case, the algorithm selects genes from the individuals in the initial population and recombines them. The algorithm cannot create any new genes because there is no mutation. The algorithm generates the best individual that it can using these genes at generation number 8, where the best fitness plot becomes level. After this, it creates new copies of the best individual, which are then are selected for the next generation. By generation number 17, all individuals in the population are the same, namely, the best individual. When this occurs, the average distance between individuals is 0. Since the algorithm cannot improve the best fitness value after generation 8, it stalls after 50 more generations, because **Stall generations** is set to 50.

**Mutation Without Crossover**

To see how the genetic algorithm performs when there is no crossover, set **Crossover fraction** to 0 and click **Start**. This returns the best fitness value of approximately 3.5 and displays the following plots.
In this case, the random changes that the algorithm applies never improve the fitness value of the best individual at the first generation. While it improves the individual genes of other individuals, as you can see in the upper plot by the decrease in the mean value of the fitness function, these improved genes are never combined with the genes of the best individual because there is no crossover. As a result, the best fitness plot is level and the algorithm stalls at generation number 50.

**Comparing Results for Varying Crossover Fractions**

The demo `deterministicstudy.m`, which is included in the software, compares the results of applying the genetic algorithm to Rastrigin’s function with **Crossover fraction** set to 0, .2, .4, .6, .8, and 1. The demo runs for 10 generations. At each generation, the demo plots the means and standard deviations of the best fitness values in all the preceding generations, for each value of the **Crossover fraction**.
To run the demo, enter
deterministicstudy

at the MATLAB prompt. When the demo is finished, the plots appear as in the following figure.

The lower plot shows the means and standard deviations of the best fitness values over 10 generations, for each of the values of the crossover fraction. The upper plot shows a color-coded display of the best fitness values in each generation.

For this fitness function, setting Crossover fraction to 0.8 yields the best result. However, for another fitness function, a different setting for Crossover fraction might yield the best result.
Example: Global vs. Local Minima with GA

Sometimes the goal of an optimization is to find the global minimum or maximum of a function—a point where the function value is smaller or larger at any other point in the search space. However, optimization algorithms sometimes return a local minimum—a point where the function value is smaller than at nearby points, but possibly greater than at a distant point in the search space. The genetic algorithm can sometimes overcome this deficiency with the right settings.

As an example, consider the following function

\[ f(x) = \begin{cases} \exp\left(-\left(\frac{x}{20}\right)^2\right) & \text{for } x \leq 20, \\ -\exp(-1) + (x - 20)(x - 22) & \text{for } x > 20. \end{cases} \]

The following figure shows a plot of the function.

The function has two local minima, one at \( x = 0 \), where the function value is \(-1\), and the other at \( x = 21 \), where the function value is \(-1 - \frac{1}{e}\). Since the latter value is smaller, the global minimum occurs at \( x = 21 \).
Running the Genetic Algorithm on the Example
To run the genetic algorithm on this example,

1. Copy and paste the following code into a new file in the MATLAB Editor.

   ```matlab
   function y = two_min(x)
   if x<=20
       y = -exp(-(x/20).^2);
   else
       y = -exp(-1)+(x-20)*(x-22);
   end
   ```

2. Save the file as `two_min.m` in a folder on the MATLAB path.

3. In the Optimization Tool,
   - Set **Fitness function** to `@two_min`.
   - Set **Number of variables** to 1.
   - Click **Start**.

The genetic algorithm returns a point very close to the local minimum at \( x = 0 \).

The following custom plot shows why the algorithm finds the local minimum rather than the global minimum. The plot shows the range of individuals in each generation and the best individual.
Note that all individuals are between -2 and 2.5. While this range is larger than the default Initial range of $[0; 1]$, due to mutation, it is not large enough to explore points near the global minimum at $x = 21$.

One way to make the genetic algorithm explore a wider range of points—that is, to increase the diversity of the populations—is to increase the Initial range. The Initial range does not have to include the point $x = 21$, but it must be large enough so that the algorithm generates individuals near $x = 21$. Set Initial range to $[0; 15]$ as shown in the following figure.
Then click **Start**. The genetic algorithm returns a point very close to 21.
This time, the custom plot shows a much wider range of individuals. By the second generation there are individuals greater than 21, and by generation 12, the algorithm finds a best individual that is approximately equal to 21.

\[ f(x_1, x_2) = 100 \left( x_2 - x_1^2 \right)^2 + (1 - x_1)^2. \]
The following figure shows a plot of Rosenbrock’s function.

Global Optimization Toolbox software contains the `dejong2fcn.m` file, which computes Rosenbrock’s function. To see a demo of this example, enter

```
hybriddemo
```

at the MATLAB prompt.

To explore the example, first enter `optimtool('ga')` to open the Optimization Tool to the `ga` solver. Enter the following settings:

- Set **Fitness function** to `@dejong2fcn`.
- Set **Number of variables** to 2.
- Set **Population size** to 10.
Before adding a hybrid function, click **Start** to run the genetic algorithm by itself. The genetic algorithm displays the following results in the **Run solver and view results** pane:

The final point is somewhat close to the true minimum at (1, 1). You can improve this result by setting **Hybrid function** to `fminunc` (in the **Hybrid function** options).

`fminunc` uses the final point of the genetic algorithm as its initial point. It returns a more accurate result, as shown in the **Run solver and view results** pane.
You can set options for the hybrid function separately from the calling function. Use optimset (or psoptimset for the patternsearch hybrid function) to create the options structure. For example:

```matlab
hybridopts = optimset('display','iter','LargeScale','off);
```

In the Optimization Tool enter the name of your options structure in the Options box under Hybrid function:

At the command line, the syntax is as follows:

```matlab
options = gaoptimset('HybridFcn',{@fminunc,hybridopts});
```

hybridopts must exist before you set options.
Setting the Maximum Number of Generations

The Generations option in Stopping criteria determines the maximum number of generations the genetic algorithm runs for—see “Stopping Conditions for the Algorithm” on page 5-25. Increasing the Generations option often improves the final result.

As an example, change the settings in the Optimization Tool as follows:

- Set Fitness function to @rastriginsfcn.
- Set Number of variables to 10.
- Select Best fitness in the Plot functions pane.
- Set Generations to Inf.
- Set Stall generations to Inf.
- Set Stall time to Inf.

Run the genetic algorithm for approximately 300 generations and click Stop. The following figure shows the resulting best fitness plot after 300 generations.
Note that the algorithm *stalls* at approximately generation number 170—that is, there is no immediate improvement in the fitness function after generation 170. If you restore **Stall generations** to its default value of 50, the algorithm would terminate at approximately generation number 230. If the genetic algorithm stalls repeatedly with the current setting for **Generations**, you can try increasing both the **Generations** and **Stall generations** options to improve your results. However, changing other options might be more effective.

**Note** When **Mutation function** is set to **Gaussian**, increasing the value of **Generations** might actually worsen the final result. This can occur because the Gaussian mutation function decreases the average amount of mutation in each generation by a factor that depends on the value specified in **Generations**. Consequently, the setting for **Generations** affects the behavior of the algorithm.

**Vectorizing the Fitness Function**

The genetic algorithm usually runs faster if you *vectorize* the fitness function. This means that the genetic algorithm only calls the fitness function once, but expects the fitness function to compute the fitness for all individuals in the current population at once. To vectorize the fitness function,

- Write the file that computes the function so that it accepts a matrix with arbitrarily many rows, corresponding to the individuals in the population. For example, to vectorize the function

  \[ f(x_1, x_2) = x_1^2 - 2x_1x_2 + 6x_1 + x_2^2 - 6x_2 \]

  write the file using the following code:

  ```
  z = x(:, 1).^2 - 2*x(:, 1).*x(:, 2) + 6*x(:, 1) + x(:, 2).^2 - 6*x(:, 2);
  ```

  The colon in the first entry of \( x \) indicates all the rows of \( x \), so that \( x(:, 1) \) is a vector. The \(^\text{^} \) and \(^\text{*} \) operators perform elementwise operations on the vectors.

- In the **User function evaluation** pane, set the **Evaluate fitness and constraint functions** option to **vectorized**.
**Note** The fitness function, and any nonlinear constraint function, must accept an arbitrary number of rows to use the Vectorize option. `ga` sometimes evaluates a single row even during a vectorized calculation.

The following comparison, run at the command line, shows the improvement in speed with Vectorize set to On.

```matlab
tic;ga(@rastriginsfcn,20);toc

elapsed_time =

4.3660

options=gaoptimset('Vectorize','on');
tic;ga(@rastriginsfcn,20,[],[],[],[],[],[],[],options);toc

elapsed_time =

0.5810
```

If there are nonlinear constraints, the objective function and the nonlinear constraints all need to be vectorized in order for the algorithm to compute in a vectorized manner.

**Constrained Minimization Using ga**

Suppose you want to minimize the simple fitness function of two variables $x_1$ and $x_2$,

$$
\min_x f(x) = 100 \left(x_1^2 - x_2\right)^2 + (1-x_1)^2
$$

subject to the following nonlinear inequality constraints and bounds

- $x_1 \cdot x_2 + x_1 - x_2 + 1.5 \leq 0$ (nonlinear constraint)
- $10 - x_1 \cdot x_2 \leq 0$ (nonlinear constraint)
- $0 \leq x_1 \leq 1$ (bound)
- $0 \leq x_2 \leq 13$ (bound)
Begin by creating the fitness and constraint functions. First, create a file named `simple_fitness.m` as follows:

```matlab
function y = simple_fitness(x)
    y = 100*(x(1)^2 - x(2))^2 + (1 - x(1))^2;
```

The genetic algorithm function, `ga`, assumes the fitness function will take one input `x`, where `x` has as many elements as the number of variables in the problem. The fitness function computes the value of the function and returns that scalar value in its one return argument, `y`.

Then create a file, `simple_constraint.m`, containing the constraints

```matlab
function [c, ceq] = simple_constraint(x)
    c = [1.5 + x(1)*x(2) + x(1) - x(2);...
         -x(1)*x(2) + 10];
    ceq = [];
```

The `ga` function assumes the constraint function will take one input `x`, where `x` has as many elements as the number of variables in the problem. The constraint function computes the values of all the inequality and equality constraints and returns two vectors, `c` and `ceq`, respectively.

To minimize the fitness function, you need to pass a function handle to the fitness function as the first argument to the `ga` function, as well as specifying the number of variables as the second argument. Lower and upper bounds are provided as `LB` and `UB` respectively. In addition, you also need to pass a function handle to the nonlinear constraint function.

```matlab
ObjectiveFunction = @simple_fitness;
nvars = 2;  % Number of variables
LB = [0 0];  % Lower bound
UB = [1 13]; % Upper bound
ConstraintFunction = @simple_constraint;
[x,fval] = ga(ObjectiveFunction,nvars,[],[],[],[],LB,UB,ConstraintFunction)
```

Optimization terminated: average change in the fitness value less than options.TolFun and constraint violation is less than options.TolCon.

```
x =
```

The genetic algorithm solver handles linear constraints and bounds differently from nonlinear constraints. All the linear constraints and bounds are satisfied throughout the optimization. However, ga may not satisfy all the nonlinear constraints at every generation. If ga converges to a solution, the nonlinear constraints will be satisfied at that solution.

ga uses the mutation and crossover functions to produce new individuals at every generation. ga satisfies linear and bound constraints by using mutation and crossover functions that only generate feasible points. For example, in the previous call to ga, the mutation function mutationguassian does not necessarily obey the bound constraints. So when there are bound or linear constraints, the default ga mutation function is mutationadaptfeasible. If you provide a custom mutation function, this custom function must only generate points that are feasible with respect to the linear and bound constraints. All the included crossover functions generate points that satisfy the linear constraints and bounds except the crossoverheuristic function.

To see the progress of the optimization, use the gaoptimset function to create an options structure that selects two plot functions. The first plot function is gaplotbestf, which plots the best and mean score of the population at every generation. The second plot function is gaplotmaxconstr, which plots the maximum constraint violation of nonlinear constraints at every generation. You can also visualize the progress of the algorithm by displaying information to the command window using the 'Display' option.

options = gaoptimset('PlotFcns',{@gaplotbestf,@gaplotmaxconstr},'Display','iter');

Rerun the ga solver.

[x,fval] = ga(ObjectiveFunction,nvars,[],[],[],[],...
LB,UB,ConstraintFunction,options)
Using the Genetic Algorithm

Optimization terminated: average change in the fitness value less than options.TolFun and constraint violation is less than options.TolCon.

\[ x = \]
\[
0.8122 \quad 12.3123
\]

\[ fval = \]
\[
1.3578\times10^4
\]

You can provide a start point for the minimization to the ga function by specifying the InitialPopulation option. The ga function will use the first individual from InitialPopulation as a start point for a constrained minimization.

\[ X0 = [0.5 \ 0.5]; \quad \text{Start point (row vector)} \]
\[ \text{options = gaoptimset(options,'InitialPopulation',X0);} \]

Now, rerun the ga solver.

\[ [x,fval] = ga(ObjectiveFunction,nvars,[],[],[],[],... \]
\[
\text{LB,UB,ConstraintFunction,options})
\]

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Genetic Algorithm Examples

Best max Stall
Generation f-count f(x) constraint Generations
1 965 13579.6 0 0
2 1728 13578.2 1.776e-015 0
3 2422 13578.2 0 0

Optimization terminated: average change in the fitness value less than options.TolFun and constraint violation is less than options.TolCon.

x =
0.8122 12.3122

fval =
1.3578e+004

Vectorized Constraints
If there are nonlinear constraints, the objective function and the nonlinear constraints all need to be vectorized in order for the algorithm to compute in a vectorized manner.

“Vectorizing the Objective and Constraint Functions” on page 4-83 contains an example of how to vectorize both for the solver patternsearch. The syntax is nearly identical for ga. The only difference is that patternsearch can have its patterns appear as either row or column vectors; the corresponding vectors for ga are the population vectors, which are always rows.
Using Simulated Annealing

- “What Is Simulated Annealing?” on page 6-2
- “Performing a Simulated Annealing Optimization” on page 6-3
- “Example: Minimizing De Jong’s Fifth Function” on page 6-8
- “Understanding Simulated Annealing Terminology” on page 6-11
- “How Simulated Annealing Works” on page 6-13
- “Using Simulated Annealing from the Command Line” on page 6-17
- “Simulated Annealing Examples” on page 6-22
What Is Simulated Annealing?

Simulated annealing is a method for solving unconstrained and bound-constrained optimization problems. The method models the physical process of heating a material and then slowly lowering the temperature to decrease defects, thus minimizing the system energy.

At each iteration of the simulated annealing algorithm, a new point is randomly generated. The distance of the new point from the current point, or the extent of the search, is based on a probability distribution with a scale proportional to the temperature. The algorithm accepts all new points that lower the objective, but also, with a certain probability, points that raise the objective. By accepting points that raise the objective, the algorithm avoids being trapped in local minima, and is able to explore globally for more possible solutions. An annealing schedule is selected to systematically decrease the temperature as the algorithm proceeds. As the temperature decreases, the algorithm reduces the extent of its search to converge to a minimum.
Performing a Simulated Annealing Optimization

Call `simulannealbnd` at the Command Line

To call the simulated annealing function at the command line, use the syntax

\[
[x \text{ } fval] = \text{simulannealbnd}(@objfun,x0,lb,ub,options)
\]

where

- `@objfun` is a function handle to the objective function.
- `x0` is an initial guess for the optimizer.
- `lb` and `ub` are lower and upper bound constraints, respectively, on `x`.
- `options` is a structure containing options for the algorithm. If you do not pass in this argument, `simulannealbnd` uses its default options.

The results are given by:

- `x` — Final point returned by the solver
- `fval` — Value of the objective function at `x`

The command-line function `simulannealbnd` is convenient if you want to

- Return results directly to the MATLAB workspace.
- Run the simulated annealing algorithm multiple times with different options by calling `simulannealbnd` from a file.

“Using Simulated Annealing from the Command Line” on page 6-17 provides a detailed description of using the function `simulannealbnd` and creating the options structure.
Using the Optimization Tool

To open the Optimization Tool, enter

\[ \text{optimtool('simulannealbnd')} \]

at the command line, or enter \text{optimtool} and then choose \text{simulannealbnd} from the \textbf{Solver} menu.
Performing a Simulated Annealing Optimization

You can also start the tool from the MATLAB Start menu as pictured:
To use the Optimization Tool, you must first enter the following information:

- **Objective function** — The objective function you want to minimize. Enter the fitness function in the form `@fitnessfun`, where `fitnessfun.m` is a file that computes the objective function. “Computing Objective Functions” on page 2-2 explains how write this file. The `@` sign creates a function handle to `fitnessfun`. 
• **Number of variables** — The length of the input vector to the fitness function. For the function `my_fun` described in “Computing Objective Functions” on page 2-2, you would enter 2.

You can enter bounds for the problem in the **Constraints** pane. If the problem is unconstrained, leave these fields blank.

To run the simulated annealing algorithm, click the **Start** button. The tool displays the results of the optimization in the **Run solver and view results** pane.

You can change the options for the simulated annealing algorithm in the **Options** pane. To view the options in one of the categories listed in the pane, click the + sign next to it.

For more information,

- See the “Graphical Optimization Tool” chapter in the Optimization Toolbox documentation.
- See “Minimizing Using the Optimization Tool” on page 6-9 for an example of using the tool with the function `simulannealbnd`. 
Example: Minimizing De Jong’s Fifth Function

In this section...

“Description” on page 6-8
“Minimizing at the Command Line” on page 6-9
“Minimizing Using the Optimization Tool” on page 6-9

Description

This section presents an example that shows how to find the minimum of the function using simulated annealing.

De Jong’s fifth function is a two-dimensional function with many (25) local minima:

dejong5fcn
Many standard optimization algorithms get stuck in local minima. Because the simulated annealing algorithm performs a wide random search, the chance of being trapped in local minima is decreased.

**Note** Because simulated annealing uses random number generators, each time you run this algorithm you can get different results. See “Reproducing Your Results” on page 6-20 for more information.

### Minimizing at the Command Line

To run the simulated annealing algorithm without constraints, call `simulannealbnd` at the command line using the objective function in `dejong5fcn.m`, referenced by anonymous function pointer:

```matlab
fun = @dejong5fcn;
[x fval] = simulannealbnd(fun, [0 0])
```

This returns

```
x =
    -31.9779  -31.9595
fval =
    0.9980
```

where

- `x` is the final point returned by the algorithm.
- `fval` is the objective function value at the final point.

### Minimizing Using the Optimization Tool

To run the minimization using the Optimization Tool,

1. Set up your problem as pictured in the Optimization Tool.
2 Click Start under Run solver and view results:
Understanding Simulated Annealing Terminology

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<tbody>
<tr>
<td>“Objective Function” on page 6-11</td>
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<tr>
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</tr>
</tbody>
</table>

Objective Function

The *objective function* is the function you want to optimize. Global Optimization Toolbox algorithms attempt to find the minimum of the objective function. Write the objective function as a file or anonymous function, and pass it to the solver as a function handle. For more information, see “Computing Objective Functions” on page 2-2.

Temperature

The *temperature* is a parameter in simulated annealing that affects two aspects of the algorithm:

- The distance of a trial point from the current point (See “Outline of the Algorithm” on page 6-13, Step 1.)
- The probability of accepting a trial point with higher objective function value (See “Outline of the Algorithm” on page 6-13, Step 2.)

Temperature can be a vector with different values for each component of the current point. Typically, the initial temperature is a scalar.

Temperature decreases gradually as the algorithm proceeds. You can specify the initial temperature as a positive scalar or vector in the InitialTemperature option. You can specify the temperature as a function of iteration number as a function handle in the TemperatureFcn option. The temperature is a function of the “Annealing Parameter” on page 6-12, which is a proxy for the iteration number. The slower the rate of temperature decrease, the better the chances are of finding an optimal solution, but the longer the
run time. For a list of built-in temperature functions and the syntax of a custom temperature function, see “Temperature Options” on page 9-58.

**Annealing Parameter**

The *annealing parameter* is a proxy for the iteration number. The algorithm can raise temperature by setting the annealing parameter to a lower value than the current iteration. (See “Reannealing” on page 6-12.) You can specify the temperature schedule as a function handle with the `TemperatureFcn` option.

**Reannealing**

*Annealing* is the technique of closely controlling the temperature when cooling a material to ensure that it reaches an optimal state. *Reannealing* raises the temperature after the algorithm accepts a certain number of new points, and starts the search again at the higher temperature. Reannealing avoids the algorithm getting caught at local minima. Specify the reannealing schedule with the `ReannealInterval` option.
How Simulated Annealing Works

In this section...

“Outline of the Algorithm” on page 6-13
“Stopping Conditions for the Algorithm” on page 6-15
“Bibliography” on page 6-15

Outline of the Algorithm

The simulated annealing algorithm performs the following steps:

1 The algorithm generates a random trial point. The algorithm chooses
the distance of the trial point from the current point by a probability
distribution with a scale depending on the current temperature. You set
the trial point distance distribution as a function with the AnnealingFcn
option. Choices:
   - @annealingfast (default) — Step length equals the current temperature,
     and direction is uniformly random.
   - @annealingboltz — Step length equals the square root of temperature,
     and direction is uniformly random.
   - @myfun — Custom annealing algorithm, myfun. For custom annealing
     function syntax, see “Algorithm Settings” on page 9-59.

2 The algorithm determines whether the new point is better or worse than
the current point. If the new point is better than the current point, it
becomes the next point. If the new point is worse than the current point,
the algorithm can still make it the next point. The algorithm accepts
a worse point based on an acceptance function. Choose the acceptance
function with the AcceptanceFcn option. Choices:
   - @acceptancesa (default) — Simulated annealing acceptance function.
     The probability of acceptance is

\[
\frac{1}{1 + \exp\left(\frac{\Delta}{\max(T)}\right)}
\]
where

\[ \Delta = \text{new objective} - \text{old objective}. \]
\[ T_0 = \text{initial temperature of component } i \]
\[ T = \text{the current temperature}. \]

Since both \( \Delta \) and \( T \) are positive, the probability of acceptance is between 0 and 1/2. Smaller temperature leads to smaller acceptance probability. Also, larger \( \Delta \) leads to smaller acceptance probability.

- @myfun — Custom acceptance function, myfun. For custom acceptance function syntax, see “Algorithm Settings” on page 9-59.

3 The algorithm systematically lowers the temperature, storing the best point found so far. The TemperatureFcn option specifies the function the algorithm uses to update the temperature. Let \( k \) denote the annealing parameter. (The annealing parameter is the same as the iteration number until reannealing.) Options:

- @temperatureexp (default) — \( T = T_0 \cdot 0.95^k \).
- @temperaturefast — \( T = T_0 / k \).
- @temperatureboltz — \( T = T_0 / \log(k) \).
- @myfun — Custom temperature function, myfun. For custom temperature function syntax, see “Temperature Options” on page 9-58.

4 `simulannealbnd` reanneals after it accepts `ReannealInterval` points. Reannealing sets the annealing parameters to lower values than the iteration number, thus raising the temperature in each dimension. The annealing parameters depend on the values of estimated gradients of the objective function in each dimension. The basic formula is

\[ k_i = \log \left( \frac{T_0 \cdot \max_j(s_j)}{T_i \cdot s_i} \right), \]

where

\[ k_i = \text{annealing parameter for component } i. \]
\[ T_0 = \text{initial temperature of component } i. \]
How Simulated Annealing Works

\[ T_i = \text{current temperature of component } i. \]
\[ s_i = \text{gradient of objective in direction } i \text{ times difference of bounds in direction } i. \]

`simulannealbnd` safeguards the annealing parameter values against `Inf` and other improper values.

5 The algorithm stops when the average change in the objective function is small relative to the `TolFun` tolerance, or when it reaches any other stopping criterion. See “Stopping Conditions for the Algorithm” on page 6-15.

For more information on the algorithm, see Ingber [1].

**Stopping Conditions for the Algorithm**

The simulated annealing algorithm uses the following conditions to determine when to stop:

- **TolFun** — The algorithm runs until the average change in value of the objective function in `StallIterLim` iterations is less than the value of `TolFun`. The default value is `1e-6`.

- **MaxIter** — The algorithm stops when the number of iterations exceeds this maximum number of iterations. You can specify the maximum number of iterations as a positive integer or `Inf`. The default value is `Inf`.

- **MaxFunEval** specifies the maximum number of evaluations of the objective function. The algorithm stops if the number of function evaluations exceeds the value of `MaxFunEval`. The default value is `3000*numberofvariables`.

- **TimeLimit** specifies the maximum time in seconds the algorithm runs before stopping. The default value is `Inf`.

- **ObjectiveLimit** — The algorithm stops when the best objective function value is less than or equal to the value of `ObjectiveLimit`. The default value is `-Inf`.

**Bibliography**

Using Simulated Annealing from the Command Line

In this section...

“Running simulannealbnd With the Default Options” on page 6-17
“Setting Options for simulannealbnd at the Command Line” on page 6-18
“Reproducing Your Results” on page 6-20

Running simulannealbnd With the Default Options

To run the simulated annealing algorithm with the default options, call simulannealbnd with the syntax

\[
[x,fval] = \text{simulannealbnd}(\text{@objfun},x0)
\]

The input arguments to simulannealbnd are

- @objfun — A function handle to the file that computes the objective function. “Computing Objective Functions” on page 2-2 explains how to write this file.
- x0 — The initial guess of the optimal argument to the objective function.

The output arguments are

- x — The final point.
- \(fval\) — The value of the objective function at \(x\).

For a description of additional input and output arguments, see the reference pages for simulannealbnd.

You can run the example described in “Example: Minimizing De Jong’s Fifth Function” on page 6-8 from the command line by entering

\[
[x,fval] = \text{simulannealbnd}(@\text{dejong5fcn}, \{0,0\})
\]

This returns

\[
x =
\begin{bmatrix}
-31.9564 \\
-15.9755
\end{bmatrix}
\]
Using Simulated Annealing

\[ fval = \\
5.9288 \]

**Additional Output Arguments**
To get more information about the performance of the algorithm, you can call `simulannealbnd` with the syntax

\[ [x,fval,exitflag,output] = simulannealbnd(@objfun,x0) \]

Besides \( x \) and \( fval \), this function returns the following additional output arguments:

- \( \text{exitflag} \) — Flag indicating the reason the algorithm terminated
- \( \text{output} \) — Structure containing information about the performance of the algorithm

See the `simulannealbnd` reference pages for more information about these arguments.

**Setting Options for simulannealbnd at the Command Line**
You can specify options by passing an options structure as an input argument to `simulannealbnd` using the syntax

\[ [x,fval] = simulannealbnd(@objfun,x0,[],[],options) \]

This syntax does not specify any lower or upper bound constraints.

You create the options structure using the `saoptimset` function:

\[ \text{options} = \text{saoptimset}('\text{simulannealbnd}') \]

This returns the structure options with the default values for its fields:

\[ \text{options} = \\
\begin{align*}
\text{AnnealingFcn: } & \text{@annealingfast} \\
\text{TemperatureFcn: } & \text{@temperatureexp} \\
\text{AcceptanceFcn: } & \text{@acceptancesa} \\
\text{TolFun: } & 1.0000e-006
\end{align*} \]
StallIterLimit: '500*numberofvariables'
MaxFunEvals: '3000*numberofvariables'
TimeLimit: Inf
MaxIter: Inf
ObjectiveLimit: -Inf
Display: 'final'
DisplayInterval: 10
HybridFcn: []
HybridInterval: 'end'
PlotFcns: []
PlotInterval: 1
OutputFcns: []
InitialTemperature: 100
ReannealInterval: 100
DataType: 'double'

The value of each option is stored in a field of the options structure, such as options.ReannealInterval. You can display any of these values by entering options followed by the name of the field. For example, to display the interval for reannealing used for the simulated annealing algorithm, enter

```matlab
options.ReannealInterval
ans =
    100
```

To create an options structure with a field value that is different from the default—for example, to set ReannealInterval to 300 instead of its default value 100—enter

```matlab
options = saoptimset('ReannealInterval', 300)
```

This creates the options structure with all values set to their defaults, except for ReannealInterval, which is set to 300.

If you now enter

```matlab
simulannealbnd(@objfun,x0,[],[],options)
```

simulannealbnd runs the simulated annealing algorithm with a reannealing interval of 300.
If you subsequently decide to change another field in the options structure, such as setting `PlotFcns` to `@saplotbestf`, which plots the best objective function value at each iteration, call `saoptimset` with the syntax

```matlab
options = saoptimset(options,'PlotFcns',@saplotbestf)
```

This preserves the current values of all fields of `options` except for `PlotFcns`, which is changed to `@saplotbestf`. Note that if you omit the input argument `options`, `saoptimset` resets `ReannealInterval` to its default value 100.

You can also set both `ReannealInterval` and `PlotFcns` with the single command

```matlab
options = saoptimset('ReannealInterval',300, ... 'PlotFcns',@saplotbestf)
```

**Reproducing Your Results**

Because the simulated annealing algorithm is stochastic—that is, it makes random choices—you get slightly different results each time you run it. The algorithm uses the default MATLAB pseudorandom number stream. For more information about random number streams, see `RandStream`. Each time the algorithm calls the stream, its state changes. So the next time the algorithm calls the stream, it returns a different random number.

If you need to reproduce your results exactly, call `simulannealbnd` with the output argument. The output structure contains the current random number generator state in the `output.rngstate` field. Reset the state before running the function again.

For example, to reproduce the output of `simulannealbnd` applied to De Jong’s fifth function, call `simulannealbnd` with the syntax

```matlab
[x,fval,exitflag,output] = simulannealbnd(@dejong5fcn,[0 0]);
```

Suppose the results are

```matlab
x =
  31.9361  -31.9457

fval =
  4.9505
```
The state of the random number generator, \texttt{rngstate}, is stored in output:

\begin{verbatim}
output = 
    iterations: 1754
    funccount: 1769
    message: 'Optimization terminated:... change in best function value less than options.TolFun.'
    rngstate: [1x1 struct]
    problemtype: 'unconstrained'
    temperature: [2x1 double]
    totaltime: 1.1094
\end{verbatim}

Reset the stream by entering

\begin{verbatim}
stream = RandStream.getGlobalStream;
stream.State = output.rngstate.state;
\end{verbatim}

If you now run \texttt{simulannealbnd} a second time, you get the same results.

You can reproduce your run in the Optimization Tool by checking the box \textbf{Use random states from previous run} in the \textbf{Run solver and view results} section.

\begin{figure}[h]
\centering
\includegraphics[scale=0.6]{random_states.png}
\caption{Check the box \textbf{Use random states from previous run} to reproduce your run.}
\end{figure}

\textbf{Note} If you do not need to reproduce your results, it is better not to set the states of \texttt{RandStream}, so that you get the benefit of the randomness in these algorithms.
Simulated Annealing Examples

If you are viewing this documentation in the Help browser, click the following link to see the demo Minimization Using Simulated Annealing. Or, from the MATLAB command line, type `echodemo('saobjective')`. 
Multiobjective Optimization

- “What Is Multiobjective Optimization?” on page 7-2
- “Using gamultiobj” on page 7-5
- “References” on page 7-14
What Is Multiobjective Optimization?

You might need to formulate problems with more than one objective, since a single objective with several constraints may not adequately represent the problem being faced. If so, there is a vector of objectives,

\[ F(x) = [F_1(x), F_2(x), \ldots, F_m(x)], \]

that must be traded off in some way. The relative importance of these objectives is not generally known until the system’s best capabilities are determined and tradeoffs between the objectives fully understood. As the number of objectives increases, tradeoffs are likely to become complex and less easily quantified. The designer must rely on his or her intuition and ability to express preferences throughout the optimization cycle. Thus, requirements for a multiobjective design strategy must enable a natural problem formulation to be expressed, and be able to solve the problem and enter preferences into a numerically tractable and realistic design problem.

Multiobjective optimization is concerned with the minimization of a vector of objectives \( F(x) \) that can be the subject of a number of constraints or bounds:

\[
\min_{x \in \mathbb{R}^n} F(x), \text{ subject to } \\
G_i(x) = 0, \quad i = 1, \ldots, k_c; \quad G_i(x) \leq 0, \quad i = k_c + 1, \ldots, k; \quad l \leq x \leq u.
\]

Note that because \( F(x) \) is a vector, if any of the components of \( F(x) \) are competing, there is no unique solution to this problem. Instead, the concept of noninferiority [4] (also called Pareto optimality [1] and [2]) must be used to characterize the objectives. A noninferior solution is one in which an improvement in one objective requires a degradation of another. To define this concept more precisely, consider a feasible region, \( \Omega \), in the parameter space. \( x \) is an element of the \( n \)-dimensional real numbers \( x \in \mathbb{R}^n \) that satisfies all the constraints, i.e.,

\[ \Omega = \{ x \in \mathbb{R}^n \}, \]

subject to
What Is Multiobjective Optimization?

\[ G_i(x) = 0, \quad i = 1, \ldots, k_e, \]
\[ G_i(x) \leq 0, \quad i = k_e + 1, \ldots, k, \]
\[ l \leq x \leq u. \]

This allows definition of the corresponding feasible region for the objective function space \( \Lambda \):

\[ \Lambda = \left\{ y \in \mathbb{R}^m : y = F(x), x \in \Omega \right\}. \]

The performance vector \( F(x) \) maps parameter space into objective function space, as represented in two dimensions in the figure Mapping from Parameter Space into Objective Function Space on page 7-3.

Figure 7-1: Mapping from Parameter Space into Objective Function Space

A noninferior solution point can now be defined.

**Definition:** Point \( x^* \in \Omega \) is a noninferior solution if for some neighborhood of \( x^* \) there does not exist a \( \Delta x \) such that \((x^* + \Delta x) \in \Omega \) and

\[ F_i(x^* + \Delta x) \leq F_i(x^*), \quad i = 1, \ldots, m, \text{ and} \]
\[ F_j(x^* + \Delta x) < F_j(x^*) \text{ for at least one } j. \]

In the two-dimensional representation of the figure Set of Noninferior Solutions on page 7-4, the set of noninferior solutions lies on the curve between \( C \) and \( D \). Points \( A \) and \( B \) represent specific noninferior points.
Figure 7-2: Set of Noninferior Solutions

A and B are clearly noninferior solution points because an improvement in one objective, $F_1$, requires a degradation in the other objective, $F_2$, i.e., $F_{1B} < F_{1A}$, $F_{2B} > F_{2A}$.

Since any point in $\Omega$ that is an inferior point represents a point in which improvement can be attained in all the objectives, it is clear that such a point is of no value. Multiobjective optimization is, therefore, concerned with the generation and selection of noninferior solution points.

Noninferior solutions are also called Pareto optima. A general goal in multiobjective optimization is constructing the Pareto optima. The algorithm used in gamultiobj is described in [3].
Using gamultiobj

In this section...

“Problem Formulation” on page 7-5
“Using gamultiobj with Optimization Tool” on page 7-6
“Example: Multiobjective Optimization” on page 7-7
“Options and Syntax: Differences With ga” on page 7-13

Problem Formulation

The gamultiobj solver attempts to create a set of Pareto optima for a multiobjective minimization. You may optionally set bounds and linear constraints on variables. gamultiobj uses the genetic algorithm for finding local Pareto optima. As in the ga function, you may specify an initial population, or have the solver generate one automatically.

The fitness function for use in gamultiobj should return a vector of type double. The population may be of type double, a bit string vector, or can be a custom-typed vector. As in ga, if you use a custom population type, you must write your own creation, mutation, and crossover functions that accept inputs of that population type, and specify these functions in the following fields, respectively:

- **Creation function** (CreationFcn)
- **Mutation function** (MutationFcn)
- **Crossover function** (CrossoverFcn)

You can set the initial population in a variety of ways. Suppose that you choose a population of size \( m \). (The default population size is 15 times the number of variables \( n \).) You can set the population:

- As an \( m \)-by-\( n \) matrix, where the rows represent \( m \) individuals.
- As a \( k \)-by-\( n \) matrix, where \( k < m \). The remaining \( m - k \) individuals are generated by a creation function.
- The entire population can be created by a creation function.
Using `gamultiobj` with Optimization Tool

You can access `gamultiobj` from the Optimization Tool GUI. Enter

```matlab
optimtool('gamultiobj')
```

at the command line, or enter `optimtool` and then choose `gamultiobj` from the **Solver** menu. You can also start the tool from the MATLAB **Start** menu as pictured:
If the Quick Reference help pane is closed, you can open it by clicking the “>>” button on the upper right of the GUI: 

All the options available are explained briefly in the help pane.

You can create an options structure in the Optimization Tool, export it to the MATLAB workspace, and use the structure at the command line. For details, see “Importing and Exporting Your Work” in the Optimization Toolbox documentation.

**Example: Multiobjective Optimization**

This example has a two-objective fitness function $f(x)$, where $x$ is also two-dimensional:

```matlab
function f = mymulti1(x)

f(1) = x(1)^4 - 10*x(1)^2 + x(1)*x(2) + x(2)^4 - (x(1)^2)*(x(2)^2);

f(2) = x(2)^4 - (x(1)^2)*(x(2)^2) + x(1)^4 + x(1)*x(2);
```

Create this function file before proceeding.

**Performing the Optimization with Optimization Tool**

1. To define the optimization problem, start the Optimization Tool, and set it as pictured.
2 Set the options for the problem as pictured.

3 Run the optimization by clicking **Start** under **Run solver and view results**.

A plot appears in a figure window.
This plot shows the tradeoff between the two components of $f$. It is plotted in objective function space; see the figure Set of Noninferior Solutions on page 7-4.

The results of the optimization appear in the following table containing both objective function values and the value of the variables.
You can sort the table by clicking a heading. Click the heading again to sort it in the reverse order. The following figures show the result of clicking the heading $f_1$. 

![Optimization Results](image)
Performing the Optimization at the Command Line

To perform the same optimization at the command line:

1 Set the options:

```matlab
options = gaoptimset('PopulationSize',60,...
                   'ParetoFraction',0.7,'PlotFcns',@gaplotpareto);
```

2 Run the optimization using the options:

```matlab
[x fval flag output population] = gamultiobj(@mymulti1,2,...
                                           [],[],[],[-5,-5],[5,5],options);
```

Alternate Views

There are other ways of regarding the problem. The following figure contains a plot of the level curves of the two objective functions, the Pareto frontier calculated by `gamultiobj` (boxes), and the x-values of the true Pareto frontier (diamonds connected by a nearly-straight line). The true Pareto frontier points are where the level curves of the objective functions are parallel. They
were calculated by finding where the gradients of the objective functions are parallel. The figure is plotted in parameter space; see the figure Mapping from Parameter Space into Objective Function Space on page 7-3.

Contours of objective functions, and Pareto frontier
gamultiobj found the ends of the line segment, meaning it found the full extent of the Pareto frontier.

**Options and Syntax: Differences With ga**

The syntax and options for gamultiobj are similar to those for ga, with the following differences:

- **gamultiobj** does not have nonlinear constraints, so its syntax has fewer inputs.
- **gamultiobj** takes an option `DistanceMeasureFcn`, a function that assigns a distance measure to each individual with respect to its neighbors.
- **gamultiobj** takes an option `ParetoFraction`, a number between 0 and 1 that specifies the fraction of the population on the best Pareto frontier to be kept during the optimization. If there is only one Pareto frontier, this option is ignored.
- **gamultiobj** uses only the Tournament selection function.
- **gamultiobj** uses elite individuals differently than ga. It sorts noninferior individuals above inferior ones, so it uses elite individuals automatically.
- **gamultiobj** has only one hybrid function, `fgoalattain`.
- **gamultiobj** does not have a stall time limit.
- **gamultiobj** has different plot functions available.
- **gamultiobj** does not have a choice of scaling function.
References


Parallel Processing

- “Background” on page 8-2
- “How to Use Parallel Processing” on page 8-11
Background

In this section...

“Types of Parallel Processing in Global Optimization Toolbox” on page 8-2
“How Toolbox Functions Distribute Processes” on page 8-3

Types of Parallel Processing in Global Optimization Toolbox

Parallel processing is an attractive way to speed optimization algorithms. To use parallel processing, you must have a Parallel Computing Toolbox license, and have a MATLAB worker pool open (matlabpool). For more information, see “How to Use Parallel Processing” on page 8-11.

Global Optimization Toolbox solvers use parallel computing in various ways.

<table>
<thead>
<tr>
<th>Solver</th>
<th>Parallel?</th>
<th>Parallel Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>GlobalSearch</td>
<td>×</td>
<td>No parallel functionality. However, fmincon can use parallel gradient estimation when run in GlobalSearch. See “Using Parallel Computing with fmincon, fgoalattain, and fminimax” in the Optimization Toolbox documentation.</td>
</tr>
<tr>
<td>MultiStart</td>
<td>✓</td>
<td>Start points distributed to multiple processors. From these points, local solvers run to completion. For more details, see “MultiStart” on page 8-5 and “How to Use Parallel Processing” on page 8-11. For fmincon, no parallel gradient estimation with parallel MultiStart.</td>
</tr>
<tr>
<td>ga, gamultiobj</td>
<td>✓</td>
<td>Population evaluated in parallel, which occurs once per iteration. For more details, see “Genetic Algorithm” on page 8-8 and “How to Use Parallel Processing” on page 8-11. No vectorization of fitness or constraint functions.</td>
</tr>
<tr>
<td>Solver</td>
<td>Parallel?</td>
<td>Parallel Characteristics</td>
</tr>
<tr>
<td>------------------</td>
<td>-----------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>patternsearch</td>
<td>✓</td>
<td>Poll points evaluated in parallel, which occurs once per iteration. For more details, see “Pattern Search” on page 8-7 and “How to Use Parallel Processing” on page 8-11.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No vectorization of fitness or constraint functions.</td>
</tr>
<tr>
<td>simulannealbnd</td>
<td>×</td>
<td>No parallel functionality. However, simulannealbnd can use a hybrid function that runs in parallel. See “Simulated Annealing” on page 8-10.</td>
</tr>
</tbody>
</table>

In addition, several solvers have hybrid functions that run after they finish. Some hybrid functions can run in parallel. Also, most patternsearch search methods can run in parallel. For more information, see “Parallel Search Functions or Hybrid Functions” on page 8-14.

**How Toolbox Functions Distribute Processes**

- “parfor Characteristics and Caveats” on page 8-3
- “MultiStart” on page 8-5
- “GlobalSearch” on page 8-6
- “Pattern Search” on page 8-7
- “Genetic Algorithm” on page 8-8
- “Parallel Computing with gamultiobj” on page 8-9
- “Simulated Annealing” on page 8-10

**parfor Characteristics and Caveats**

**No Nested parfor Loops.** Solvers employ the Parallel Computing Toolbox parfor function to perform parallel computations.

**Note** parfor does not work in parallel when called from within another parfor loop.
Suppose, for example, your objective function \texttt{userfcn} calls \texttt{parfor}, and you want to call \texttt{fmincon} using \texttt{MultiStart} and parallel processing. Suppose also that the conditions for parallel gradient evaluation of \texttt{fmincon} are satisfied, as given in “Parallel Optimization Functionality”. The figure When \texttt{parfor} Runs In Parallel on page 8-4 shows three cases:

1. The outermost loop is parallel \texttt{MultiStart}. Only that loop runs in parallel.

2. The outermost \texttt{parfor} loop is in \texttt{fmincon}. Only \texttt{fmincon} runs in parallel.

3. The outermost \texttt{parfor} loop is in \texttt{userfcn}. In this case, \texttt{userfcn} can use \texttt{parfor} in parallel.

\textbf{Bold} indicates the function that runs in parallel

```matlab
problem = createOptimProblem(fmincon,'objective',@userfcn,...)
ms = MultiStart('UseParallel','always');
x = run(ms,problem,10)
```

- Only the outermost \texttt{parfor} loop runs in parallel

- If \texttt{fmincon} UseParallel option = 'always', \texttt{fmincon} estimates gradients in parallel

```matlab
x = fmincon(@userfcn,...)
```

- If \texttt{fmincon} UseParallel option = 'never', \texttt{userfcn} can use \texttt{parfor} in parallel

**When \texttt{parfor} Runs In Parallel**

**Parallel Random Numbers Are Not Reproducible.** Random number sequences in MATLAB are pseudorandom, determined from a \textit{seed}, or an initial setting. Parallel computations use seeds that are not necessarily controllable or reproducible. For example, each instance of MATLAB has a default global setting that determines the current seed for random sequences.

For \texttt{patternsearch}, if you select MADS as a poll or search method, parallel pattern search does not have reproducible runs. If you select the genetic
algorithm or Latin hypercube as search methods, parallel pattern search does not have reproducible runs.

For `ga` and `gamultiobj`, parallel population generation gives nonreproducible results.

`MultiStart` is different. You can have reproducible runs from parallel `MultiStart`. Runs are reproducible because `MultiStart` generates pseudorandom start points locally, and then distributes the start points to parallel processors. Therefore, the parallel processors do not use random numbers. For more details, see “Parallel Processing and Random Number Streams” on page 3-69.

**Limitations and Performance Considerations.** More caveats related to `parfor` appear in the “Limitations” section of the Parallel Computing Toolbox documentation.

For information on factors that affect the speed of parallel computations, and factors that affect the results of parallel computations, see “Improving Performance with Parallel Computing” in the Optimization Toolbox documentation. The same considerations apply to parallel computing with Global Optimization Toolbox functions.

**MultiStart**

`MultiStart` can automatically distribute a problem and start points to multiple processes or processors. The problems run independently, and `MultiStart` combines the distinct local minima into a vector of `GlobalOptimSolution` objects. `MultiStart` uses parallel computing when you:

- Have a license for Parallel Computing Toolbox software.
- Enable parallel computing with `matlabpool`, a Parallel Computing Toolbox function.
- Set the `UseParallel` property to `'always'` in the `MultiStart` object:

  ```matlab
  ms = MultiStart('UseParallel','always');
  ```

  When these conditions hold, `MultiStart` distributes a problem and start points to processes or processors one at a time. The algorithm halts when it
reaches a stopping condition or runs out of start points to distribute. If the MultiStart Display property is 'iter', then MultiStart displays:

Running the local solvers in parallel.

For an example of parallel MultiStart, see “Example: Parallel MultiStart” on page 3-81.

**Implementation Issues in Parallel MultiStart.** \(f\)mincon cannot estimate gradients in parallel when used with parallel MultiStart. This lack of parallel gradient estimation is due to the limitation of \texttt{parfor} described in “No Nested parfor Loops” on page 8-3.

\(f\)mincon can take longer to estimate gradients in parallel rather than in serial. In this case, using MultiStart with parallel gradient estimation in \(f\)mincon amplifies the slowdown. For example, suppose the ms MultiStart object has UseParallel set to 'never'. Suppose \(f\)mincon takes 1 s longer to solve problem with problem.options.UseParallel set to 'always'. Then run(ms,problem,200) takes 200 s longer than the same run with problem.options.UseParallel set to 'never'.

**Note** When executing serially, \texttt{parfor} loops run slower than \texttt{for} loops. Therefore, for best performance, set your local solver UseParallel option to 'never' when the MultiStart UseParallel property is 'always'.

**GlobalSearch**

GlobalSearch does not distribute a problem and start points to multiple processes or processors. However, when GlobalSearch runs the \(f\)mincon local solver, \(f\)mincon can estimate gradients by parallel finite differences. \(f\)mincon uses parallel computing when you:

- Have a license for Parallel Computing Toolbox software.
- Enable parallel computing with \texttt{matlabpool}, a Parallel Computing Toolbox function.
- Set the UseParallel option to 'always' with \texttt{optimset}. Set this option in the problem structure:
opts = optimset('UseParallel','always','Algorithm','sqp');
problem = createOptimProblem('fmincon','objective',@myobj,...
    'x0',startpt,'options',opts);

For more details, see “Using Parallel Computing with fmincon, fgoalattain,
and fminimax” in the Optimization Toolbox documentation.

**Pattern Search**

patternsearch can automatically distribute the evaluation of objective and
constraint functions associated with the points in a pattern to multiple
processes or processors. patternsearch uses parallel computing when you:

- Have a license for Parallel Computing Toolbox software.
- Enable parallel computing with matlabpool, a Parallel Computing Toolbox
  function.
- Set the following options using psoptimset or the Optimization Tool:
  - CompletePoll is 'on'.
  - Vectorized is 'off' (default).
  - UseParallel is 'always'.

When these conditions hold, the solver computes the objective function and
constraint values of the pattern search in parallel during a poll. Furthermore,
patternsearch overrides the setting of the Cache option, and uses the default
'off' setting.

**Parallel Search Function.** patternsearch can optionally call a search
function at each iteration. The search is parallel when you:

- Set CompleteSearch to 'on'.
- Do not set the search method to @searchneldermead or custom.
- Set the search method to a patternsearch poll method or Latin hypercube
  search, and set UseParallel to 'always'.
- Or, if you set the search method to ga, create a search method option
  structure with UseParallel set to 'always'.

8-7
Implementation Issues in Parallel Pattern Search. The limitations on patternsearch options, listed in “Pattern Search” on page 8-7, arise partly from the limitations of parfor, and partly from the nature of parallel processing:

- Cache is overridden to be 'off' — patternsearch implements Cache as a persistent variable. parfor does not handle persistent variables, because the variable could have different settings at different processors.

- CompletePoll is 'on' — CompletePoll determines whether a poll stops as soon as patternsearch finds a better point. When searching in parallel, parfor schedules all evaluations simultaneously, and patternsearch continues after all evaluations complete. patternsearch cannot halt evaluations after they start.

- Vectorized is 'off' — Vectorized determines whether patternsearch evaluates all points in a pattern with one function call in a vectorized fashion. If Vectorized is 'on', patternsearch does not distribute the evaluation of the function, so does not use parfor.

Genetic Algorithm

ga and gamultiobj can automatically distribute the evaluation of objective and nonlinear constraint functions associated with a population to multiple processors. ga uses parallel computing when you:

- Have a license for Parallel Computing Toolbox software.
- Enable parallel computing with matlabpool, a Parallel Computing Toolbox function.
- Set the following options using gaoptimset or the Optimization Tool:
  - Vectorized is 'off' (default).
  - UseParallel is 'always'.

When these conditions hold, the solver computes the objective function and nonlinear constraint values of the individuals in a population in parallel.

Implementation Issues in Parallel Genetic Algorithm. The limitations on options, listed in “Genetic Algorithm” on page 8-8, arise partly from limitations of parfor, and partly from the nature of parallel processing:
• Vectorized is 'off' — Vectorized determines whether `ga` evaluates an entire population with one function call in a vectorized fashion. If Vectorized is 'on', `ga` does not distribute the evaluation of the function, so does not use `parfor`.

`ga` can have a hybrid function that runs after it finishes; see “Using a Hybrid Function” on page 5-87. If you want the hybrid function to take advantage of parallel computation, set its options separately so that `UseParallel` is 'always'. If the hybrid function is `patternsearch`, set `CompletePoll` to 'on' so that `patternsearch` runs in parallel.

If the hybrid function is `fmincon`, set the following options with `optimset` to have parallel gradient estimation:

• `GradObj` must not be 'on' — it can be 'off' or `[]`.
• Or, if there is a nonlinear constraint function, `GradConstr` must not be 'on' — it can be 'off' or `[]`.

To find out how to write options for the hybrid function, see “Parallel Hybrid Functions” on page 8-15.

**Parallel Computing with `gamultiobj`**

Parallel computing with `gamultiobj` works almost the same as with `ga`. For detailed information, see “Genetic Algorithm” on page 8-8.

The difference between parallel computing with `gamultiobj` and `ga` has to do with the hybrid function. `gamultiobj` allows only one hybrid function, `fgoalattain`. This function optionally runs after `gamultiobj` finishes its run. Each individual in the calculated Pareto frontier, that is, the final population found by `gamultiobj`, becomes the starting point for an optimization using `fgoalattain`. These optimizations run in parallel. The number of processors performing these optimizations is the smaller of the number of individuals and the size of your `matlabpool`.

For `fgoalattain` to run in parallel, set its options correctly:

```matlab
fgoalopts = optimset('UseParallel','always');
gaoptions = gaoptimset('HybridFcn',{@fgoalattain,fgoalopts});
```
Run `gamultiobj` with `gaoptions`, and `fgoalattain` runs in parallel. For more information about setting the hybrid function, see “Hybrid Function Options” on page 9-50.

`gamultiobj` calls `fgoalattain` using a `parfor` loop, so `fgoalattain` does not estimate gradients in parallel when used as a hybrid function with `gamultiobj`. For more information, see “No Nested `parfor` Loops” on page 8-3.

**Simulated Annealing**

`simulannealbnd` does not run in parallel automatically. However, it can call hybrid functions that take advantage of parallel computing. To find out how to write options for the hybrid function, see “Parallel Hybrid Functions” on page 8-15.
How to Use Parallel Processing

In this section...

“Multicore Processors” on page 8-11
“Processor Network” on page 8-12
“Parallel Search Functions or Hybrid Functions” on page 8-14

Multicore Processors

If you have a multicore processor, you might see speedup using parallel processing. You can establish a `matlabpool` of several parallel workers with a Parallel Computing Toolbox license. For a description of Parallel Computing Toolbox software, and the maximum number of parallel workers, see “Product Overview”.

Suppose you have a dual-core processor, and want to use parallel computing:

- Enter
  
  \[
  \text{matlabpool open 2}
  \]

  at the command line. The 2 specifies the number of MATLAB processes to start.

- Set your solver to use parallel processing.

<table>
<thead>
<tr>
<th>MultiStart</th>
<th>Patternsearch</th>
<th>GA</th>
</tr>
</thead>
</table>
| ms = MultiStart('UseParallel', 'always');  
or  
ms.UseParallel = 'always' | options = psoptimset('UseParallel', 'always', 'CompletePoll', 'on', 'Vectorized', 'off'); | options = gaoptimset('UseParallel', 'always', 'Vectorized', 'off'); |

For Optimization Tool:

- Options > User function evaluation > Evaluate

For Optimization Tool:

- Options > User function evaluation > Evaluate
When you run an applicable solver with options, applicable solvers automatically use parallel computing.

To stop computing optimizations in parallel, set UseParallel to 'never', or set the Optimization Tool not to compute in parallel. To halt all parallel computation, enter

```
matlabpool close
```

**Processor Network**

If you have multiple processors on a network, use Parallel Computing Toolbox functions and MATLAB® Distributed Computing Server™ software to establish parallel computation. Here are the steps to take:

1. Make sure your system is configured properly for parallel computing. Check with your systems administrator, or refer to the Parallel Computing Toolbox documentation, or the Administrator Guide documentation for MATLAB Distributed Computing Server.

To perform a basic check:

- **a** At the command line, enter
  
  ```
  matlabpool open conf
  ```

  or

  ```
  matlabpool open conf n
  ```

  where `conf` is your configuration, and `n` is the number of processors you want to use.

- **b** If `network_file_path` is the network path to your objective or constraint functions, enter
How to Use Parallel Processing

```matlab
pctRunOnAll('addpath network_file_path')
```

so the worker processors can access your objective or constraint files.

Check whether a file is on the path of every worker by entering

```matlab
pctRunOnAll('which filename')
```

If any worker does not have a path to the file, it reports

```
filename not found.
```

2. Set your solver to use parallel processing.

<table>
<thead>
<tr>
<th>MultiStart</th>
<th>Patternsearch</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>ms = MultiStart('UseParallel', 'always');</code></td>
<td><code>options = psoptimset('UseParallel', 'always', 'CompletePoll', 'on', 'Vectorized', 'off');</code></td>
<td><code>options = gaoptimset('UseParallel', 'always', 'Vectorized', 'off');</code></td>
</tr>
<tr>
<td>or</td>
<td><code>ms.UseParallel = 'always'</code></td>
<td></td>
</tr>
</tbody>
</table>

For Optimization Tool:

- Options > User function evaluation > Evaluate objective and constraint functions > in parallel
- Options > Complete poll > on

For Optimization Tool:

- Options > User function evaluation > Evaluate fitness and constraint functions > in parallel

After you establish your parallel computing environment, applicable solvers automatically use parallel computing whenever you call them with `options`.

To stop computing optimizations in parallel, set `UseParallel` to 'never', or set the Optimization Tool not to compute in parallel. To halt all parallel computation, enter

```
matlabpool close
```
Parallel Search Functions or Hybrid Functions

To have a `patternsearch` search function run in parallel, or a hybrid function for `ga` or `simulannealbnd` run in parallel, do the following.

1. Set up parallel processing as described in “Multicore Processors” on page 8-11 or “Processor Network” on page 8-12.

2. Ensure that your search function or hybrid function has the conditions outlined in these sections:

   - “patternsearch Search Function” on page 8-14
   - “Parallel Hybrid Functions” on page 8-15

**patternsearch Search Function**

`patternsearch` uses a parallel search function under the following conditions:

- `CompleteSearch` is 'on'.
- The search method is not `@searchneldermead` or `custom`.
- If the search method is a `patternsearch` poll method or Latin hypercube search, `UseParallel` is 'always'. Set at the command line with `psoptimset`:

  ```matlab
  options = psoptimset('UseParallel','always',...'
  'CompleteSearch','on','SearchMethod',@GPSPositiveBasis2N);
  ```

Or you can use the Optimization Tool.
• If the search method is `ga`, the search method option structure has `UseParallel` set to `'always'`. Set at the command line with `psoptimset` and `gaoptimset`:

```matlab
iterlim = 1; % iteration limit, specifies # ga runs
gaopt = gaoptimset('UseParallel','always');
options = psoptimset('SearchMethod',... 
    {@searchga,iterlim,gaopt});
```

In the Optimization Tool, first create the `gaopt` structure as above, and then use these settings:

For more information about search functions, see “Using a Search Method” on page 4-62.

**Parallel Hybrid Functions**

g`a` and `simulannealbnd` can have other solvers run after or interspersed with their iterations. These other solvers are called hybrid functions. For information on using a hybrid function with `gamultiobj`, see “Parallel Computing with `gamultiobj” on page 8-9. Both `patternsearch` and `fmincon` can be hybrid functions. You can set options so that `patternsearch` runs in parallel, or `fmincon` estimates gradients in parallel.

Set the options for the hybrid function as described in “Hybrid Function Options” on page 9-50 for `ga`, or “Hybrid Function Options” on page 9-61 for `simulannealbnd`. To summarize:
• If your hybrid function is `patternsearch`:

1 Create a `patternsearch` options structure:

   ```
   hybridopts = psoptimset('UseParallel','always',...
   'CompletePoll','on');
   ```

2 Set the `ga` or `simulannealbnd` options to use `patternsearch` as a hybrid function:

   ```
   options = gaoptimset('UseParallel','always'); % for ga
   options = gaoptimset(options,...
   'HybridFcn',{@patternsearch,hybridopts});
   % or, for `simulannealbnd':
   options = saoptimset('HybridFcn',{@patternsearch,hybridopts});
   ```

Or use the Optimization Tool.

For more information on parallel `patternsearch`, see “Pattern Search” on page 8-7.

• If your hybrid function is `fmincon`:

1 Create a `fmincon` options structure:

   ```
   hybridopts = optimset('UseParallel','always',...
   'Algorithm','interior-point');
   % You can use any Algorithm except trust-region-reflective
   ```

2 Set the `ga` or `simulannealbnd` options to use `fmincon` as a hybrid function:

   ```
   options = gaoptimset('UseParallel','always');
   options = gaoptimset(options,'HybridFcn',{@fmincon,hybridopts});
   % or, for `simulannealbnd':
   ```
options = saoptimset('HybridFcn',{@fmincon,hybridopts});

Or use the Optimization Tool.

For more information on parallel fmincon, see “Parallel Computing for Optimization” in the Optimization Toolbox documentation.
Options Reference

- “GlobalSearch and MultiStart Properties (Options)” on page 9-2
- “Pattern Search Options” on page 9-9
- “Genetic Algorithm Options” on page 9-31
- “Simulated Annealing Options” on page 9-56
GlobalSearch and MultiStart Properties (Options)

How to Set Properties
To create a GlobalSearch or MultiStart object with nondefault properties, use name-value pairs. For example, to create a GlobalSearch object that has iterative display and runs only from feasible points with respect to bounds and inequalities, enter

```matlab
gs = GlobalSearch('Display','iter', ...
    'StartPointsToRun','bounds-ineqs');
```

To set a property of an existing GlobalSearch or MultiStart object, use dot addressing. For example, if `ms` is a MultiStart object, and you want to set the Display property to 'iter', enter

```matlab
ms.Display = 'iter';
```

To set multiple properties of an existing object simultaneously, use the constructor (GlobalSearch or MultiStart) with name-value pairs. For example, to set the Display property to 'iter' and the MaxTime property to 100, enter

```matlab
ms = MultiStart(ms,'Display','iter','MaxTime',100);
```

For more information on setting properties, see “Changing Global Options” on page 3-65.

Properties of Both Objects
You can create a MultiStart object from a GlobalSearch object and vice-versa.
The syntax for creating a new object from an existing object is:

```matlab
ms = MultiStart(gs);
or
gs = GlobalSearch(ms);
```

The new object contains the properties that apply of the old object. This section describes those shared properties:

- “Display” on page 9-3
- “MaxTime” on page 9-3
- “OutputFcns” on page 9-3
- “PlotFcns” on page 9-5
- “StartPointsToRun” on page 9-6
- “TolX” on page 9-6
- “TolFun” on page 9-6

**Display**

Values for the `Display` property are:

- `'final'` (default) — Summary results to command line after last solver run.
- `'off'` — No output to command line.
- `'iter'` — Summary results to command line after each local solver run.

**MaxTime**

The `MaxTime` property describes a tolerance on the number of seconds since the solver began its run. Solvers halt when they see `MaxTime` seconds have passed since the beginning of the run. Time means *wall clock* as opposed to processor cycles. The default is `Inf`.

**OutputFcns**

The `OutputFcns` property directs the global solver to run one or more output functions after each local solver run completes. The output functions also
run when the global solver starts and ends. Include a handle to an output function written in the appropriate syntax, or include a cell array of such handles. The default is empty ([]).

The syntax of an output function is:

\[
\text{stop} = \text{outFcn}(\text{optimValues}, \text{state})
\]

- \text{stop} is a Boolean. When true, the algorithm stops. When false, the algorithm continues.

\textbf{Note} A local solver can have an output function. The global solver does not necessarily stop when a local solver output function causes a local solver run to stop. If you want the global solver to stop in this case, have the global solver output function stop when \text{optimValues.localsolution.exitflag} = -1.

- \text{optimValues} is a structure, described in “optimvalues Structure” on page 9-5.
- \text{state} is a string that indicates the current state of the global algorithm:
  - 'init' — The global solver has not called the local solver. The fields in the \text{optimValues} structure are empty, except for \text{localrunindex}, which is 0, and \text{funcount}, which contains the number of objective and constraint function evaluations.
  - 'iter' — The global solver calls output functions after each local solver run.
  - 'done' — The global solver finished calling local solvers. The fields in \text{optimValues} generally have the same values as the ones from the final output function call with \text{state} = 'iter'. However, the value of \text{optimValues.funcount} for \text{GlobalSearch} can be larger than the value in the last function call with 'iter', because the \text{GlobalSearch} algorithm might have performed some function evaluations that were not part of a local solver. For more information, see “GlobalSearch Algorithm” on page 3-45.
For an example using an output function, see “Example: GlobalSearch Output Function” on page 3-35.

**Note** Output and plot functions do not run when MultiStart has the UseParallel option set to 'always' and there is an open matlabpool.

**optimvalues Structure.** The optimValues structure contains the following fields:

- **bestx** — The current best point
- **bestfval** — Objective function value at bestx
- **funccount** — Total number of function evaluations
- **localrunindex** — Index of the local solver run
- **localsolution** — A structure containing part of the output of the local solver call: X, Fval and Exitflag

**PlotFcns**

The PlotFcns property directs the global solver to run one or more plot functions after each local solver run completes. Include a handle to a plot function written in the appropriate syntax, or include a cell array of such handles. The default is empty ([]).

The syntax of a plot function is the same as that of an output function. For details, see “OutputFcns” on page 9-3.

There are two predefined plot functions for the global solvers:

- @gsplotbestf plots the best objective function value.
- @gsplotfunccount plots the number of function evaluations.

For an example using a plot function, see “Example: MultiStart Plot Function” on page 3-39.
Note Output and plot functions do not run when MultiStart has the UseParallel option set to 'always' and there is an open matlabpool.

StartPointsToRun
The StartPointsToRun property directs the solver to exclude certain start points from being run:

- all — Accept all start points.
- bounds — Reject start points that do not satisfy bounds.
- bounds-ineqs — Reject start points that do not satisfy bounds or inequality constraints.

TolX
The TolX property describes how close two points must be for solvers to consider them identical for creating the vector of local solutions. Set TolX to 0 to obtain the results of every local solver run. Set TolX to a larger value to have fewer results. Solvers compute the distance between a pair of points with norm, the Euclidean distance.

Solvers consider two solutions identical if they are within TolX distance of each other and have objective function values within TolFun of each other. If both conditions are not met, solvers report the solutions as distinct.

TolFun
The TolFun property describes how close two objective function values must be for solvers to consider them identical for creating the vector of local solutions. Set TolFun to 0 to obtain the results of every local solver run. Set TolFun to a larger value to have fewer results.

Solvers consider two solutions identical if they are within TolX distance of each other and have objective function values within TolFun of each other. If both conditions are not met, solvers report the solutions as distinct.
GlobalSearch Properties

- “NumTrialPoints” on page 9-7
- “NumStageOnePoints” on page 9-7
- “MaxWaitCycle” on page 9-7
- “BasinRadiusFactor” on page 9-8
- “DistanceThresholdFactor” on page 9-8
- “PenaltyThresholdFactor” on page 9-8

NumTrialPoints
Number of potential start points to examine in addition to x0 from the problem structure. GlobalSearch runs only those potential start points that pass several tests. For more information, see “GlobalSearch Algorithm” on page 3-45.

Default: 1000

NumStageOnePoints
Number of start points in Stage 1. For details, see “Obtain Stage 1 Start Point, Run” on page 3-46.

Default: 200

MaxWaitCycle
A positive integer tolerance appearing in several points in the algorithm.

- If the observed penalty function of MaxWaitCycle consecutive trial points is at least the penalty threshold, then raise the penalty threshold (see “PenaltyThresholdFactor” on page 9-8).
- If MaxWaitCycle consecutive trial points are in a basin, then update that basin’s radius (see “BasinRadiusFactor” on page 9-8).

Default: 20
**BasinRadiusFactor**
A basin radius decreases after `MaxWaitCycle` consecutive start points are within the basin. The basin radius decreases by a factor of 
1–`BasinRadiusFactor`.

Default: 0.2

**DistanceThresholdFactor**
A multiplier for determining whether a trial point is in an existing basin of attraction. For details, see “Examine Stage 2 Trial Point to See if fmincon Runs” on page 3-47. Default: 0.75

**PenaltyThresholdFactor**
Determines increase in penalty threshold. For details, see React to Large Counter Values.

Default: 0.2

**MultiStart Properties**

**UseParallel**
The `UseParallel` property determines whether the solver distributes start points to multiple processors:

- 'never' (default) — Do not run in parallel.
- 'always' — Run in parallel.

For the solver to run in parallel you must set up a parallel environment with `matlabpool`. For details, see “How to Use Parallel Processing” on page 8-11.
Pattern Search Options

### In this section...

- “Optimization Tool vs. Command Line” on page 9-9
- “Plot Options” on page 9-10
- “Poll Options” on page 9-12
- “Search Options” on page 9-14
- “Mesh Options” on page 9-19
- “Constraint Parameters” on page 9-20
- “Cache Options” on page 9-21
- “Stopping Criteria” on page 9-21
- “Output Function Options” on page 9-22
- “Display to Command Window Options” on page 9-24
- “Vectorize and Parallel Options (User Function Evaluation)” on page 9-25
- “Options Table for Pattern Search Algorithms” on page 9-26

### Optimization Tool vs. Command Line

There are two ways to specify options for pattern search, depending on whether you are using the Optimization Tool or calling the function `patternsearch` at the command line:

- If you are using the Optimization Tool, you specify the options by selecting an option from a drop-down list or by entering the value of the option in the text field.

- If you are calling `patternsearch` from the command line, you specify the options by creating an options structure using the function `psoptimset`, as follows:

  ```
  options = psoptimset('Param1',value1,'Param2',value2,...);
  ```

See “Setting Options for patternsearch at the Command Line” on page 4-45 for examples.
In this section, each option is listed in two ways:

- By its label, as it appears in the Optimization Tool
- By its field name in the options structure

For example:

- **Poll method** refers to the label of the option in the Optimization Tool.
- **PollMethod** refers to the corresponding field of the options structure.

### Plot Options

Plot options enable you to plot data from the pattern search while it is running. When you select plot functions and run the pattern search, a plot window displays the plots on separate axes. You can stop the algorithm at any time by clicking the **Stop** button on the plot window.

**Plot interval** (PlotInterval) specifies the number of iterations between consecutive calls to the plot function.

You can select any of the following plots in the **Plot functions** pane.

- **Best function value** (@psplotbestf) plots the best objective function value.
- **Function count** (@psplotfuncount) plots the number of function evaluations.
- **Mesh size** (@psplotmeshsize) plots the mesh size.
- **Best point** (@psplotbestx) plots the current best point.
- **Max constraint** (@psplotmaxconstr) plots the maximum nonlinear constraint violation.
- **Custom** enables you to use your own plot function. To specify the plot function using the Optimization Tool,
  - Select **Custom function**.
  - Enter @myfun in the text box, where myfun is the name of your function.
“Structure of the Plot Functions” on page 9-11 describes the structure of a plot function.

To display a plot when calling `patternsearch` from the command line, set the `PlotFcns` field of `options` to be a function handle to the plot function. For example, to display the best function value, set `options` as follows

```matlab
options = psoptimset('PlotFcns', @psplotbestf);
```

To display multiple plots, use the syntax

```matlab
options = psoptimset('PlotFcns', {@plotfun1, @plotfun2, ...});
```

where `@plotfun1`, `@plotfun2`, and so on are function handles to the plot functions (listed in parentheses in the preceding list).

**Structure of the Plot Functions**

The first line of a plot function has the form

```matlab
function stop = plotfun(optimvalues, flag)
```

The input arguments to the function are

- `optimvalues` — Structure containing information about the current state of the solver. The structure contains the following fields:
  - `x` — Current point
  - `iteration` — Iteration number
  - `fval` — Objective function value
  - `meshsize` — Current mesh size
  - `funccount` — Number of function evaluations
  - `method` — Method used in last iteration
  - `TolFun` — Tolerance on function value in last iteration
  - `TolX` — Tolerance on `x` value in last iteration
  - `nonlinineq` — Nonlinear inequality constraints, displayed only when a nonlinear constraint function is specified
- **nonlineq** — Nonlinear equality constraints, displayed only when a nonlinear constraint function is specified

- **flag** — Current state in which the plot function is called. The possible values for **flag** are:
  - **init** — Initialization state
  - **iter** — Iteration state
  - **interrupt** — Intermediate stage
  - **done** — Final state

“Passing Extra Parameters” in the Optimization Toolbox documentation explains how to provide additional parameters to the function.

The output argument **stop** provides a way to stop the algorithm at the current iteration. **stop** can have the following values:

- **false** — The algorithm continues to the next iteration.
- **true** — The algorithm terminates at the current iteration.

**Poll Options**

Poll options control how the pattern search polls the mesh points at each iteration.

**Poll method** (**PollMethod**) specifies the pattern the algorithm uses to create the mesh. There are two patterns for each of the classes of direct search algorithms: the generalized pattern search (GPS) algorithm, the generating set search (GSS) algorithm, and the mesh adaptive direct search (MADS) algorithm. These patterns are the Positive basis \(2N\) and the Positive basis \(N+1\):

- The default pattern, **GPS Positive basis 2N** (**GPSPositiveBasis2N**), consists of the following \(2N\) vectors, where \(N\) is the number of independent variables for the objective function.

  \[
  \begin{bmatrix}
  1 & 0 & 0 & \ldots & 0 \\
  0 & 1 & 0 & \ldots & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  0 & 0 & 0 & \ldots & -1
  \end{bmatrix}
  \begin{bmatrix}
  0 & 1 & 0 & \ldots & 0 \\
  -1 & 0 & 0 & \ldots & 0 \\
  0 & -1 & 0 & \ldots & 0 \\
  0 & 0 & 0 & \ldots & -1
  \end{bmatrix}
  \]
For example, if the optimization problem has three independent variables, the pattern consists of the following six vectors.

\[ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \]

- The GSS Positive basis 2N pattern (\texttt{GSSPositiveBasis2N}) is similar to GPS Positive basis 2N, but adjusts the basis vectors to account for linear constraints. GSS Positive basis 2N is more efficient than GPS Positive basis 2N when the current point is near a linear constraint boundary.

- The MADS Positive basis 2N pattern (\texttt{MADSPositiveBasis2N}) consists of \(2N\) randomly generated vectors, where \(N\) is the number of independent variables for the objective function. This is done by randomly generating \(N\) vectors which form a linearly independent set, then using this first set and the negative of this set gives \(2N\) vectors. As shown above, the GPS Positive basis 2N pattern is formed using the positive and negative of the linearly independent identity, however, with the MADS Positive basis 2N, the pattern is generated using a random permutation of an \(N\text{-by-}N\) linearly independent lower triangular matrix that is regenerated at each iteration.

- The GPS Positive basis NP1 pattern consists of the following \(N+1\) vectors.

\[ \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -1 & -1 & -1 & \ldots & -1 \end{bmatrix} \]

For example, if the objective function has three independent variables, the pattern consists of the following four vectors.

\[ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} \]

- The GSS Positive basis Np1 pattern (\texttt{GSSPositiveBasisNp1}) is similar to GPS Positive basis Np1, but adjusts the basis vectors to account for linear constraints. GSS Positive basis Np1 is more efficient than GPS Positive basis Np1 when the current point is near a linear constraint boundary.

- The MADS Positive basis Np1 pattern (\texttt{MADSPositiveBasisNp1}) consists of \(N\) randomly generated vectors to form the positive basis, where \(N\) is the number of independent variables for the objective function. Then, one more random vector is generated, giving \(N+1\) randomly generated vectors.
Each iteration generates a new pattern when the MADS Positive basis N+1 is selected.

**Complete poll** (CompletePoll) specifies whether all the points in the current mesh must be polled at each iteration. **Complete Poll** can have the values `On` or `Off`.

- If you set **Complete poll** to `On`, the algorithm polls all the points in the mesh at each iteration and chooses the point with the smallest objective function value as the current point at the next iteration.
- If you set **Complete poll** to `Off`, the default value, the algorithm stops the poll as soon as it finds a point whose objective function value is less than that of the current point. The algorithm then sets that point as the current point at the next iteration.

**Polling order** (PollingOrder) specifies the order in which the algorithm searches the points in the current mesh. The options are

- **Random** — The polling order is random.
- **Success** — The first search direction at each iteration is the direction in which the algorithm found the best point at the previous iteration. After the first point, the algorithm polls the mesh points in the same order as **Consecutive**.
- **Consecutive** — The algorithm polls the mesh points in *consecutive* order, that is, the order of the pattern vectors as described in “Poll Method” on page 4-49.

**Search Options**

Search options specify an optional search that the algorithm can perform at each iteration prior to the polling. If the search returns a point that improves the objective function, the algorithm uses that point at the next iteration and omits the polling. Please note, if you have selected the same **Search method** and **Poll method**, only the option selected in the Poll method will be used, although both will be used when the options selected are different.

**Complete search** (CompleteSearch) applies when you set **Search method** to GPS Positive basis Np1, GPS Positive basis 2N, GSS Positive basis Np1, GSS Positive basis 2N, MADS Positive basis Np1, MADS Positive
basis 2N, or Latin hypercube. **Complete search** can have the values On or Off.

For GPS Positive basis Np1, MADS Positive basis Np1, GPS Positive basis 2N, or MADS Positive basis 2N, **Complete search** has the same meaning as the poll option **Complete poll**.

**Search method** (SearchMethod) specifies the optional search step. The options are

- None ([ ]) (the default) specifies no search step.
- GPS Positive basis 2N (@GPSPositiveBasis2N)
- GPS Positive basis Np1 (@GPSPositiveBasisNp1)
- GSS Positive basis 2N (@GSSPositiveBasis2N)
- GSS Positive basis Np1 (@GSSPositiveBasisNp1)
- MADS Positive basis 2N (@MADSPositiveBasis2N)
- MADS Positive basis Np1 (@MADSPositiveBasisNp1)
- Genetic Algorithm (@searchga) specifies a search using the genetic algorithm. If you select Genetic Algorithm, two other options appear:
  - **Iteration limit** — Positive integer specifying the number of iterations of the pattern search for which the genetic algorithm search is performed. The default for **Iteration limit** is 1.
  - **Options** — Options structure for the genetic algorithm, which you can set using gaoptimset.

To change the default values of **Iteration limit** and **Options** at the command line, use the syntax

```matlab
options = psoptimset('SearchMethod',... @{searchga,iterlim,optionsGA})
```

where `iterlim` is the value of **Iteration limit** and `optionsGA` is the genetic algorithm options structure.
**Note** If you set **Search method** to Genetic algorithm or Nelder-Mead, we recommend that you leave **Iteration limit** set to the default value 1, because performing these searches more than once is not likely to improve results.

- **Latin hypercube (@searchlhs)** specifies a Latin hypercube search. `patternsearch` generates each point for the search as follows. For each component, take a random permutation of the vector $[1, 2, \ldots, k]$ minus $\text{rand}(1,k)$, divided by $k$. ($k$ is the number of points.) This yields $k$ points, with each component close to evenly spaced. The resulting points are then scaled to fit any bounds. **Latin hypercube** uses default bounds of -1 and 1.

The way the search is performed depends on the setting for **Complete search**:

- If you set **Complete search** to **On**, the algorithm polls all the points that are randomly generated at each iteration by the Latin hypercube search and chooses the one with the smallest objective function value.

- If you set **Complete search** to **Off** (the default), the algorithm stops the poll as soon as it finds one of the randomly generated points whose objective function value is less than that of the current point, and chooses that point for the next iteration.

If you select **Latin hypercube**, two other options appear:

- **Iteration limit** — Positive integer specifying the number of iterations of the pattern search for which the Latin hypercube search is performed. The default for **Iteration limit** is 1.

- **Design level** — The **Design level** is the number of points `patternsearch` searches, a positive integer. The default for **Design level** is 15 times the number of dimensions.

To change the default values of **Iteration limit** and **Design level** at the command line, use the syntax

```matlab
options=psoptimset('SearchMethod', {@searchlhs, iterlim, level})
```

where `iterlim` is the value of **Iteration limit** and `level` is the value of **Design level**.

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Pattern Search Options

- Nelder-Mead (@searchneldermead) specifies a search using fminsearch, which uses the Nelder-Mead algorithm. If you select Nelder-Mead, two other options appear:
  - **Iteration limit** — Positive integer specifying the number of iterations of the pattern search for which the Nelder-Mead search is performed. The default for **Iteration limit** is 1.
  - **Options** — Options structure for the function fminsearch, which you can create using the function optimset.

To change the default values of **Iteration limit** and **Options** at the command line, use the syntax

```matlab
options = psoptimset('SearchMethod', ... 
    {@searchneldermead, iterlim, optionsNM})
```

where `iterlim` is the value of **Iteration limit** and `optionsNM` is the options structure.

- **Custom** enables you to write your own search function. To specify the search function using the Optimization Tool,
  - Set **Search function** to Custom.
  - Set **Function name** to @myfun, where `myfun` is the name of your function.

If you are using `patternsearch`, set

```matlab
options = psoptimset('SearchMethod', @myfun);
```

To see a template that you can use to write your own search function, enter

```matlab
edit searchfcntemplate
```

The following section describes the structure of the search function.

**Structure of the Search Function**

Your search function must have the following calling syntax.

```matlab
function [successSearch, xBest, fBest, funccount] = 
    searchfcntemplate(fun, x, A, b, Aeq, beq, lb, ub, ... 
        optimValues, options)
```
The search function has the following input arguments:

- **fun** — Objective function
- **x** — Current point
- **A,b** — Linear inequality constraints
- **Aeq,beq** — Linear equality constraints
- **lb,ub** — Lower and upper bound constraints
- **optimValues** — Structure that enables you to set search options. The structure contains the following fields:
  - **x** — Current point
  - **fval** — Objective function value at x
  - **iteration** — Current iteration number
  - **funccount** — Counter for user function evaluation
  - **scale** — Scale factor used to scale the design points
  - **problemtype** — Flag passed to the search routines, indicating whether the problem is 'unconstrained', 'boundconstraints', or 'linearconstraints'. This field is a subproblem type for nonlinear constrained problems.
  - **meshsize** — Current mesh size used in search step
  - **method** — Method used in last iteration
- **options** — Pattern search options structure

The function has the following output arguments:

- **successSearch** — A Boolean identifier indicating whether the search is successful or not
- **xBest,fBest** — Best point and best function value found by search method
- **funccount** — Number of user function evaluation in search method

See “Using a Search Method” on page 4-62 for an example.
Mesh Options

Mesh options control the mesh that the pattern search uses. The following options are available.

**Initial size** (InitialMeshSize) specifies the size of the initial mesh, which is the length of the shortest vector from the initial point to a mesh point. **Initial size** should be a positive scalar. The default is 1.0.

**Max size** (MaxMeshSize) specifies a maximum size for the mesh. When the maximum size is reached, the mesh size does not increase after a successful iteration. **Max size** must be a positive scalar, and is only used when a GPS or GSS algorithm is selected as the Poll or Search method. The default value is Inf. MADS uses a maximum size of 1.

**Accelerator** (MeshAccelerator) specifies whether, when the mesh size is small, the **Contraction factor** is multiplied by 0.5 after each unsuccessful iteration. **Accelerator** can have the values *On* or *Off*, the default. **Accelerator** applies to the GPS and GSS algorithms.

**Rotate** (MeshRotate) is only applied when **Poll method** is set to GPS Positive basis Np1 or GSS Positive basis Np1. It specifies whether the mesh vectors are multiplied by –1 when the mesh size is less than 1/100 of the mesh tolerance (minimum mesh size TolMesh) after an unsuccessful poll. In other words, after the first unsuccessful poll with small mesh size, instead of polling in directions $e_i$ (unit positive directions) and $-\Sigma e_i$, the algorithm polls in directions $-e_i$ and $\Sigma e_i$. **Rotate** can have the values *Off* or *On* (the default). When the problem has equality constraints, **Rotate** is disabled.

**Rotate** is especially useful for discontinuous functions.

---

**Note** Changing the setting of **Rotate** has no effect on the poll when **Poll method** is set to GPS Positive basis 2N, GSS Positive basis 2N, MADS Positive basis 2N, or MADS Positive basis Np1.

**Scale** (ScaleMesh) specifies whether the algorithm scales the mesh points by carefully multiplying the pattern vectors by constants proportional to the logarithms of the absolute values of components of the current point (or, for unconstrained problems, of the initial point). **Scale** can have the values
Off or On (the default). When the problem has equality constraints, Scale is disabled.

**Expansion factor** (MeshExpansion) specifies the factor by which the mesh size is increased after a successful poll. The default value is 2.0, which means that the size of the mesh is multiplied by 2.0 after a successful poll. **Expansion factor** must be a positive scalar and is only used when a GPS or GSS method is selected as the Poll or Search method. MADS uses a factor of 4.0.

**Contraction factor** (MeshContraction) specifies the factor by which the mesh size is decreased after an unsuccessful poll. The default value is 0.5, which means that the size of the mesh is multiplied by 0.5 after an unsuccessful poll. **Contraction factor** must be a positive scalar and is only used when a GPS or GSS method is selected as the Poll or Search method. MADS uses a factor of 0.25.

See “Mesh Expansion and Contraction” on page 4-66 for more information.

**Constraint Parameters**

For information on the meaning of penalty parameters, see “Description of the Nonlinear Constraint Solver” on page 4-30.

- **Initial penalty** (InitialPenalty) — Specifies an initial value of the penalty parameter that is used by the nonlinear constraint algorithm. **Initial penalty** must be greater than or equal to 1, and has a default of 10.

- **Penalty factor** (PenaltyFactor) — Increases the penalty parameter when the problem is not solved to required accuracy and constraints are not satisfied. **Penalty factor** must be greater than 1, and has a default of 100.

**Bind tolerance** (TolBind) specifies the tolerance for the distance from the current point to the boundary of the feasible region with respect to linear constraints. This means **Bind tolerance** specifies when a linear constraint is active. **Bind tolerance** is not a stopping criterion. Active linear constraints change the pattern of points patternsearch uses for polling or searching. patternsearch always uses points that satisfy linear constraints to within **Bind tolerance**. The default value of **Bind tolerance** is 1e-3.
Cache Options

The pattern search algorithm can keep a record of the points it has already polled, so that it does not have to poll the same point more than once. If the objective function requires a relatively long time to compute, the cache option can speed up the algorithm. The memory allocated for recording the points is called the cache. This option should only be used for deterministic objective functions, but not for stochastic ones.

**Cache** (Cache) specifies whether a cache is used. The options are On and Off, the default. When you set Cache to On, the algorithm does not evaluate the objective function at any mesh points that are within Tolerance of a point in the cache.

**Tolerance** (CacheTol) specifies how close a mesh point must be to a point in the cache for the algorithm to omit polling it. Tolerance must be a positive scalar. The default value is eps.

**Size** (CacheSize) specifies the size of the cache. Size must be a positive scalar. The default value is 1e4.

See “Using Cache” on page 4-75 for more information.

Stopping Criteria

Stopping criteria determine what causes the pattern search algorithm to stop. Pattern search uses the following criteria:

**Mesh tolerance** (TolMesh) specifies the minimum tolerance for mesh size. The GPS and GSS algorithms stop if the mesh size becomes smaller than Mesh tolerance. MADS 2N stops when the mesh size becomes smaller than TolMesh^2. MADS Np1 stops when the mesh size becomes smaller than (TolMesh/nVar)^2, where nVar is the number of elements of x0. The default value of TolMesh is 1e-6.

**Max iteration** (MaxIter) specifies the maximum number of iterations the algorithm performs. The algorithm stops if the number of iterations reaches Max iteration. You can select either

- 100*numberOfVariables — Maximum number of iterations is 100 times the number of independent variables (the default).
• **Specify** — A positive integer for the maximum number of iterations

**Max function evaluations** (MaxFunEval) specifies the maximum number of evaluations of the objective function. The algorithm stops if the number of function evaluations reaches **Max function evaluations**. You can select either

• **2000*numberOfVariables** — Maximum number of function evaluations is 2000 times the number of independent variables.

• **Specify** — A positive integer for the maximum number of function evaluations

**Time limit** (TimeLimit) specifies the maximum time in seconds the pattern search algorithm runs before stopping. This also includes any specified pause time for pattern search algorithms.

**X tolerance** (TolX) specifies the minimum distance between the current points at two consecutive iterations. The algorithm stops if the distance between two consecutive points is less than **X tolerance**. The default value is $1e^{-6}$.

**Function tolerance** (TolFun) specifies the minimum tolerance for the objective function. After a successful poll, if the difference between the function value at the previous best point and function value at the current best point is less than TolFun, the algorithm halts. The default value is $1e^{-6}$.

See “Setting Tolerances for the Solver” on page 4-79 for an example.

**Nonlinear constraint tolerance** (TolCon) — The **Nonlinear constraint tolerance** is not used as stopping criterion. It is used to determine the feasibility with respect to nonlinear constraints.

### Output Function Options

Output functions are functions that the pattern search algorithm calls at each generation. To specify the output function using the Optimization Tool,

• **Select Custom function**.

• Enter @myfun in the text box, where myfun is the name of your function.
• To pass extra parameters in the output function, use “Anonymous Functions”.
• For multiple output functions, enter a cell array of output function handles:
  {@myfun1,@myfun2,...}.

At the command line, set

\[
\text{options} = \text{psoptimset('OutputFcn',@myfun)};
\]

For multiple output functions, enter a cell array:

\[
\text{options} = \text{psoptimset('OutputFcn',{@myfun1,@myfun2,...})};
\]

To see a template that you can use to write your own output function, enter

\[
\text{edit psoutputfcntemplate}
\]

at the MATLAB command prompt.

The following section describes the structure of the output function.

**Structure of the Output Function**

Your output function must have the following calling syntax:

\[
[\text{stop, options, optchanged}] = \text{myfun(optimvalues, options, flag)}
\]

The function has the following input arguments:

• **optimvalues** — Structure containing information about the current state of the solver. The structure contains the following fields:
  - x — Current point
  - iteration — Iteration number
  - fval — Objective function value
  - meshsize — Current mesh size
  - funcount — Number of function evaluations
  - method — Method used in last iteration
  - TolFun — Tolerance on function value in last iteration
- **TolX** — Tolerance on x value in last iteration
- **nonlinineq** — Nonlinear inequality constraints, displayed only when a nonlinear constraint function is specified
- **nonlineq** — Nonlinear equality constraints, displayed only when a nonlinear constraint function is specified

• **options** — Options structure
• **flag** — Current state in which the output function is called. The possible values for `flag` are
  - **init** — Initialization state
  - **iter** — Iteration state
  - **interrupt** — Intermediate stage
  - **done** — Final state

“Passing Extra Parameters” in the Optimization Toolbox documentation explains how to provide additional parameters to the output function.

The output function returns the following arguments to `ga`:

- **stop** — Provides a way to stop the algorithm at the current iteration. `stop` can have the following values.
  - **false** — The algorithm continues to the next iteration.
  - **true** — The algorithm terminates at the current iteration.
- **options** — Options structure.
- **optchanged** — Flag indicating changes to `options`.

**Display to Command Window Options**

**Level of display** (`'Display'`) specifies how much information is displayed at the command line while the pattern search is running. The available options are

- **Off** (`'off'`) — No output is displayed.
- **Iterative** (`'iter'`) — Information is displayed for each iteration.
• **Diagnose** ('diagnose') — Information is displayed for each iteration. In addition, the diagnostic lists some problem information and the options that are changed from the defaults.

• **Final** ('final') — The reason for stopping is displayed.

Both **Iterative** and **Diagnose** display the following information:

• **Iter** — Iteration number

• **FunEval** — Cumulative number of function evaluations

• **MeshSize** — Current mesh size

• **FunVal** — Objective function value of the current point

• **Method** — Outcome of the current poll (with no nonlinear constraint function specified). With a nonlinear constraint function, **Method** displays the update method used after a subproblem is solved.

• **Max Constraint** — Maximum nonlinear constraint violation (displayed only when a nonlinear constraint function has been specified)

The default value of **Level of display** is

• **Off** in the Optimization Tool

• **'final'** in an options structure created using `psoptimset`

---

**Vectorize and Parallel Options (User Function Evaluation)**

You can choose to have your objective and constraint functions evaluated in serial, parallel, or in a vectorized fashion. These options are available in the **User function evaluation** section of the **Options** pane of the Optimization Tool, or by setting the 'Vectorized' and 'UseParallel' options with `psoptimset`.

• When **Evaluate objective and constraint functions** ('Vectorized') is in **serial** ('Off'), `patternsearch` calls the objective function on one point at a time as it loops through all of the mesh points. (At the command line, this assumes 'UseParallel' is at its default value of 'never'.)
• When **Evaluate objective and constraint functions** ('Vectorized') is **vectorized** ('On'), `patternsearch` calls the objective function on all the points in the mesh at once, i.e., in a single call to the objective function if either **Complete Poll** or **Complete Search** is On.

If there are nonlinear constraints, the objective function and the nonlinear constraints all need to be vectorized in order for the algorithm to compute in a vectorized manner.

• When **Evaluate objective and constraint functions** (**UseParallel**) is **in parallel** ('always') `patternsearch` calls the objective function in parallel, using the parallel environment you established (see “How to Use Parallel Processing” on page 8-11). At the command line, set **UseParallel** to 'never' to compute serially.

**Note** You cannot simultaneously use vectorized and parallel computations. If you set 'UseParallel' to 'always' and 'Vectorized' to 'On', `patternsearch` evaluates your objective and constraint functions in a vectorized manner, not in parallel.

### How Objective and Constraint Functions Are Evaluated

<table>
<thead>
<tr>
<th><strong>UseParallel</strong></th>
<th><strong>Vectorized</strong> = 'Off'</th>
<th><strong>Vectorized</strong> = 'On'</th>
</tr>
</thead>
<tbody>
<tr>
<td>'Never'</td>
<td>Serial</td>
<td>Vectorized</td>
</tr>
<tr>
<td>'Always'</td>
<td>Parallel</td>
<td>Vectorized</td>
</tr>
</tbody>
</table>

### Options Table for Pattern Search Algorithms

**Option Availability Table for All Algorithms**

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
<th>Algorithm Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cache</td>
<td>With Cache set to 'on', <code>patternsearch</code> keeps a history of the mesh points it polls and does</td>
<td>All</td>
</tr>
</tbody>
</table>
## Option Availability Table for All Algorithms (Continued)

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
<th>Algorithm Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>not poll points close to them again at subsequent iterations. Use this option if <code>patternsearch</code> runs slowly because it is taking a long time to compute the objective function. If the objective function is stochastic, it is advised not to use this option.</td>
<td>All</td>
<td></td>
</tr>
<tr>
<td>CacheSize</td>
<td>Size of the cache, in number of points.</td>
<td>All</td>
</tr>
<tr>
<td>CacheTol</td>
<td>Positive scalar specifying how close the current mesh point must be to a point in the cache in order for <code>patternsearch</code> to avoid polling it. Available if 'Cache' option is set to 'on'.</td>
<td>All</td>
</tr>
<tr>
<td>CompletePoll</td>
<td>Complete poll around current iterate. Evaluate all the points in a poll step.</td>
<td>All</td>
</tr>
<tr>
<td>CompleteSearch</td>
<td>Complete search around current iterate when the search method is a poll method. Evaluate all the points in a search step.</td>
<td>All</td>
</tr>
<tr>
<td>Display</td>
<td>Level of display to Command Window.</td>
<td>All</td>
</tr>
<tr>
<td>InitialMeshSize</td>
<td>Initial mesh size used in pattern search algorithms.</td>
<td>All</td>
</tr>
</tbody>
</table>
### Option Availability Table for All Algorithms (Continued)

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
<th>Algorithm Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>InitialPenalty</td>
<td>Initial value of the penalty parameter.</td>
<td>All</td>
</tr>
<tr>
<td>MaxFunEvals</td>
<td>Maximum number of objective function evaluations.</td>
<td>All</td>
</tr>
<tr>
<td>MaxIter</td>
<td>Maximum number of iterations.</td>
<td>All</td>
</tr>
<tr>
<td>MaxMeshSize</td>
<td>Maximum mesh size used in a poll/search step.</td>
<td>GPS and GSS</td>
</tr>
<tr>
<td>MeshAccelerator</td>
<td>Accelerate mesh size contraction.</td>
<td>GPS and GSS</td>
</tr>
<tr>
<td>MeshContraction</td>
<td>Mesh contraction factor, used when iteration is unsuccessful.</td>
<td>GPS and GSS</td>
</tr>
<tr>
<td>MeshExpansion</td>
<td>Mesh expansion factor, expands mesh when iteration is successful.</td>
<td>GPS and GSS</td>
</tr>
<tr>
<td>MeshRotate</td>
<td>Rotate the pattern before declaring a point to be optimum.</td>
<td>GPS Np1 and GSS Np1</td>
</tr>
<tr>
<td>OutputFcn</td>
<td>User-specified function that a pattern search calls at each iteration.</td>
<td>All</td>
</tr>
<tr>
<td>PenaltyFactor</td>
<td>Penalty update parameter.</td>
<td>All</td>
</tr>
<tr>
<td>PlotFcn</td>
<td>Specifies function to plot at runtime.</td>
<td>All</td>
</tr>
<tr>
<td>PlotInterval</td>
<td>Specifies that plot functions will be called at every interval.</td>
<td>All</td>
</tr>
</tbody>
</table>
# Option Availability Table for All Algorithms (Continued)

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
<th>Algorithm Availability</th>
</tr>
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<tbody>
<tr>
<td>PollingOrder</td>
<td>Order in which search directions are polled.</td>
<td>GPS and GSS</td>
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<tr>
<td>PollMethod</td>
<td>Polling strategy used in pattern search.</td>
<td>All</td>
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<tr>
<td>ScaleMesh</td>
<td>Automatic scaling of variables.</td>
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<td>SearchMethod</td>
<td>Specifies search method used in pattern search.</td>
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</tr>
<tr>
<td>TimeLimit</td>
<td>Total time (in seconds) allowed for optimization. Also includes any specified pause time for pattern search algorithms.</td>
<td>All</td>
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<td>TolCon</td>
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</tr>
<tr>
<td>Option</td>
<td>Description</td>
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<tr>
<td>UseParallel</td>
<td>When ‘always’, compute objective functions of a poll or search in parallel. Disable by setting to ‘never’.</td>
<td>All</td>
</tr>
<tr>
<td>Vectorized</td>
<td>Specifies whether objective and constraint functions are vectorized.</td>
<td>All</td>
</tr>
</tbody>
</table>
Optimization Tool vs. Command Line

There are two ways to specify options for the genetic algorithm, depending on whether you are using the Optimization Tool or calling the functions `ga` or `gamultiobj` from the command line:

- If you are using the Optimization Tool (`optimtool`), select an option from a drop-down list or enter the value of the option in a text field.
- If you are calling `ga` or `gamultiobj` from the command line, create an options structure using the function `gaoptimset`, as follows:

  ```
  options = gaoptimset('Param1', value1, 'Param2', value2, ...);
  ```
See “Setting Options for ga at the Command Line” on page 5-51 for examples.

In this section, each option is listed in two ways:

- By its label, as it appears in the Optimization Tool
- By its field name in the options structure

For example:

- **Population type** is the label of the option in the Optimization Tool.
- **PopulationType** is the corresponding field of the options structure.

**Plot Options**

Plot options enable you to plot data from the genetic algorithm while it is running. When you select plot functions and run the genetic algorithm, a plot window displays the plots on separate axes. Click on any subplot to view a larger version of the plot in a separate figure window. You can stop the algorithm at any time by clicking the **Stop** button on the plot window.

**Plot interval** (**PlotInterval**) specifies the number of generations between consecutive calls to the plot function.

You can select any of the following plot functions in the **Plot functions** pane:

- **Best fitness** (**@gaplotbestf**) plots the best function value versus generation.
- **Expectation** (**@gaplotexpectation**) plots the expected number of children versus the raw scores at each generation.
- **Score diversity** (**@gaplotscorediversity**) plots a histogram of the scores at each generation.
- **Stopping** (**@gaplotstopping**) plots stopping criteria levels.
- **Best individual** (**@gaplotbestindiv**) plots the vector entries of the individual with the best fitness function value in each generation.
• **Genealogy** (@gaplotgenealogy) plots the genealogy of individuals. Lines from one generation to the next are color-coded as follows:
  - Red lines indicate mutation children.
  - Blue lines indicate crossover children.
  - Black lines indicate elite individuals.

• **Scores** (@gaplotscores) plots the scores of the individuals at each generation.

• **Max constraint** (@gaplotmaxconstr) plots the maximum nonlinear constraint violation at each generation.

• **Distance** (@gaplotdistance) plots the average distance between individuals at each generation.

• **Range** (@gaplotrange) plots the minimum, maximum, and mean fitness function values in each generation.

• **Selection** (@gaplotselection) plots a histogram of the parents.

• **Custom function** enables you to use plot functions of your own. To specify the plot function if you are using the Optimization Tool:
   - Select **Custom function**.
   - Enter `@myfun` in the text box, where `myfun` is the name of your function.

See “Structure of the Plot Functions” on page 9-34.

**gamultiobj** allows **Distance**, **Genealogy**, **Score diversity**, **Selection**, **Stopping**, and **Custom function**, as well as the following additional choices:

• **Pareto front** (@gaplotpareto) plots the Pareto front for the first two objective functions.

• **Average Pareto distance** (@gaplotparetodistance) plots a bar chart of the distance of each individual from its neighbors.

• **Rank histogram** (@gaplotrankhist) plots a histogram of the ranks of the individuals. Individuals of rank 1 are on the Pareto frontier. Individuals of rank 2 are lower than at least one rank 1 individual, but are not lower than any individuals from other ranks, etc.
- **Average Pareto spread** (@gaplotspread) plots the average spread as a function of iteration number.

To display a plot when calling `ga` from the command line, set the `PlotFcns` field of `options` to be a function handle to the plot function. For example, to display the best fitness plot, set `options` as follows

```matlab
options = gaoptimset('PlotFcns', @gaplotbestf);
```

To display multiple plots, use the syntax

```matlab
options = gaoptimset('PlotFcns', {{@plotfun1, @plotfun2, ...}});
```

where `@plotfun1`, `@plotfun2`, and so on are function handles to the plot functions.

**Structure of the Plot Functions**

The first line of a plot function has the form

```matlab
function stop = plotfun(options,state,flag)
```

The input arguments to the function are

- `options` — Structure containing all the current options settings.
- `flag` — String that tells what stage the algorithm is currently in.

“Passing Extra Parameters” in the Optimization Toolbox documentation explains how to provide additional parameters to the function.

The output argument `stop` provides a way to stop the algorithm at the current iteration. `stop` can have the following values:

- `false` — The algorithm continues to the next iteration.
- `true` — The algorithm terminates at the current iteration.
The State Structure

**ga.** The state structure for ga, which is an input argument to plot, mutation, and output functions, contains the following fields:

- Population — Population in the current generation
- Score — Scores of the current population
- Generation — Current generation number
- StartTime — Time when genetic algorithm started
- StopFlag — String containing the reason for stopping
- Selection — Indices of individuals selected for elite, crossover and mutation
- Expectation — Expectation for selection of individuals
- Best — Vector containing the best score in each generation
- LastImprovement — Generation at which the last improvement in fitness value occurred
- LastImprovementTime — Time at which last improvement occurred
- NonlinIneq — Nonlinear inequality constraints, displayed only when a nonlinear constraint function is specified
- NonlinEq — Nonlinear equality constraints, displayed only when a nonlinear constraint function is specified

**gamultiobj.** The state structure for gamultiobj, which is an input argument to plot, mutation, and output functions, contains the following fields:

- Population — Population in the current generation
- Score — Scores of the current population, a Population-by-nObjectives matrix, where nObjectives is the number of objectives
- Generation — Current generation number
- StartTime — Time when genetic algorithm started
- StopFlag — String containing the reason for stopping
• **Selection** — Indices of individuals selected for elite, crossover and mutation

• **Rank** — Vector of the ranks of members in the population

• **Distance** — Vector of distances of each member of the population to the nearest neighboring member

• **AverageDistance** — The average of Distance

• **Spread** — Vector whose entries are the spread in each generation

### Population Options

Population options enable you to specify the parameters of the population that the genetic algorithm uses.

**Population type** (`PopulationType`) specifies the type of the input to the fitness function. Types and their restrictions:

• **Double vector** ('doubleVector') — Use this option if the individuals in the population have type `double`. This is the default. Use this option for mixed integer programming.

• **Bit string** ('bitstring') — Use this option if the individuals in the population are bit strings. For **Creation function** (`CreationFcn`) and **Mutation function** (`MutationFcn`), use Uniform (`mutationuniform`) or Custom. For **Crossover function** (`CrossoverFcn`), use Scattered (@crossoverscattered), Single point (@crossoversinglepoint), Two point (@crossovertwopoint), or Custom. You cannot use a Hybrid function or Nonlinear constraint function.

• **Custom** — For **Crossover function** and **Mutation function**, use Custom. For **Creation function**, either use Custom, or provide an Initial population. You cannot use a Hybrid function or Nonlinear constraint function.

**Population size** (`PopulationSize`) specifies how many individuals there are in each generation. With a large population size, the genetic algorithm searches the solution space more thoroughly, thereby reducing the chance that the algorithm will return a local minimum that is not a global minimum. However, a large population size also causes the algorithm to run more slowly.
If you set Population size to a vector, the genetic algorithm creates multiple subpopulations, the number of which is the length of the vector. The size of each subpopulation is the corresponding entry of the vector.

**Creation function** (CreationFcn) specifies the function that creates the initial population for `ga`. Do not use with integer problems. You can choose from the following functions:

- **Uniform** (@gacreationuniform) creates a random initial population with a uniform distribution. This is the default if there are no constraints or bound constraints.

- **Feasible population** (@gacreationlinearfeasible) creates a random initial population that satisfies all bounds and linear constraints. It is biased to create individuals that are on the boundaries of the constraints, and to create well-dispersed populations. This is the default if there are linear constraints.

- **Custom** enables you to write your own creation function, which must generate data of the type that you specify in Population type. To specify the creation function if you are using the Optimization Tool,
  - Set **Creation function** to Custom.
  - Set **Function name** to @myfun, where myfun is the name of your function.

If you are using `ga`, set

```matlab
options = gaoptimset('CreationFcn', @myfun);
```

Your creation function must have the following calling syntax.

```matlab
function Population = myfun(GenomeLength, FitnessFcn, options)
```

The input arguments to the function are

- **GenomeLength** — Number of independent variables for the fitness function
- **FitnessFcn** — Fitness function
- **options** — Options structure
The function returns Population, the initial population for the genetic algorithm.

“Passing Extra Parameters” in the Optimization Toolbox documentation explains how to provide additional parameters to the function.

**Initial population** (InitialPopulation) specifies an initial population for the genetic algorithm. The default value is [], in which case ga uses the default **Creation function** to create an initial population. If you enter a nonempty array in the Initial population field, the array must have no more than Population size rows, and exactly Number of variables columns. In this case, the genetic algorithm calls a **Creation function** to generate the remaining individuals, if required.

**Initial scores** (InitialScores) specifies initial scores for the initial population. The initial scores can also be partial. Do not use with integer problems.

**Initial range** (PopInitRange) specifies the range of the vectors in the initial population that is generated by a creation function. Do not use with integer problems. You can set Initial range to be a matrix with two rows and Number of variables columns, each column of which has the form [lb; ub], where lb is the lower bound and ub is the upper bound for the entries in that coordinate. If you specify Initial range to be a 2-by-1 vector, each entry is expanded to a constant row of length Number of variables.

See “Example: Setting the Initial Range” on page 5-61 for an example.

**Fitness Scaling Options**

Fitness scaling converts the raw fitness scores that are returned by the fitness function to values in a range that is suitable for the selection function. You can specify options for fitness scaling in the **Fitness scaling** pane.

**Scaling function** (FitnessScalingFcn) specifies the function that performs the scaling. The options are

- **Rank (@fitscalingrank)** — The default fitness scaling function, Rank, scales the raw scores based on the rank of each individual instead of its score. The rank of an individual is its position in the sorted scores. An
individual with rank $r$ has scaled score proportional to $1/\sqrt{r}$. So the scaled score of the most fit individual is proportional to 1, the scaled score of the next most fit is proportional to $1/\sqrt{2}$, and so on. Rank fitness scaling removes the effect of the spread of the raw scores. The square root makes poorly ranked individuals more nearly equal in score, compared to rank scoring. For more information, see “Fitness Scaling” on page 5-70.

- **Proportional** (@fitscalingprop) — Proportional scaling makes the scaled value of an individual proportional to its raw fitness score.

- **Top** (@fitscalingtop) — Top scaling scales the top individuals equally. Selecting Top displays an additional field, **Quantity**, which specifies the number of individuals that are assigned positive scaled values. **Quantity** can be an integer between 1 and the population size or a fraction between 0 and 1 specifying a fraction of the population size. The default value is 0.4. Each of the individuals that produce offspring is assigned an equal scaled value, while the rest are assigned the value 0. The scaled values have the form $[01/n 1/n 0 0 1/n 0 0 1/n ...]$. To change the default value for **Quantity** at the command line, use the following syntax

  ```matlab
  options = gaoptimset('FitnessScalingFcn', {@fitscalingtop, quantity})
  ```

  where `quantity` is the value of **Quantity**.

- **Shift linear** (@fitscalingshiftlinear) — Shift linear scaling scales the raw scores so that the expectation of the fittest individual is equal to a constant multiplied by the average score. You specify the constant in the **Max survival rate** field, which is displayed when you select Shift linear. The default value is 2. To change the default value of **Max survival rate** at the command line, use the following syntax

  ```matlab
  options = gaoptimset('FitnessScalingFcn',
                      {@fitscalingshiftlinear, rate})
  ```

  where `rate` is the value of **Max survival rate**.

- **Custom** enables you to write your own scaling function. To specify the scaling function using the Optimization Tool,
- Set **Scaling function** to **Custom**.
- Set **Function name** to @myfun, where myfun is the name of your function.

If you are using ga at the command line, set

```matlab
options = gaoptimset('FitnessScalingFcn', @myfun);
```

Your scaling function must have the following calling syntax:

```matlab
function expectation = myfun(scores, nParents)
```

The input arguments to the function are
- **scores** — A vector of scalars, one for each member of the population
- **nParents** — The number of parents needed from this population

The function returns **expectation**, a column vector of scalars of the same length as **scores**, giving the scaled values of each member of the population. The sum of the entries of **expectation** must equal **nParents**.

“Passing Extra Parameters” in the Optimization Toolbox documentation explains how to provide additional parameters to the function.

See “Fitness Scaling” on page 5-70 for more information.

**Selection Options**
Selection options specify how the genetic algorithm chooses parents for the next generation. You can specify the function the algorithm uses in the **Selection function** (SelectionFcn) field in the **Selection** options pane. Do not use with integer problems. The options are

- **Stochastic uniform** (@selectionstochunif) — The default selection function, Stochastic uniform, lays out a line in which each parent corresponds to a section of the line of length proportional to its scaled value. The algorithm moves along the line in steps of equal size. At each step, the algorithm allocates a parent from the section it lands on. The first step is a uniform random number less than the step size.
- **Remainder** (@selectionremainder) — Remainder selection assigns parents deterministically from the integer part of each individual’s scaled value and
then uses roulette selection on the remaining fractional part. For example, if the scaled value of an individual is 2.3, that individual is listed twice as a parent because the integer part is 2. After parents have been assigned according to the integer parts of the scaled values, the rest of the parents are chosen stochastically. The probability that a parent is chosen in this step is proportional to the fractional part of its scaled value.

- **Uniform (@selectionuniform)** — Uniform selection chooses parents using the expectations and number of parents. Uniform selection is useful for debugging and testing, but is not a very effective search strategy.

- **Roulette (@selectionroulette)** — Roulette selection chooses parents by simulating a roulette wheel, in which the area of the section of the wheel corresponding to an individual is proportional to the individual’s expectation. The algorithm uses a random number to select one of the sections with a probability equal to its area.

- **Tournament (@selectiontournament)** — Tournament selection chooses each parent by choosing **Tournament size** players at random and then choosing the best individual out of that set to be a parent. **Tournament size** must be at least 2. The default value of **Tournament size** is 4.

To change the default value of **Tournament size** at the command line, use the syntax

```matlab
options = gaoptimset('SelectionFcn',...{@selectttournament,size})
```

where `size` is the value of **Tournament size**.

- **Custom** enables you to write your own selection function. To specify the selection function using the Optimization Tool,

  - Set **Selection function** to Custom.
  
  - Set **Function name** to `@myfun`, where `myfun` is the name of your function.

If you are using `ga` at the command line, set

```matlab
options = gaoptimset('SelectionFcn', @myfun);
```

Your selection function must have the following calling syntax:

```matlab
function parents = myfun(expectation, nParents, options)
```

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The input arguments to the function are

- **expectation** — Expected number of children for each member of the population
- **nParents** — Number of parents to select
- **options** — Genetic algorithm options structure

The function returns **parents**, a row vector of length **nParents** containing the indices of the parents that you select.

“Passing Extra Parameters” in the Optimization Toolbox documentation explains how to provide additional parameters to the function.

See “Selection” on page 5-73 for more information.

**Reproduction Options**

Reproduction options specify how the genetic algorithm creates children for the next generation.

**Elite count** (**EliteCount**) specifies the number of individuals that are guaranteed to survive to the next generation. Set **Elite count** to be a positive integer less than or equal to the population size. The default value is 2 for noninteger problems, and \(0.05 \times \min(\max(10 \times nvars, 40), 100)\) for integer problems.

**Crossover fraction** (**CrossoverFraction**) specifies the fraction of the next generation, other than elite children, that are produced by crossover. Set **Crossover fraction** to be a fraction between 0 and 1, either by entering the fraction in the text box or moving the slider. The default value is 0.8.

See “Setting the Crossover Fraction” on page 5-77 for an example.

**Mutation Options**

Mutation options specify how the genetic algorithm makes small random changes in the individuals in the population to create mutation children. Mutation provides genetic diversity and enable the genetic algorithm to search a broader space. You can specify the mutation function in the
**Mutation function** (MutationFcn) field in the **Mutation** options pane. Do not use with integer problems. You can choose from the following functions:

- **Gaussian** (mutationgaussian) — The default mutation function, Gaussian, adds a random number taken from a Gaussian distribution with mean 0 to each entry of the parent vector. The standard deviation of this distribution is determined by the parameters **Scale** and **Shrink**, which are displayed when you select Gaussian, and by the **Initial range** setting in the **Population** options.

  - The **Scale** parameter determines the standard deviation at the first generation. If you set **Initial range** to be a 2-by-1 vector \( v \), the initial standard deviation is the same at all coordinates of the parent vector, and is given by \( \text{Scale} \cdot (v(2) - v(1)) \).

    If you set **Initial range** to be a vector \( v \) with two rows and **Number of variables** columns, the initial standard deviation at coordinate \( i \) of the parent vector is given by \( \text{Scale} \cdot (v(i,2) - v(i,1)) \).

  - The **Shrink** parameter controls how the standard deviation shrinks as generations go by. If you set **Initial range** to be a 2-by-1 vector, the standard deviation at the \( k \)th generation, \( \sigma_k \), is the same at all coordinates of the parent vector, and is given by the recursive formula

    \[
    \sigma_k = \sigma_{k-1} \left(1 - \text{Shrink} \cdot \frac{k}{\text{Generations}} \right)
    \]

    If you set **Initial range** to be a vector with two rows and **Number of variables** columns, the standard deviation at coordinate \( i \) of the parent vector at the \( k \)th generation, \( \sigma_{i,k} \), is given by the recursive formula

    \[
    \sigma_{i,k} = \sigma_{i,k-1} \left(1 - \text{Shrink} \cdot \frac{k}{\text{Generations}} \right)
    \]

    If you set **Shrink** to 1, the algorithm shrinks the standard deviation in each coordinate linearly until it reaches 0 at the last generation is reached. A negative value of **Shrink** causes the standard deviation to grow.

The default value of both **Scale** and **Shrink** is 1. To change the default values at the command line, use the syntax
options = gaoptimset('MutationFcn', ...  
{@mutationgaussian, scale, shrink})

where scale and shrink are the values of Scale and Shrink, respectively.

- **Uniform (mutationuniform)** — Uniform mutation is a two-step process. First, the algorithm selects a fraction of the vector entries of an individual for mutation, where each entry has a probability Rate of being mutated. The default value of Rate is 0.01. In the second step, the algorithm replaces each selected entry by a random number selected uniformly from the range for that entry.

To change the default value of Rate at the command line, use the syntax

    options = gaoptimset('MutationFcn', {'mutationuniform', rate})

where rate is the value of Rate.

- **Adaptive Feasible (mutationadaptfeasible)** randomly generates directions that are adaptive with respect to the last successful or unsuccessful generation. The feasible region is bounded by the constraints and inequality constraints. A step length is chosen along each direction so that linear constraints and bounds are satisfied.

- **Custom** enables you to write your own mutation function. To specify the mutation function using the Optimization Tool,

  - Set **Mutation function** to Custom.
  - Set **Function name** to @myfun, where myfun is the name of your function.

If you are using ga, set

    options = gaoptimset('MutationFcn', @myfun);

Your mutation function must have this calling syntax:

    function mutationChildren = myfun(parents, options, nvars,  
    FitnessFcn, state, thisScore, thisPopulation)

The arguments to the function are

- **parents** — Row vector of parents chosen by the selection function
- options — Options structure
- nvars — Number of variables
- FitnessFcn — Fitness function
- thisScore — Vector of scores of the current population
- thisPopulation — Matrix of individuals in the current population

The function returns mutationChildren—the mutated offspring—as a matrix whose rows correspond to the children. The number of columns of the matrix is Number of variables.

“Passing Extra Parameters” in the Optimization Toolbox documentation explains how to provide additional parameters to the function.

**Crossover Options**

Crossover options specify how the genetic algorithm combines two individuals, or parents, to form a crossover child for the next generation.

**Crossover function** (CrossoverFcn) specifies the function that performs the crossover. Do not use with integer problems. You can choose from the following functions:

- **Scattered (@crossoverscattered)**, the default crossover function, creates a random binary vector and selects the genes where the vector is a 1 from the first parent, and the genes where the vector is a 0 from the second parent, and combines the genes to form the child. For example, if p1 and p2 are the parents

  \[ p1 = [a \ b \ c \ d \ e \ f \ g \ h] \]
  \[ p2 = [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8] \]

  and the binary vector is [1 1 0 0 1 0 0 0], the function returns the following child:

  \[ \text{child1} = [a \ b \ 3 \ 4 \ e \ 6 \ 7 \ 8] \]
• **Single point** (@crossoversinglepoint) chooses a random integer \( n \) between 1 and **Number of variables** and then
  - Selects vector entries numbered less than or equal to \( n \) from the first parent.
  - Selects vector entries numbered greater than \( n \) from the second parent.
  - Concatenates these entries to form a child vector.

For example, if \( p_1 \) and \( p_2 \) are the parents

\[
p_1 = [a\ b\ c\ d\ e\ f\ g\ h]\n\]
\[
p_2 = [1\ 2\ 3\ 4\ 5\ 6\ 7\ 8]\n\]

and the crossover point is 3, the function returns the following child.

\[
\text{child} = [a\ b\ c\ 4\ 5\ 6\ 7\ 8]\n\]

• **Two point** (@crossovertwopoint) selects two random integers \( m \) and \( n \) between 1 and **Number of variables**. The function selects
  - Vector entries numbered less than or equal to \( m \) from the first parent
  - Vector entries numbered from \( m+1 \) to \( n \), inclusive, from the second parent
  - Vector entries numbered greater than \( n \) from the first parent.

The algorithm then concatenates these genes to form a single gene. For example, if \( p_1 \) and \( p_2 \) are the parents

\[
p_1 = [a\ b\ c\ d\ e\ f\ g\ h]\n\]
\[
p_2 = [1\ 2\ 3\ 4\ 5\ 6\ 7\ 8]\n\]

and the crossover points are 3 and 6, the function returns the following child.

\[
\text{child} = [a\ b\ c\ 4\ 5\ 6\ g\ h]\n\]

• **Intermediate** (@crossoverintermediate) creates children by taking a weighted average of the parents. You can specify the weights by a single parameter, **Ratio**, which can be a scalar or a row vector of length **Number of variables**. The default is a vector of all 1’s. The function creates the child from \( \text{parent1} \) and \( \text{parent2} \) using the following formula.
child = parent1 + rand * Ratio * (parent2 - parent1)

If all the entries of Ratio lie in the range [0, 1], the children produced are within the hypercube defined by placing the parents at opposite vertices. If Ratio is not in that range, the children might lie outside the hypercube. If Ratio is a scalar, then all the children lie on the line between the parents.

To change the default value of Ratio at the command line, use the syntax

```matlab
options = gaoptimset('CrossoverFcn', ...  
{@crossoverintermediate, ratio});
```

where ratio is the value of Ratio.

- **Heuristic (@crossoverheuristic)** returns a child that lies on the line containing the two parents, a small distance away from the parent with the better fitness value in the direction away from the parent with the worse fitness value. You can specify how far the child is from the better parent by the parameter Ratio, which appears when you select Heuristic. The default value of Ratio is 1.2. If parent1 and parent2 are the parents, and parent1 has the better fitness value, the function returns the child

  \[
  child = parent2 + R * (parent1 - parent2);
  \]

To change the default value of Ratio at the command line, use the syntax

```matlab
options = gaoptimset('CrossoverFcn', ...  
{@crossoverheuristic, ratio});
```

where ratio is the value of Ratio.

- **Arithmetic (@crossoverarithmetic)** creates children that are the weighted arithmetic mean of two parents. Children are always feasible with respect to linear constraints and bounds.

- **Custom** enables you to write your own crossover function. To specify the crossover function using the Optimization Tool,
  - Set Crossover function to Custom.
  - Set Function name to @myfun, where myfun is the name of your function.

If you are using ga, set
options = gaoptimset('CrossoverFcn',@myfun);

Your crossover function must have the following calling syntax.

\[
\text{xoverKids} = \text{myfun}(\text{parents, options, nvars, FitnessFcn, ... unused, thisPopulation})
\]

The arguments to the function are
- parents — Row vector of parents chosen by the selection function
- options — options structure
- nvars — Number of variables
- FitnessFcn — Fitness function
- unused — Placeholder not used
- thisPopulation — Matrix representing the current population. The number of rows of the matrix is Population size and the number of columns is Number of variables.

The function returns xoverKids—the crossover offspring—as a matrix whose rows correspond to the children. The number of columns of the matrix is Number of variables.

"Passing Extra Parameters" in the Optimization Toolbox documentation explains how to provide additional parameters to the function.

**Migration Options**

Migration options specify how individuals move between subpopulations. Migration occurs if you set Population size to be a vector of length greater than 1. When migration occurs, the best individuals from one subpopulation replace the worst individuals in another subpopulation. Individuals that migrate from one subpopulation to another are copied. They are not removed from the source subpopulation.

You can control how migration occurs by the following three fields in the Migration options pane:

- **Direction** (MigrationDirection) — Migration can take place in one or both directions.
- If you set Direction to Forward ('forward'), migration takes place toward the last subpopulation. That is, the $n$th subpopulation migrates into the $(n+1)$th subpopulation.

- If you set Direction to Both ('both'), the $n$th subpopulation migrates into both the $(n-1)$th and the $(n+1)$th subpopulation.

Migration wraps at the ends of the subpopulations. That is, the last subpopulation migrates into the first, and the first may migrate into the last.

- **Interval** (MigrationInterval) — Specifies how many generation pass between migrations. For example, if you set **Interval** to 20, migration takes place every 20 generations.

- **Fraction** (MigrationFraction) — Specifies how many individuals move between subpopulations. **Fraction** specifies the fraction of the smaller of the two subpopulations that moves. For example, if individuals migrate from a subpopulation of 50 individuals into a subpopulation of 100 individuals and you set **Fraction** to 0.1, the number of individuals that migrate is $0.1 \times 50 = 5$.

### Constraint Parameters

Constraint parameters refer to the nonlinear constraint solver. For more information on the algorithm, see “Description of the Nonlinear Constraint Solver” on page 5-38.

- **Initial penalty** (InitialPenalty) — Specifies an initial value of the penalty parameter that is used by the nonlinear constraint algorithm. **Initial penalty** must be greater than or equal to 1, and has a default of 10.

- **Penalty factor** (PenaltyFactor) — Increases the penalty parameter when the problem is not solved to required accuracy and constraints are not satisfied. **Penalty factor** must be greater than 1, and has a default of 100.

### Multiobjective Options

Multiobjective options define parameters characteristic of the multiobjective genetic algorithm. You can specify the following parameters:

- **DistanceMeasureFcn** — Defines a handle to the function that computes distance measure of individuals, computed in decision variable or design
space (genotype) or in function space (phenotype). For example, the default distance measure function is \texttt{distancecrowding} in function space, or \{\texttt{distancecrowding},\texttt{phenotype}\}.

- \texttt{ParetoFraction} — Sets the fraction of individuals to keep on the first Pareto front while the solver selects individuals from higher fronts. This option is a scalar between 0 and 1.

### Hybrid Function Options

A hybrid function is another minimization function that runs after the genetic algorithm terminates. You can specify a hybrid function in \texttt{Hybrid function} (\texttt{HybridFcn}) options. Do not use with integer problems. The choices are

- \texttt{[]} — No hybrid function.
- \texttt{fminsearch (@fminsearch)} — Uses the MATLAB function \texttt{fminsearch} to perform unconstrained minimization.
- \texttt{patternsearch (@patternsearch)} — Uses a pattern search to perform constrained or unconstrained minimization.
- \texttt{fminunc (@fminunc)} — Uses the Optimization Toolbox function \texttt{fminunc} to perform unconstrained minimization.
- \texttt{fmincon (@fmincon)} — Uses the Optimization Toolbox function \texttt{fmincon} to perform constrained minimization.

You can set a separate options structure for the hybrid function. Use \texttt{psoptimset} or \texttt{optimset} to create the structure, depending on whether the hybrid function is \texttt{patternsearch} or not:

```matlab
hybridopts = optimset('display','iter','LargeScale','off');
```

Include the hybrid options in the Genetic Algorithm options structure as follows:

```matlab
options = gaoptimset(options,'HybridFcn',{@fminunc,hybridopts});
```

\texttt{hybridopts} must exist before you set \texttt{options}.

See “Using a Hybrid Function” on page 5-87 for an example.
Stopping Criteria Options

Stopping criteria determine what causes the algorithm to terminate. You can specify the following options:

- **Generations** (Generations) — Specifies the maximum number of iterations for the genetic algorithm to perform. The default is 100.
- **Time limit** (TimeLimit) — Specifies the maximum time in seconds the genetic algorithm runs before stopping.
- **Fitness limit** (FitnessLimit) — The algorithm stops if the best fitness value is less than or equal to the value of Fitness limit.
- **Stall generations** (StallGenLimit) — The algorithm stops if the weighted average change in the fitness function value over Stall generations is less than Function tolerance.
- **Stall time limit** (StallTimeLimit) — The algorithm stops if there is no improvement in the best fitness value for an interval of time in seconds specified by Stall time.
- **Function tolerance** (TolFun) — The algorithm runs until the cumulative change in the fitness function value over Stall generations is less than or equal to Function Tolerance.
- **Nonlinear constraint tolerance** (TolCon) — The Nonlinear constraint tolerance is not used as stopping criterion. It is used to determine the feasibility with respect to nonlinear constraints.

See “Setting the Maximum Number of Generations” on page 5-91 for an example.

Output Function Options

Output functions are functions that the genetic algorithm calls at each generation. To specify the output function using the Optimization Tool,

- Select **Custom function**.
- Enter @myfun in the text box, where myfun is the name of your function.
- To pass extra parameters in the output function, use “Anonymous Functions”.
For multiple output functions, enter a cell array of output function handles: `{@myfun1,@myfun2,...}`.

At the command line, set

```matlab
options = gaoptimset('OutputFcn',@myfun);
```

For multiple output functions, enter a cell array:

```matlab
options = gaoptimset('OutputFcn',{@myfun1,@myfun2,...});
```

To see a template that you can use to write your own output functions, enter

```matlab
edit gaoutputfcntemplate
```

at the MATLAB command line.

**Structure of the Output Function**

The output function has the following calling syntax.

```matlab
[state,options,optchanged] = myfun(options,state,flag)
```

The function has the following input arguments:

- **options** — Options structure
- **flag** — String indicating the current status of the algorithm as follows:
  - 'init' — Initial stage
  - 'iter' — Algorithm running
  - 'interrupt' — Intermediate stage
  - 'done' — Algorithm terminated

“Passing Extra Parameters” in the Optimization Toolbox documentation explains how to provide additional parameters to the function.

The output function returns the following arguments to **ga**:
• state — Structure containing information about the current generation. "The State Structure" on page 9-35 describes the fields of state. To stop the iterations, set state.StopFlag to a nonempty string.

• options — Options structure modified by the output function. This argument is optional.

• optchanged — Flag indicating changes to options

**Display to Command Window Options**

**Level of display** ('Display') specifies how much information is displayed at the command line while the genetic algorithm is running. The available options are

• Off ('off') — No output is displayed.
• Iterative ('iter') — Information is displayed at each iteration.
• Diagnose ('diagnose') — Information is displayed at each iteration. In addition, the diagnostic lists some problem information and the options that have been changed from the defaults.
• Final ('final') — The reason for stopping is displayed.

Both Iterative and Diagnose display the following information:

• Generation — Generation number
• f-count — Cumulative number of fitness function evaluations
• Best $f(x)$ — Best fitness function value
• Mean $f(x)$ — Mean fitness function value
• Stall generations — Number of generations since the last improvement of the fitness function

When a nonlinear constraint function has been specified, Iterative and Diagnose will not display the Mean $f(x)$, but will additionally display:

• Max Constraint — Maximum nonlinear constraint violation

The default value of **Level of display** is
• Off in the Optimization Tool
• 'final' in an options structure created using `gaoptimset`

**Vectorize and Parallel Options (User Function Evaluation)**

You can choose to have your fitness and constraint functions evaluated in serial, parallel, or in a vectorized fashion. These options are available in the User function evaluation section of the Options pane of the Optimization Tool, or by setting the 'Vectorized' and 'UseParallel' options with `gaoptimset`.

• When Evaluate fitness and constraint functions ('Vectorized') is in serial ('Off'), `ga` calls the fitness function on one individual at a time as it loops through the population. (At the command line, this assumes 'UseParallel' is at its default value of 'never'.)

• When Evaluate fitness and constraint functions ('Vectorized') is vectorized ('On'), `ga` calls the fitness function on the entire population at once, i.e., in a single call to the fitness function.

  If there are nonlinear constraints, the fitness function and the nonlinear constraints all need to be vectorized in order for the algorithm to compute in a vectorized manner.

  See “Vectorizing the Fitness Function” on page 5-92 for an example.

• When Evaluate fitness and constraint functions (UseParallel) is in parallel ('always'), `ga` calls the fitness function in parallel, using the parallel environment you established (see “How to Use Parallel Processing” on page 8-11). At the command line, set `UseParallel` to 'never' to compute serially.

**Note** You cannot simultaneously use vectorized and parallel computations. If you set 'UseParallel' to 'always' and 'Vectorized' to 'On', `ga` evaluates your fitness and constraint functions in a vectorized manner, not in parallel.
### How Fitness and Constraint Functions Are Evaluated

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<td>'Always'</td>
<td>Parallel</td>
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Simulated Annealing Options

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“Plot Options” on page 9-56
“Temperature Options” on page 9-58
“Algorithm Settings” on page 9-59
“Hybrid Function Options” on page 9-61
“Stopping Criteria Options” on page 9-62
“Output Function Options” on page 9-62
“Display Options” on page 9-64

saoptimset At The Command Line

Specify options by creating an options structure using the saoptimset function as follows:

```
options = saoptimset('Param1',value1,'Param2',value2, ...);
```

See “Setting Options for simulannealbnd at the Command Line” on page 6-18 for examples.

Each option in this section is listed by its field name in the options structure. For example, InitialTemperature refers to the corresponding field of the options structure.

Plot Options

Plot options enable you to plot data from the simulated annealing solver while it is running. When you specify plot functions and run the algorithm, a plot window displays the plots on separate axes. Right-click on any subplot to view a larger version of the plot in a separate figure window.

PlotInterval specifies the number of iterations between consecutive calls to the plot function.
To display a plot when calling `simulannealbnd` from the command line, set the `PlotFcns` field of `options` to be a function handle to the plot function. You can specify any of the following plots:

- `@saplotbestf` plots the best objective function value.
- `@saplotbestx` plots the current best point.
- `@saplotf` plots the current function value.
- `@saplotx` plots the current point.
- `@saplotstopping` plots stopping criteria levels.
- `@saplottemperature` plots the temperature at each iteration.
- `@myfun` plots a custom plot function, where `myfun` is the name of your function. See “Structure of the Plot Functions” on page 9-11 for a description of the syntax.

For example, to display the best objective plot, set `options` as follows:

```matlab
options = saoptimset('PlotFcns',@saplotbestf);
```

To display multiple plots, use the cell array syntax:

```matlab
options = saoptimset('PlotFcns',{@plotfun1,@plotfun2, ...});
```

where `@plotfun1`, `@plotfun2`, and so on are function handles to the plot functions.

**Structure of the Plot Functions**

The first line of a plot function has the form:

```matlab
function stop = plotfun(options,optimvalues,flag)
```

The input arguments to the function are:

- `options` — Options structure created using `saoptimset`.
- `optimvalues` — Structure containing information about the current state of the solver. The structure contains the following fields:
  - `x` — Current point
- `fval` — Objective function value at x
- `bestx` — Best point found so far
- `bestfval` — Objective function value at best point
- `temperature` — Current temperature
- `iteration` — Current iteration
- `funcCount` — Number of function evaluations
- `t0` — Start time for algorithm
- `k` — Annealing parameter

- `flag` — Current state in which the plot function is called. The possible values for `flag` are
  - `'init'` — Initialization state
  - `'iter'` — Iteration state
  - `'done'` — Final state

The output argument `stop` provides a way to stop the algorithm at the current iteration. `stop` can have the following values:

- `false` — The algorithm continues to the next iteration.
- `true` — The algorithm terminates at the current iteration.

**Temperature Options**

Temperature options specify how the temperature will be lowered at each iteration over the course of the algorithm.

- `InitialTemperature` — Initial temperature at the start of the algorithm. The default is 100.
- `TemperatureFcn` — Function used to update the temperature schedule. Let $k$ denote the annealing parameter. (The annealing parameter is the same as the iteration number until reannealing.) The options are:
  - `@temperatureexp` — The temperature is equal to $\text{InitialTemperature} \times 0.95^k$. This is the default.
Simulated Annealing Options

- @temperaturefast — The temperature is equal to InitialTemperature / \( k \).
- @temperatureboltz — The temperature is equal to InitialTemperature / \( \ln(k) \).
- @myfun — Uses a custom function, myfun, to update temperature. The syntax is:

\[
\text{temperature} = \text{myfun(optimValues,options)}
\]

where optimValues is a structure described in “Structure of the Plot Functions” on page 9-57. options is either the structure created with saoptimset, or the structure of default options, if you did not create an options structure. Both the annealing parameter optimValues.k and the temperature optimValues.temperature are vectors with length equal to the number of elements of the current point \( x \). For example, the function temperaturefast is:

\[
\text{temperature} = \text{options.InitialTemperature}./\text{optimValues.k};
\]

Algorithm Settings

Algorithm settings define algorithmic specific parameters used in generating new points at each iteration.

Parameters that can be specified for simulannealbnd are:

- **DataType** — Type of data to use in the objective function. Choices:
  - 'double' (default) — A vector of type double.
  - 'custom' — Any other data type. You must provide a 'custom' annealing function. You cannot use a hybrid function.

- **AnnealingFcn** — Function used to generate new points for the next iteration. The choices are:
  - @annealingfast — The step has length temperature, with direction uniformly at random. This is the default.
  - @annealingboltz — The step has length square root of temperature, with direction uniformly at random.
  - @myfun — Uses a custom annealing algorithm, myfun. The syntax is:
newx = myfun(optimValues,problem)

where optimValues is a structure described in “Structure of the Output Function” on page 9-63, and problem is a structure containing the following information:

- **objective**: function handle to the objective function
- **x0**: the start point
- **nvar**: number of decision variables
- **lb**: lower bound on decision variables
- **ub**: upper bound on decision variables

For example, the current position is optimValues.x, and the current objective function value is problem.objective(optimValues.x).

- **ReannealInterval** — Number of points accepted before reannealing. The default value is 100.
- **AcceptanceFcn** — Function used to determine whether a new point is accepted or not. The choices are:
  - @acceptancesa — Simulated annealing acceptance function, the default. If the new objective function value is less than the old, the new point is always accepted. Otherwise, the new point is accepted at random with a probability depending on the difference in objective function values and on the current temperature. The acceptance probability is

\[
\frac{1}{1 + \exp\left(\frac{\Delta}{\max(T)}\right)}
\]

where \(\Delta = \text{new objective} - \text{old objective}\), and \(T\) is the current temperature. Since both \(\Delta\) and \(T\) are positive, the probability of acceptance is between 0 and 1/2. Smaller temperature leads to smaller acceptance probability. Also, larger \(\Delta\) leads to smaller acceptance probability.

- @myfun — A custom acceptance function, myfun. The syntax is:

\[
\text{acceptpoint} = \text{myfun}(\text{optimValues}, \text{newx}, \text{newfval});
\]
where `optimValues` is a structure described in “Structure of the Output Function” on page 9-63, `newx` is the point being evaluated for acceptance, and `newfval` is the objective function at `newx`. `acceptpoint` is a Boolean, with value `true` to accept `newx`, and `false` to reject `newx`.

**Hybrid Function Options**

A hybrid function is another minimization function that runs during or at the end of iterations of the solver. `HybridInterval` specifies the interval (if not `never` or `end`) at which the hybrid function is called. You can specify a hybrid function using the `HybridFcn` option. The choices are:

- `[]` — No hybrid function.
- `@fminsearch` — Uses the MATLAB function `fminsearch` to perform unconstrained minimization.
- `@patternsearch` — Uses `patternsearch` to perform constrained or unconstrained minimization.
- `@fminunc` — Uses the Optimization Toolbox function `fminunc` to perform unconstrained minimization.
- `@fmincon` — Uses the Optimization Toolbox function `fmincon` to perform constrained minimization.

You can set a separate options structure for the hybrid function. Use `psoptimset` or `optimset` to create the structure, depending on whether the hybrid function is `patternsearch` or not:

```matlab
hybridopts = optimset('display','iter','LargeScale','off');
```

Include the hybrid options in the `simulannealbnd` options structure as follows:

```matlab
options = saoptimset(options,'HybridFcn',{@fminunc,hybridopts});
```

`hybridopts` must exist before you set `options`.

See “Using a Hybrid Function” on page 5-87 for an example.
Stopping Criteria Options

Stopping criteria determine what causes the algorithm to terminate. You can specify the following options:

- **TolFun** — The algorithm runs until the average change in value of the objective function in `StallIterLim` iterations is less than `TolFun`. The default value is $1e^{-6}$.

- **MaxIter** — The algorithm stops if the number of iterations exceeds this maximum number of iterations. You can specify the maximum number of iterations as a positive integer or `Inf`. `Inf` is the default.

- **MaxFunEval** specifies the maximum number of evaluations of the objective function. The algorithm stops if the number of function evaluations exceeds the maximum number of function evaluations. The allowed maximum is $3000 \times \text{number of variables}$.

- **TimeLimit** specifies the maximum time in seconds the algorithm runs before stopping.

- **ObjectiveLimit** — The algorithm stops if the best objective function value is less than or equal to the value of `ObjectiveLimit`.

Output Function Options

Output functions are functions that the algorithm calls at each iteration. The default value is to have no output function, `[]`. You must first create an output function using the syntax described in “Structure of the Output Function” on page 9-63.

Using the Optimization Tool:

- Specify **Output function** as `@myfun`, where `myfun` is the name of your function.

- To pass extra parameters in the output function, use “Anonymous Functions”.

- For multiple output functions, enter a cell array of output function handles: `{@myfun1, @myfun2, ...}`.
• options = saoptimset('OutputFcns',@myfun);
• For multiple output functions, enter a cell array:

    options = saoptimset('OutputFcn',{@myfun1,@myfun2,...});

To see a template that you can use to write your own output functions, enter

    edit saoutputfcntemplate

at the MATLAB command line.

**Structure of the Output Function**
The output function has the following calling syntax.

    [stop,options,optchanged] = myfun(options,optimvalues,flag)

The function has the following input arguments:

• options — Options structure created using saoptimset.
• optimvalues — Structure containing information about the current state of the solver. The structure contains the following fields:
  - x — Current point
  - fval — Objective function value at x
  - bestx — Best point found so far
  - bestfval — Objective function value at best point
  - temperature — Current temperature, a vector the same length as x
  - iteration — Current iteration
  - funccount — Number of function evaluations
  - t0 — Start time for algorithm
  - k — Annealing parameter, a vector the same length as x
• flag — Current state in which the output function is called. The possible values for flag are
  - 'init' — Initialization state
- 'iter' — Iteration state
- 'done' — Final state

“ Passing Extra Parameters” in the Optimization Toolbox documentation explains how to provide additional parameters to the output function.

The output function returns the following arguments:

- **stop** — Provides a way to stop the algorithm at the current iteration. **stop** can have the following values:
  - false — The algorithm continues to the next iteration.
  - true — The algorithm terminates at the current iteration.
- **options** — Options structure modified by the output function.
- **optchanged** — A boolean flag indicating changes were made to **options**. This must be set to true if options are changed.

**Display Options**

Use the **Display** option to specify how much information is displayed at the command line while the algorithm is running. The available options are

- off — No output is displayed. This is the default value for an options structure created using **saoptimset**.
- iter — Information is displayed at each iteration.
- diagnose — Information is displayed at each iteration. In addition, the diagnostic lists some problem information and the options that have been changed from the defaults.
- final — The reason for stopping is displayed. This is the default.

Both iter and diagnose display the following information:

- **Iteration** — Iteration number
- **f-count** — Cumulative number of objective function evaluations
- **Best f(x)** — Best objective function value
- **Current f(x)** — Current objective function value
• Mean Temperature — Mean temperature function value
Function Reference

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**GlobalSearch**

createOptimProblem  
Create optimization problem structure

GlobalSearch  
Find global minimum

run (GlobalSearch)  
Find global minimum

**MultiStart**

createOptimProblem  
Create optimization problem structure

CustomStartPointSet  
User-supplied start points

list (CustomStartPointSet)  
List custom start points in set

list (RandomStartPointSet)  
Generate start points

MultiStart  
Find multiple local minima

RandomStartPointSet  
Random start points

run (MultiStart)  
Run local solver from multiple points

**Genetic Algorithm**

ga  
Find minimum of function using genetic algorithm

gamultiobj  
Find minima of multiple functions using genetic algorithm

gaoptimget  
Obtain values of genetic algorithm options structure

gaoptimset  
Create genetic algorithm options structure
Direct Search

- **patternsearch**: Find minimum of function using pattern search
- **psoptimget**: Obtain values of pattern search options structure
- **psoptimset**: Create pattern search options structure

Simulated Annealing

- **saoptimget**: Values of simulated annealing options structure
- **saoptimset**: Create simulated annealing options structure
- **simulannealbnd**: Find unconstrained or bound-constrained minimum of function of several variables using simulated annealing algorithm
# Class Reference

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Functions — Alphabetical List
Purpose

Create optimization problem structure

Syntax

```matlab
problem = createOptimProblem('solverName')
problem = createOptimProblem('solverName','ParameterName',ParameterValue,...)
```

Description

`problem = createOptimProblem('solverName')` creates an empty optimization problem structure for the `solverName` solver.

`problem = createOptimProblem('solverName','ParameterName',ParameterValue,...)` accepts one or more comma-separated parameter name/value pairs. Specify `ParameterName` inside single quotes.

Input Arguments

- **solverName**
  
  Name of the solver. For a `GlobalSearch` problem, use `'fmincon'`. For a `MultiStart` problem, use `'fmincon'`, `'fminunc'`, `'lsqcurvefit'` or `'lsqnonlin'`.

Parameter Name/Value Pairs

- **Aeq**
  
  Matrix for linear equality constraints. The constraints have the form:
  
  \[ A_{eq} \mathbf{x} = \mathbf{b}_{eq} \]

- **Aineq**
  
  Matrix for linear inequality constraints. The constraints have the form:
  
  \[ A_{ineq} \mathbf{x} \leq \mathbf{b}_{ineq} \]

- **beq**
  
  Vector for linear equality constraints. The constraints have the form:
  
  \[ A_{eq} \mathbf{x} = \mathbf{b}_{eq} \]
bineq
Vector for linear inequality constraints. The constraints have the form:
\[ A_{ineq} x \leq b_{ineq} \]

lb
Vector of lower bounds.

nonlcon
Function handle to the nonlinear constraint function. The constraint function must accept a vector \( x \) and return two vectors: \( c \), the nonlinear inequality constraints, and \( ceq \), the nonlinear equality constraints. If one of these constraint functions is empty, nonlcon must return [ ] for that function.

If the GradConstr option is 'on', then in addition nonlcon must return two additional outputs, gradc and gradceq. The gradc parameter is a matrix with one column for the gradient of each constraint, as is gradceq.

For more information, see “Constraints” on page 2-6.

objective
Function handle to the objective function. For all solvers except lsqnonlin and lsqcurvefit, the objective function must accept a vector \( x \) and return a scalar. If the GradObj option is 'on', then the objective function must return a second output, a vector, representing the gradient of the objective. For lsqnonlin, the objective function must accept a vector \( x \) and return a vector. lsqnonlin sums the squares of the objective function values. For lsqcurvefit, the objective function must accept two inputs, \( x \) and \( xdata \), and return a vector.

For more information, see “Computing Objective Functions” on page 2-2.

options
createOptimProblem

Options structure. Create this structure with `optimset`, or by exporting from the Optimization Tool.

ub
Vector of upper bounds.

x0
A vector, a potential starting point for the optimization. Gives the dimensionality of the problem.

xdata
Vector of data points for `lsqcurvefit`.

ydata
Vector of data points for `lsqcurvefit`.

Output Arguments

problem
Optimization problem structure.

Examples

Create a problem structure using Rosenbrock's function as objective (see “Using a Hybrid Function” on page 5-87), the interior-point algorithm for `fmincon`, and bounds with absolute value 2:

```matlab
anonrosen = @(x)(100*(x(2) - x(1)^2)^2 + (1-x(1))^2);
opts = optimset('Algorithm','interior-point');
problem = createOptimProblem('fmincon','x0',randn(2,1),...
    'objective',anonrosen,'lb',[-2;-2],
    'ub',[2;2],
    'options',opts);
```

Alternatives

You can create a problem structure by exporting from the Optimization Tool (`optimtool`), as described in “Exporting from the Optimization Tool” on page 3-8.

See Also

`optimtool` | `MultiStart` | `GlobalSearch`

Tutorials

• “Create a Problem Structure” on page 3-4
CustomStartPointSet

**Purpose**
User-supplied start points

**Description**
An object wrapper of a matrix whose rows represent start points for MultiStart.

**Construction**
`tpoints = CustomStartPointSet(ptmatrix)` generates a CustomStartPointSet object from the `ptmatrix` matrix. Each row of `ptmatrix` represents one start point.

**Properties**

- `DimStartPoints`
  Dimension of each start point, a read-only property. `DimStartPoints` is the number of columns in `ptmatrix`.
  
  `DimStartPoints` should be the same as the number of elements in `problem.x0`, the problem structure you pass to `run`.

- `NumStartPoints`
  Number of start points, a read-only property. This is the number of rows in `ptmatrix`.

**Methods**

- `list`
  List custom start points in set

**Copy Semantics**
Value. To learn how value classes affect copy operations, see Copying Objects in the MATLAB Programming Fundamentals documentation.

**Examples**
Create a `CustomStartPointSet` object with 40 three-dimensional rows. Each row represents a normally distributed random variable with mean `[10,10,10]` and variance `diag([4,4,4])`:

```matlab
fortypts = 10*ones(40,3) + 4*randn(40,3); % a matrix
startpts = CustomStartPointSet(fortypts);
```

`startpts` is the `fortypts` matrix in an object wrapper.
Get the `fortypts` matrix from the `startpts` object of the previous example:

```matlab
fortypts = list(startpts);
```

**See Also**
RandomStartPointSet | MultiStart | list

**Tutorials**
- “CustomStartPointSet Object for Start Points” on page 3-18

**How To**
- Class Attributes
- Property Attributes
Purpose

Find minimum of function using genetic algorithm

Syntax

\[ x = \text{ga}(\text{fitnessfcn},nvars) \]
\[ x = \text{ga}(\text{fitnessfcn},nvars,A,b) \]
\[ x = \text{ga}(\text{fitnessfcn},nvars,A,b,Aeq,beq) \]
\[ x = \text{ga}(\text{fitnessfcn},nvars,A,b,Aeq,beq,\text{LB},\text{UB}) \]
\[ x = \text{ga}(\text{fitnessfcn},nvars,A,b,Aeq,beq,\text{LB},\text{UB},\text{nonlcon}) \]
\[ x = \text{ga}(\text{fitnessfcn},nvars,A,b,Aeq,beq,\text{LB},\text{UB},\text{nonlcon},\text{options}) \]
\[ x = \text{ga}(\text{fitnessfcn},nvars,A,b,[],[],\text{LB},\text{UB},\text{nonlcon},\text{IntCon}) \]
\[ x = \text{ga}(\text{fitnessfcn},nvars,A,b,[],[],\text{LB},\text{UB},\text{nonlcon},\text{IntCon},\text{options}) \]
\[ x = \text{ga}(\text{problem}) \]
\[ [x,fval] = \text{ga}(\text{fitnessfcn},nvars,...) \]
\[ [x,fval,\text{exitflag}] = \text{ga}(\text{fitnessfcn},nvars,...) \]
\[ [x,fval,\text{exitflag},\text{output}] = \text{ga}(\text{fitnessfcn},nvars,...) \]
\[ [x,fval,\text{exitflag},\text{output},\text{population}] = \text{ga}(\text{fitnessfcn},\text{nvars},...) \]
\[ [x,fval,\text{exitflag},\text{output},\text{population},\text{scores}] = \text{ga}(\text{fitnessfcn},\text{nvars},...) \]

Description

\[ x = \text{ga}(\text{fitnessfcn},\text{nvars}) \] finds a local unconstrained minimum, \( x \), to the objective function, \( \text{fitnessfcn} \). \( \text{nvars} \) is the dimension (number of design variables) of \( \text{fitnessfcn} \). The objective function, \( \text{fitnessfcn} \), accepts a vector \( x \) of size 1-by-\( \text{nvars} \), and returns a scalar evaluated at \( x \).

\[ x = \text{ga}(\text{fitnessfcn},\text{nvars},\text{A},\text{b}) \] finds a local minimum \( x \) to \( \text{fitnessfcn} \), subject to the linear inequalities \( A \times x \leq b \). \( \text{fitnessfcn} \) accepts input \( x \) and returns a scalar function value evaluated at \( x \).

\[ x = \text{ga}(\text{fitnessfcn},\text{nvars},\text{A},\text{b},\text{Aeq},\text{beq}) \] finds a local minimum \( x \) to \( \text{fitnessfcn} \), subject to the linear equalities \( \text{Aeq} \times x = \text{beq} \) as well as \( A \times x \leq b \). (Set \( \text{A}=[\] and \( \text{b}=[\) if no inequalities exist.)

\[ x = \text{ga}(\text{fitnessfcn},\text{nvars},\text{A},\text{b},\text{Aeq},\text{beq},\text{LB},\text{UB}) \] defines a set of lower and upper bounds on the design variables, \( x \), so that a solution is found in the range \( \text{LB} \leq x \leq \text{UB} \). Use empty matrices for \( \text{LB} \) and \( \text{UB} \) if no bounds exist.
The function `ga` minimizes the function `fitnessfcn` such that \( C(x) \leq 0 \) and \( Ceq(x) = 0 \).

When there are integer constraints, `ga` does not accept linear or nonlinear equality constraints, only inequality constraints.

```
x = ga(fitnessfcn,nvars,A,b,Aeq,beq,lb,ub,nonlcon) subjects
the minimization to the constraints defined in nonlcon. The function
nonlcon accepts x and returns vectors C and Ceq, representing the
nonlinear inequalities and equalities respectively. `ga` minimizes the
fitnessfcn such that C(x) \( \leq 0 \) and Ceq(x) = 0. (Set LB=\( [] \) and UB=\( [] \)
if no bounds exist.)
```

```
x = ga(fitnessfcn,nvars,A,b,Aeq,beq,lb,ub,nonlcon,options) 
minimizes with the default optimization parameters replaced by values
in the structure options, which can be created using the gaoptimset
function.
```

```
x = ga(fitnessfcn,nvars,A,b,[],[],lb,ub,nonlcon,IntCon)
requires that the variables listed in IntCon take integer values.
```

### Note
When there are integer constraints, `ga` does not accept linear or nonlinear equality constraints, only inequality constraints.

```
x = 
ga(fitnessfcn,nvars,A,b,[],[],lb,ub,nonlcon,IntCon,options)
minimizes with integer constraints and with the default optimization
parameters replaced by values in the options structure.
```

```
x = ga(problem) finds the minimum for problem, where problem is
a structure.
```

```
[x,fval] = ga(fitnessfcn,nvars,...) returns fval, the value of
the fitness function at x.
```

```
[x,fval,exitflag] = ga(fitnessfcn,nvars,...) returns exitflag, an integer identifying the reason the algorithm terminated.
```

```
[x,fval,exitflag,output] = ga(fitnessfcn,nvars,...) returns
output, a structure that contains output from each generation and
other information about the performance of the algorithm.
```
[x,fval,exitflag,output,population] = ga(fitnessfcn,nvars,...) returns the matrix, population, whose rows are the final population.

[x,fval,exitflag,output,population,scores] = ga(fitnessfcn,nvars,...) returns scores the scores of the final population.

**Tips**

- To write a function with additional parameters to the independent variables that can be called by ga, see “Passing Extra Parameters” in the Optimization Toolbox documentation.

- For problems that use the population type Double Vector (the default), ga does not accept functions whose inputs are of type complex. To solve problems involving complex data, write your functions so that they accept real vectors, by separating the real and imaginary parts.

**Input Arguments**

fitnessfcn

Handle to the fitness function. The fitness function should accept a row vector of length nvars and return a scalar value.

When the 'Vectorized' option is 'on', fitnessfcn should accept a pop-by-nvars matrix, where pop is the current population size. In this case fitnessfcn should return a vector the same length as pop containing the fitness function values. fitnessfcn should not assume any particular size for pop, since ga can pass a single member of a population even in a vectorized calculation.

nvars

Positive integer representing the number of variables in the problem.

A

Matrix for linear inequality constraints of the form

A x ≤ b.
If the problem has $m$ linear inequality constraints and $nvars$ variables, then

- $A$ is a matrix of size $m$-by-$nvars$.
- $b$ is a vector of length $m$.

**Note** ga does not enforce linear constraints to be satisfied when the PopulationType option is 'bitString' or 'custom'.

$b$

Vector for linear inequality constraints of the form

$$A \ x \leq \ b.$$ 

If the problem has $m$ linear inequality constraints and $nvars$ variables, then

- $A$ is a matrix of size $m$-by-$nvars$.
- $b$ is a vector of length $m$.

$A_{eq}$

Matrix for linear equality constraints of the form

$$A_{eq} \ x = \ beq.$$ 

If the problem has $m$ linear equality constraints and $nvars$ variables, then

- $A_{eq}$ is a matrix of size $m$-by-$nvars$.
- $beq$ is a vector of length $m$. 
**Note** ga does not enforce linear constraints to be satisfied when the PopulationType option is 'bitString' or 'custom'.

**beq**

Vector for linear equality constraints of the form

\[ A_{eq} \times = b_{eq}. \]

If the problem has \( m \) linear equality constraints and \( nvars \) variables, then

- \( A_{eq} \) is a matrix of size \( m \)-by-\( nvars \).
- \( b_{eq} \) is a vector of length \( m \).

**LB**

Vector of lower bounds. ga enforces that iterations stay above \( LB \). Set \( LB(i) = -\infty \) if \( x(i) \) is unbounded below.

**UB**

Vector of upper bounds. ga enforces that iterations stay below \( UB \). Set \( UB(i) = \infty \) if \( x(i) \) is unbounded above.

**nonlcon**

Function handle that returns two outputs:

\[ [c, ceq] = \text{nonlcon}(x) \]

ga attempts to achieve \( c \leq 0 \) and \( ceq = 0 \). \( c \) and \( ceq \) are row vectors when there are multiple constraints. Set unused outputs to \([]\).

You can write nonlcon as a function handle to a file, such as

\[ \text{nonlcon} = @\text{constraintfile} \]

where constraintfile.m is a file on your MATLAB path.
To learn how to use vectorized constraints, see “Vectorized Constraints” on page 2-7.

**Note** ga does not enforce nonlinear constraints to be satisfied when the PopulationType option is set to ‘bitString’ or ‘custom’.

If IntCon is not empty, the second output of nonlcon (ceq) must be empty ([ ]).

For information on how ga uses nonlcon, see “Description of the Nonlinear Constraint Solver” on page 5-38.

**options**

Structure containing optimization options. Create options using gaoptimset, or by exporting options from the Optimization Tool as described in “Importing and Exporting Your Work” in the Optimization Toolbox documentation.

**IntCon**

Vector of positive integers taking values from 1 to nvars. Each value in IntCon represents an x component that is integer-valued.

**Note** When IntCon is nonempty, Aeq and beq must be empty ([ ]), and nonlcon must return empty for ceq. For more information on integer programming, see “Mixed Integer Optimization” on page 5-28.

**problem**

Structure containing the following fields:
<table>
<thead>
<tr>
<th><strong>fitnessfcn</strong></th>
<th>Fitness function</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>nvars</strong></td>
<td>Number of design variables</td>
</tr>
<tr>
<td><strong>Aineq</strong></td>
<td>A matrix for linear inequality constraints</td>
</tr>
<tr>
<td><strong>Bineq</strong></td>
<td>b vector for linear inequality constraints</td>
</tr>
<tr>
<td><strong>Aeq</strong></td>
<td>Aeq matrix for linear equality constraints</td>
</tr>
<tr>
<td><strong>Beq</strong></td>
<td>beq vector for linear equality constraints</td>
</tr>
<tr>
<td><strong>lb</strong></td>
<td>Lower bound on $x$</td>
</tr>
<tr>
<td><strong>ub</strong></td>
<td>Upper bound on $x$</td>
</tr>
<tr>
<td><strong>nonlcon</strong></td>
<td>Nonlinear constraint function</td>
</tr>
<tr>
<td><strong>rngstate</strong></td>
<td>Optional field to reset the state of the random number generator</td>
</tr>
<tr>
<td><strong>intcon</strong></td>
<td>Index vector of integer variables</td>
</tr>
<tr>
<td><strong>solver</strong></td>
<td>'ga'</td>
</tr>
<tr>
<td><strong>options</strong></td>
<td>Options structure created using gaoptimset or the Optimization Tool</td>
</tr>
</tbody>
</table>

Create problem by exporting a problem from the Optimization Tool, as described in “Importing and Exporting Your Work” in the Optimization Toolbox documentation.

### Output Arguments

- **$x$**
  - Best point that `ga` located during its iterations.

- **fval**
  - Fitness function evaluated at $x$.

- **exitflag**
  - Integer giving the reason `ga` stopped iterating:
<table>
<thead>
<tr>
<th>Exit Flag</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>Without nonlinear constraints</strong> — Average cumulative change in value of the fitness function over StallGenLimit generations is less than TolFun, and the constraint violation is less than TolCon.</td>
</tr>
<tr>
<td></td>
<td><strong>With nonlinear constraints</strong> — Magnitude of the complementarity measure (see “Definitions” on page 12-15) is less than $\sqrt{TolCon}$, the subproblem is solved using a tolerance less than TolFun, and the constraint violation is less than TolCon.</td>
</tr>
<tr>
<td>2</td>
<td>Fitness limit reached and the constraint violation is less than TolCon.</td>
</tr>
<tr>
<td>3</td>
<td>Value of the fitness function did not change in StallGenLimit generations and the constraint violation is less than TolCon.</td>
</tr>
<tr>
<td>4</td>
<td>Magnitude of step smaller than machine precision and the constraint violation is less than TolCon.</td>
</tr>
<tr>
<td>5</td>
<td>Minimum fitness limit FitnessLimit reached and the constraint violation is less than TolCon.</td>
</tr>
<tr>
<td>0</td>
<td>Maximum number of generations Generations exceeded.</td>
</tr>
<tr>
<td>-1</td>
<td>Optimization terminated by the output or plot function.</td>
</tr>
<tr>
<td>-2</td>
<td>No feasible point found.</td>
</tr>
<tr>
<td>-4</td>
<td>Stall time limit StallTimeLimit exceeded.</td>
</tr>
<tr>
<td>-5</td>
<td>Time limit TimeLimit exceeded.</td>
</tr>
</tbody>
</table>

When there are integer constraints, ga uses the penalty fitness value instead of the fitness value for stopping criteria.
output

Structure containing output from each generation and other information about algorithm performance. The output structure contains the following fields:

- **problemtype** — String describing the type of problem, one of:
  - 'unconstrained'
  - 'boundconstraints'
  - 'linearconstraints'
  - 'nonlinearconstr'
  - 'integerconstraints'

- **rngstate** — State of the MATLAB random number generator, just before the algorithm started. You can use the values in rngstate to reproduce the output of ga. See “Reproducing Your Results” on page 5-55.

- **generations** — Number of generations computed.
- **funccount** — Number of evaluations of the fitness function.
- **message** — Reason the algorithm terminated.
- **maxconstraint** — Maximum constraint violation, if any.

population

Matrix whose rows contain the members of the final population.

scores

Column vector of the fitness values (scores for integerconstraints problems) of the final population.

**Definitions**  
**Complementarity Measure**

In the nonlinear constraint solver, the complementarity measure is the norm of the vector whose elements are \( c_i \lambda_i \), where \( c_i \) is the nonlinear
inequality constraint violation, and $\lambda_i$ is the corresponding Lagrange multiplier.

**Examples**

Given the following inequality constraints and lower bounds

\[
\begin{bmatrix}
1 & 1 \\
-1 & 2 \\
2 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\leq
\begin{bmatrix}
2 \\
2 \\
3
\end{bmatrix},
\]

$x_1 \geq 0$, $x_2 \geq 0$,

use this code to find the minimum of the `lincontest6` function, which is provided in your software:

```matlab
A = [1 1; -1 2; 2 1];
b = [2; 2; 3];
lb = zeros(2,1);
[x,fval,exitflag] = ga(@lincontest6,...
    2,A,b,[],[],lb)
```

Optimization terminated: average change in the fitness value less than options.TolFun.

\[
x =
\begin{bmatrix}
0.6700 \\
1.3310
\end{bmatrix}
\]

\[
fval =
-8.2218
\]

\[
exitflag =
1
\]

Optimize a function where some variables must be integers:

```matlab
fun = @(x) (x(1) - 0.2)^2 + ...
    (x(2) - 1.7)^2 + (x(3) - 5.1)^2;
```
x = ga(fun,3,[],[],[],[],[],[],...[2 3]) % variables 2 and 3 are integers

Optimization terminated: average change in the penalty fitness value less than options.TolFun and constraint violation is less than options.TolCon.

x =
0.2000 2.0000 5.0000

**Algorithms**

For a description of the genetic algorithm, see “How the Genetic Algorithm Works” on page 5-21.

For a description of the mixed integer programming algorithm, see “Integer ga Algorithm” on page 5-36.

For a description of the nonlinear constraint algorithm, see “Description of the Nonlinear Constraint Solver” on page 5-38.

**Alternatives**

For problems without integer constraints, consider using patternsearch instead of ga.

**See Also**

gamultiobj | gaoptimset | patternsearch

**How To**

- Chapter 5, “Using the Genetic Algorithm”
- Chapter 1, “Introducing Global Optimization Toolbox Functions”
- Chapter 2, “Writing Files for Optimization Functions”
**Purpose**
Find minima of multiple functions using genetic algorithm

**Syntax**

\[
X = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS})
\]

\[
X = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS}, A, b)
\]

\[
X = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS}, A, b, Aeq, beq)
\]

\[
X = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS}, A, b, Aeq, beq, LB, UB)
\]

\[
X = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS}, A, b, Aeq, beq, LB, UB, \text{options})
\]

\[
X = \text{gamultiobj}(	ext{problem})
\]

\[
[X, FVAL] = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS}, ...)
\]

\[
[X, FVAL, EXITFLAG] = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS}, ...)
\]

\[
[X, FVAL, EXITFLAG, OUTPUT] = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS}, ...)
\]

\[
[X, FVAL, EXITFLAG, OUTPUT, \text{POPULATION}] = \text{gamultiobj}(	ext{FITNESSFCN}, ...)
\]

\[
[X, FVAL, EXITFLAG, OUTPUT, \text{POPULATION}, \text{SCORE}] = \text{gamultiobj}(	ext{FITNESSFCN}, ...)
\]

**Description**

gamultiobj implements the genetic algorithm at the command line to minimize a multicomponent objective function.

\[X = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS})\]
finds a local Pareto set \(X\) of the objective functions defined in \(\text{FITNESSFCN}\). \(\text{NVARS}\) is the dimension of the optimization problem (number of decision variables). \(\text{FITNESSFCN}\) accepts a vector \(X\) of size 1-by-\(\text{NVARS}\) and returns a vector of size 1-by-numberOfObjectives evaluated at a decision variable. \(X\) is a matrix with \(\text{NVARS}\) columns. The number of rows in \(X\) is the same as the number of Pareto solutions. All solutions in a Pareto set are equally optimal; it is up to the designer to select a solution in the Pareto set depending on the application.

\[X = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS}, A, b)\]
finds a local Pareto set \(X\) of the objective functions defined in \(\text{FITNESSFCN}\), subject to the linear inequalities \(A \ast x \leq b\). Linear constraints are supported only for the default PopulationType option ('doubleVector'). Other population types, e.g., 'bitString' and 'custom', are not supported.

\[X = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS}, A, b, Aeq, beq)\]
finds a local Pareto set \(X\) of the objective functions defined in \(\text{FITNESSFCN}\), subject
to the linear equalities \( Aeq \cdot x = beq \) as well as the linear inequalities \( A \cdot x \leq b \). (Set \( A=[] \) and \( b=[] \) if no inequalities exist.) Linear constraints are supported only for the default \texttt{PopulationType} option (\texttt{'doubleVector'}). Other population types, e.g., \texttt{'bitString'} and \texttt{'custom'}, are not supported.

\[
X = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS}, A, b, Aeq, beq, LB, UB)
\]
defines a set of lower and upper bounds on the design variables \( X \) so that a local Pareto set is found in the range \( LB \leq x \leq UB \). Use empty matrices for \( LB \) and \( UB \) if no bounds exist. Set \( LB(i) = -\text{Inf} \) if \( X(i) \) is unbounded below; set \( UB(i) = \text{Inf} \) if \( X(i) \) is unbounded above. Bound constraints are supported only for the default \texttt{PopulationType} option (\texttt{'doubleVector'}). Other population types, e.g., \texttt{'bitString'} and \texttt{'custom'}, are not supported.

\[
X = \text{gamultiobj}(	ext{FITNESSFCN}, \text{NVARS}, A, b, Aeq, beq, LB, UB, \text{options})
\]
finds a Pareto set \( X \) with the default optimization parameters replaced by values in the structure \texttt{options}. \texttt{options} can be created with the \texttt{gaoptimset} function.

\[
X = \text{gamultiobj}(\text{problem})
\]
finds the Pareto set for \texttt{problem}, where \texttt{problem} is a structure containing the following fields:

<table>
<thead>
<tr>
<th>Field</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{fitnessfcn}</td>
<td>Fitness functions</td>
</tr>
<tr>
<td>\texttt{nvars}</td>
<td>Number of design variables</td>
</tr>
<tr>
<td>\texttt{Aineq}</td>
<td>A matrix for linear inequality constraints</td>
</tr>
<tr>
<td>\texttt{bineq}</td>
<td>b vector for linear inequality constraints</td>
</tr>
<tr>
<td>\texttt{Aeq}</td>
<td>A matrix for linear equality constraints</td>
</tr>
<tr>
<td>\texttt{beq}</td>
<td>b vector for linear equality constraints</td>
</tr>
<tr>
<td>\texttt{lb}</td>
<td>Lower bound on ( x )</td>
</tr>
<tr>
<td>\texttt{ub}</td>
<td>Upper bound on ( x )</td>
</tr>
</tbody>
</table>
rngstate | Optional field to reset the state of the random number generator
---|---
options | Options structure created using gaoptimset

Create the structure problem by exporting a problem from Optimization Tool, as described in “Importing and Exporting Your Work” in the Optimization Toolbox documentation.

\[
[X,FVAL] = \text{gamultiobj}(\text{FITNESSFCN}, \text{NVARS}, \ldots)
\]
returns a matrix \(FVAL\), the value of all the objective functions defined in \(\text{FITNESSFCN}\) at all the solutions in \(X\). \(FVAL\) has \(\text{numberOfObjectives}\) columns and same number of rows as does \(X\).

\[
[X,FVAL,\text{EXITFLAG}] = \text{gamultiobj}(\text{FITNESSFCN}, \text{NVARS}, \ldots)
\]
returns \(\text{EXITFLAG}\), which describes the exit condition of \(\text{gamultiobj}\). Possible values of \(\text{EXITFLAG}\) and the corresponding exit conditions are listed in this table.

<table>
<thead>
<tr>
<th>EXITFLAG Value</th>
<th>Exit Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Average change in value of the spread of Pareto set over options.StallGenLimit generations less than options.TolFun</td>
</tr>
<tr>
<td>0</td>
<td>Maximum number of generations exceeded</td>
</tr>
<tr>
<td>-1</td>
<td>Optimization terminated by the output or by the plot function</td>
</tr>
<tr>
<td>-2</td>
<td>No feasible point found</td>
</tr>
<tr>
<td>-5</td>
<td>Time limit exceeded</td>
</tr>
</tbody>
</table>

\[
[X,FVAL,\text{EXITFLAG},\text{OUTPUT}] = \text{gamultiobj}(\text{FITNESSFCN}, \text{NVARS}, \ldots)
\]
returns a structure \(\text{OUTPUT}\) with the following fields:
<table>
<thead>
<tr>
<th>OUTPUT Field</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>rngstate</td>
<td>State of the MATLAB random number generator, just before the algorithm started. You can use the values in rngstate to reproduce the output of ga. See “Reproducing Your Results” on page 5-55.</td>
</tr>
<tr>
<td>generations</td>
<td>Total number of generations, excluding HybridFcn iterations</td>
</tr>
<tr>
<td>funccount</td>
<td>Total number of function evaluations</td>
</tr>
<tr>
<td>maxconstraint</td>
<td>Maximum constraint violation, if any</td>
</tr>
<tr>
<td>message</td>
<td>gamultiobj termination message</td>
</tr>
</tbody>
</table>

\[ [X,FVAL,EXITFLAG,OUTPUT,POPULATION] = \]
\[ \text{gamultiobj(FITNESSFCN, ...)} \]
\[ \text{returns the final POPULATION at termination.} \]

\[ [X,FVAL,EXITFLAG,OUTPUT,POPULATION,SCORE] = \]
\[ \text{gamultiobj(FITNESSFCN, ...)} \]
\[ \text{returns the SCORE of the final POPULATION.} \]

### Examples

This example optimizes two objectives defined by Schaffer’s second function: a vector-valued function of two components and one input argument. The Pareto front is disconnected. Define this function in a file:

```matlab
function y = schaffer2(x) % y has two columns

% Initialize y for two objectives and for all x
y = zeros(length(x),2);

% Evaluate first objective.
% This objective is piecewise continuous.
for i = 1:length(x)
    if x(i) <= 1
        y(i,1) = -x(i);
    elseif x(i) <= 3
        y(i,1) = 12 - 21 * (x(i) - 1.5)^2 + (x(i) - 4)^2;
    else
        y(i,1) = 4 - 2.1 * (x(i) - 4)^2 + (x(i) - 4)^2;
    end
end
```

12-21
\begin{verbatim}
    y(i,1) = x(i) - 2;
    elseif x(i) <= 4
        y(i,1) = 4 - x(i);
    else
        y(i,1) = x(i) - 4;
    end

    % Evaluate second objective
    y(:,2) = (x - 5).^2;

    First, plot the two objectives:
    x = -1:0.1:8;
    y = schaffer2(x);
    plot(x,y(:,1),'.r'); hold on
    plot(x,y(:,2),'.b');

    The two component functions compete in the range [1, 3] and [4, 5]. But
    the Pareto-optimal front consists of only two disconnected regions: [1, 2]
    and [4, 5]. This is because the region [2, 3] is inferior to [1, 2].

    Next, impose a bound constraint on x, \(-5 \leq x \leq 10\) setting
    lb = -5;
    ub = 10;

    The best way to view the results of the genetic algorithm is to visualize
    the Pareto front directly using the @gaplotpareto option. To optimize
    Schaffer's function, a larger population size than the default (15) is
    needed, because of the disconnected front. This example uses 60. Set
    the optimization options as:
    options = gaoptimset('PopulationSize',60,'PlotFcns',...
                        @gaplotpareto);
\end{verbatim}
Now call `gamultiobj`, specifying one independent variable and only the bound constraints:

```matlab
[x,f,exitflag] = gamultiobj(@schaffer2,1,[],[],[],[],... lb,ub,options);
```

Optimization terminated: average change in the spread of Pareto solutions less than options.TolFun.

```matlab
exitflag = 1
```

The vectors `x`, `f(:,1)`, and `f(:,2)` respectively contain the Pareto set and both objectives evaluated on the Pareto set.

**Demos**

The `gamultiobjfitness` demo solves a simple problem with one decision variable and two objectives.

The `gamultiobjoptionsdemo` demo shows how to set options for multiobjective optimization.

**Algorithms**

`gamultiobj` uses a controlled elitist genetic algorithm (a variant of NSGA-II [1]). An elitist GA always favors individuals with better fitness value (rank). A controlled elitist GA also favors individuals that can help increase the diversity of the population even if they have a lower fitness value. It is important to maintain the diversity of population for convergence to an optimal Pareto front. Diversity is maintained by controlling the elite members of the population as the algorithm progresses. Two options, `ParetoFraction` and `DistanceFcn`, control the elitism. `ParetoFraction` limits the number of individuals on the Pareto front (elite members). The distance function, selected by `DistanceFcn`, helps to maintain diversity on a front by favoring individuals that are relatively far away on the front.

**References**

See Also
ga | gaoptimget | gaoptimset | patternsearch | Special
Characters | rand | randn
**Purpose**
Obtain values of genetic algorithm options structure

**Syntax**
val = gaoptimget(options, 'name')
val = gaoptimget(options, 'name', default)

**Description**
val = gaoptimget(options, 'name') returns the value of the parameter name from the genetic algorithm options structure options. gaoptimget(options, 'name') returns an empty matrix [] if the value of name is not specified in options. It is only necessary to type enough leading characters of name to uniquely identify it. gaoptimget ignores case in parameter names.

val = gaoptimget(options, 'name', default) returns the 'name' parameter, but will return the default value if the name parameter is not specified (or is []) in options.

**See Also**
ga | gamultiobj | gaoptimset

**How To**
- “Genetic Algorithm Options” on page 9-31
Purpose

Create genetic algorithm options structure

Syntax

gaoptimset
options = gaoptimset
options = gaoptimset(@ga)
options = gaoptimset(@gamultiobj)
options = gaoptimset('param1',value1,'param2',value2,...)
options = gaoptimset(oldopts,'param1',value1,...)
options = gaoptimset(oldopts,newopts)

Description

gaoptimset with no input or output arguments displays a complete list of parameters with their valid values.

options = gaoptimset (with no input arguments) creates a structure called options that contains the options, or parameters, for the genetic algorithm and sets parameters to [], indicating default values will be used.

options = gaoptimset(@ga) creates a structure called options that contains the default options for the genetic algorithm.

options = gaoptimset(@gamultiobj) creates a structure called options that contains the default options for gamultiobj.

options = gaoptimset('param1',value1,'param2',value2,...) creates a structure called options and sets the value of 'param1' to value1, 'param2' to value2, and so on. Any unspecified parameters are set to their default values. It is sufficient to type only enough leading characters to define the parameter name uniquely. Case is ignored for parameter names.

options = gaoptimset(oldopts,'param1',value1,...) creates a copy of oldopts, modifying the specified parameters with the specified values.

options = gaoptimset(oldopts,newopts) combines an existing options structure, oldopts, with a new options structure, newopts. Any parameters in newopts with nonempty values overwrite the corresponding old parameters in oldopts.
The following table lists the options you can set with `gaoptimset`. See “Genetic Algorithm Options” on page 9-31 for a complete description of these options and their values. Values in {} denote the default value. You can also view the optimization parameters and defaults by typing `gaoptimset` at the command line. I* indicates that `ga` ignores or overwrites the option for mixed integer optimization problems.

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>CreationFcn</td>
<td>I* Handle to the function that creates the initial population</td>
<td><code>@gacreationuniform</code></td>
</tr>
<tr>
<td>CrossoverFcn</td>
<td>I* Handle to the function that the algorithm uses to create crossover children</td>
<td><code>@crossoverheuristic</code></td>
</tr>
<tr>
<td>CrossoverFraction</td>
<td>The fraction of the population at the next generation, not including elite children, that is created by the crossover function</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>Display</td>
<td>Level of display</td>
<td>'off'</td>
</tr>
<tr>
<td>DistanceMeasureFcn</td>
<td>I* Handle to the function that computes distance measure of individuals, computed in decision variable or design space (genotype) or in function space (phenotype)</td>
<td><code>{@distancecrowding,'phenotype'}</code></td>
</tr>
<tr>
<td>Option</td>
<td>Description</td>
<td>Values</td>
</tr>
<tr>
<td>----------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td>EliteCount</td>
<td>Positive integer specifying how many individuals in the current generation are guaranteed to survive to the next generation. Not used in gamultiobj.</td>
<td>Positive integer</td>
</tr>
<tr>
<td>FitnessLimit</td>
<td>Scalar. If the fitness function attains the value of FitnessLimit, the algorithm halts.</td>
<td>Scalar</td>
</tr>
<tr>
<td>FitnessScalingFcn</td>
<td>Handle to the function that scales the values of the fitness function</td>
<td>@fitscalingshiftlinear</td>
</tr>
<tr>
<td>Generations</td>
<td>Positive integer specifying the maximum number of iterations before the algorithm halts</td>
<td>Positive integer</td>
</tr>
<tr>
<td>HybridFcn</td>
<td>I* Handle to a function that continues the optimization after ga terminates or Cell array specifying the hybrid function and its options structure</td>
<td>Function handle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>or</td>
</tr>
<tr>
<td>InitialPenalty</td>
<td>I* Initial value of penalty parameter</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>InitialPopulation</td>
<td>Initial population used to seed the genetic algorithm; can be partial</td>
<td>Matrix</td>
</tr>
<tr>
<td>Option</td>
<td>Description</td>
<td>Values</td>
</tr>
<tr>
<td>--------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>InitialScores</td>
<td>I* Initial scores used to determine fitness; can be partial</td>
<td>Column vector</td>
</tr>
<tr>
<td>MigrationDirection</td>
<td>Direction of migration</td>
<td>'both'</td>
</tr>
<tr>
<td>MigrationFraction</td>
<td>Scalar between 0 and 1 specifying the fraction of individuals in each</td>
<td>Scalar</td>
</tr>
<tr>
<td>MigrationInterval</td>
<td>Positive integer specifying the number of generations that take place</td>
<td>Positive integer</td>
</tr>
<tr>
<td></td>
<td>between migrations of individuals between subpopulations</td>
<td></td>
</tr>
<tr>
<td>MutationFcn</td>
<td>I* Handle to the function that produces mutation children</td>
<td>@mutationuniform</td>
</tr>
<tr>
<td>OutputFcns</td>
<td>Functions that ga calls at each iteration</td>
<td>Function handle or cell array of</td>
</tr>
<tr>
<td></td>
<td>function handles</td>
<td></td>
</tr>
<tr>
<td>ParetoFraction</td>
<td>I* Scalar between 0 and 1 specifying the fraction of individuals to keep</td>
<td>Scalar</td>
</tr>
<tr>
<td></td>
<td>on the first Pareto front while the solver selects individuals from</td>
<td></td>
</tr>
<tr>
<td></td>
<td>higher fronts</td>
<td></td>
</tr>
<tr>
<td>PenaltyFactor</td>
<td>I* Penalty update parameter</td>
<td>Positive scalar</td>
</tr>
<tr>
<td><strong>Option</strong></td>
<td><strong>Description</strong></td>
<td><strong>Values</strong></td>
</tr>
<tr>
<td>--------------</td>
<td>----------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td><code>PlotFcns</code></td>
<td>Array of handles to functions that plot data computed by the algorithm</td>
<td>@gaplotbestf</td>
</tr>
<tr>
<td></td>
<td></td>
<td>For <code>gamultiobj</code> there are additional choices: @gaplotpareto</td>
</tr>
<tr>
<td><code>PlotInterval</code></td>
<td>Positive integer specifying the number of generations between consecutive calls to the plot functions</td>
<td>Positive integer</td>
</tr>
<tr>
<td><code>PopInitRange</code></td>
<td>I* Matrix or vector specifying the range of the individuals in the initial population</td>
<td>Matrix or vector</td>
</tr>
<tr>
<td><code>PopulationSize</code></td>
<td>Size of the population</td>
<td>Positive integer</td>
</tr>
</tbody>
</table>
### Option: PopulationType
String describing the data type of the population — must be 'doubleVector' for mixed integer problems.
- 'bitstring' | 'custom' | {'doubleVector'}
  Note that linear and nonlinear constraints are not satisfied when PopulationType is set to 'bitString' or 'custom'.

### Option: SelectionFcn
Handle to the function that selects parents of crossover and mutation children.
- @selectionremainder | @selectionuniform | {@selectionstochunif} | @selectionroulette | @selectiontournament

### Option: StallGenLimit
Positive integer. The algorithm stops if there is no improvement in the objective function for StallGenLimit consecutive generations.
- Positive integer | {50}

### Option: StallTimeLimit
Positive scalar. The algorithm stops if there is no improvement in the objective function for StallTimeLimit seconds.
- Positive scalar | {Inf}

### Option: TimeLimit
Positive scalar. The algorithm stops after running for TimeLimit seconds.
- Positive scalar | {Inf}

### Option: TolCon
Positive scalar. TolCon is used to determine the feasibility with respect to nonlinear constraints.
- Positive scalar | {1e-6}
<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>TolFun</td>
<td>Positive scalar. The algorithm runs until the cumulative change in the fitness function value over StallGenLimit is less than TolFun.</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>UseParallel</td>
<td>Compute fitness functions of a population in parallel.</td>
<td>'always'</td>
</tr>
<tr>
<td>Vectorized</td>
<td>String specifying whether the computation of the fitness function is vectorized</td>
<td>'on'</td>
</tr>
</tbody>
</table>

**See Also**

ga | gamultiobj | gaoptimget

**How To**

- “Genetic Algorithm Options” on page 9-31
Purpose  Optimization solution

Description  Information on a local minimum, including location, objective function value, and start point or points that lead to the minimum.

GlobalSearch and MultiStart generate a vector of GlobalOptimSolution objects. The vector is ordered by objective function value, from lowest (best) to highest (worst).

Construction  When you run them, GlobalSearch and MultiStart create GlobalOptimSolution objects as output.

Properties  Exitflag

An integer describing the result of the local solver run.

For the meaning of the exit flag, see the description in the appropriate local solver function reference page:

- fmincon
- fminunc
- lsqcurvefit
- lsqnonlin

Fval

Objective function value at the solution.

Output

Output structure returned by the local solver.

X

Solution point, with the same dimensions as the initial point.

X0

Cell array of start points that led to the solution.
GlobalOptimSolution

Copy Semantics

Value. To learn how value classes affect copy operations, see Copying Objects in the MATLAB Programming Fundamentals documentation.

Examples

Use MultiStart to create a vector of GlobalOptimSolution objects:

```matlab
ms = MultiStart;
sixmin = @(x)(4*x(1)^2 - 2.1*x(1)^4 + x(1)^6/3 ... + x(1)*x(2) - 4*x(2)^2 + 4*x(2)^4);
problem = createOptimProblem('fmincon','x0',[-1,2],... 'objective',sixmin,'lb',[-3,-3],'ub',[3,3]);
[xmin,fmin,flag,outpt,allmins] = run(ms,problem,30);
```

allmins is the vector of GlobalOptimSolution objects:

```matlab
allmins
```

```matlab
allmins =

1x30 GlobalOptimSolution

Properties:
  X
  Fval
  Exitflag
  Output
  X0
```

See Also

MultiStart | MultiStart.run | GlobalSearch | GlobalSearch.run

Tutorials

- “Multiple Solutions” on page 3-24
- “Example: Visualizing the Basins of Attraction” on page 3-32

How To

- Class Attributes
- Property Attributes
Purpose
Find global minimum

Description
A GlobalSearch object contains properties (options) that affect how the run method searches for a global minimum, or generates a GlobalOptimSolution object.

Construction
$gs = \text{GlobalSearch}$ constructs a new global search optimization solver with its properties set to the defaults.

$gs = \text{GlobalSearch}('Property\ Name', Property\ Value, \ldots)$ constructs the object using options, specified as property name and value pairs.

$gs = \text{GlobalSearch}(oldgs, 'Property\ Name', Property\ Value, \ldots)$ constructs a copy of the GlobalSearch solver oldgs. The gs object has the named properties altered with the specified values.

$gs = \text{GlobalSearch}(ms)$ constructs $gs$, a GlobalSearch solver, with common parameter values from the ms MultiStart solver.

Properties
BasinRadiusFactor
A basin radius decreases after MaxWaitCycle consecutive start points are within the basin. The basin radius decreases by a factor of $1 - \text{BasinRadiusFactor}$.

Set BasinRadiusFactor to 0 to disable updates of the basin radius.

Default: 0.2

Display
Detail level of iterative display. Possible values:

- 'final' — Report summary results after run finishes.
- 'iter' — Report results after the initial fmincon run, after Stage 1, after every 200 start points, and after every run of fmincon, in addition to the final summary.
GlobalSearch

- 'off' — No display.

**Default:** 'final'

**DistanceThresholdFactor**

A multiplier for determining whether a trial point is in an existing basin of attraction. For details, see “Examine Stage 2 Trial Point to See if fmincon Runs” on page 3-47.

**Default:** 0.75

**MaxTime**

Time in seconds for a run. GlobalSearch halts when it sees MaxTime seconds have passed since the beginning of the run.

**Default:** Inf

**MaxWaitCycle**

A positive integer tolerance appearing in several points in the algorithm:

- If the observed penalty function of MaxWaitCycle consecutive trial points is at least the penalty threshold, then raise the penalty threshold (see PenaltyThresholdFactor).
- If MaxWaitCycle consecutive trial points are in a basin, then update that basin’s radius (see BasinRadiusFactor).

**Default:** 20

**NumStageOnePoints**

Number of start points in Stage 1. For details, see “Obtain Stage 1 Start Point, Run” on page 3-46.

**Default:** 200
GlobalSearch

NumTrialPoints

Number of potential start points to examine in addition to \( x_0 \) from the problem structure. GlobalSearch runs only those potential start points that pass several tests. For more information, see “GlobalSearch Algorithm” on page 3-45.

**Default:** 1000

OutputFcns

A function handle or cell array of function handles to output functions. Output functions run after each local solver call. They also run when the global solver starts and ends. Write your output functions using the syntax described in “OutputFcns” on page 9-3. See “Example: GlobalSearch Output Function” on page 3-35.

**Default:** []

PenaltyThresholdFactor

Determines increase in the penalty threshold. For details, see React to Large Counter Values.

**Default:** 0.2

PlotFcns

A function handle or cell array of function handles to plot functions. Plot functions run after each local solver call. They also run when the global solver starts and ends. Write your plot functions using the syntax described in “OutputFcns” on page 9-3. There are two built-in plot functions:

- @gsplotbestf plots the best objective function value.
- @gsplotfunccount plots the number of function evaluations.

See “Example: MultiStart Plot Function” on page 3-39.
GlobalSearch

Default: []

StartPointsToRun

Directs the solver to exclude certain start points from being run:

- all — Accept all start points.
- bounds — Reject start points that do not satisfy bounds.
- bounds-ineqs — Reject start points that do not satisfy bounds or inequality constraints.

GlobalSearch checks the StartPointsToRun property only during Stage 2 of the GlobalSearch algorithm (the main loop). For more information, see “GlobalSearch Algorithm” on page 3-45.

Default: 'all'

TolFun

Describes how close two objective function values must be for solvers to consider them identical for creating the vector of local solutions. Solvers consider two solutions identical if they are within TolX distance of each other and have objective function values within TolFun of each other. If both conditions are not met, solvers report the solutions as distinct. Set TolFun to 0 to obtain the results of every local solver run. Set TolFun to a larger value to have fewer results.

Default: 1e-6

TolX

Describes how close two points must be for solvers to consider them identical for creating the vector of local solutions. Solvers compute the distance between a pair of points with norm, the Euclidean distance. Solvers consider two solutions identical if they are within TolX distance of each other and have objective
function values within `TolFun` of each other. If both conditions are not met, solvers report the solutions as distinct. Set `TolX` to 0 to obtain the results of every local solver run. Set `TolX` to a larger value to have fewer results.

**Default:** 1e-6

### Methods

**run**

Find global minimum

### Copy Semantics

Value. To learn how value classes affect copy operations, see Copying Objects in the MATLAB Programming Fundamentals documentation.

### Examples

Solve a problem using a default `GlobalSearch` object:

```matlab
opts = optimset('Algorithm','interior-point');
problem = createOptimProblem('fmincon','objective', @(x) x.^2 + 4*sin(5*x), 'x0', 3, 'lb', -5, 'ub', 5, 'options', opts);
gs = GlobalSearch;
[x,f] = run(gs,problem)
```

### Algorithms

A detailed description of the algorithm appears in “GlobalSearch Algorithm” on page 3-45. Ugray et al. [1] describes both the algorithm and the scatter-search method of generating trial points.

### References


### See Also

`MultiStart` | `createOptimProblem` | `GlobalOptimSolution`

### Tutorials

- Chapter 3, “Using GlobalSearch and MultiStart”
- “GlobalSearch and MultiStart Examples” on page 3-70
GlobalSearch

How To

• Class Attributes
• Property Attributes
Purpose

List custom start points in set

Syntax

tpts = list(CS)

Description

`tpts = list(CS)` returns the matrix of start points in the `CS` CustomStartPoints object.

Input Arguments

CS

A CustomStartPointSet object.

Output Arguments

`tpts`

Matrix of start points. The rows of `tpts` represent the start points.

Examples

Create a CustomStartPointSet containing 40 seven-dimensional normally distributed points, then use `list` to get the matrix of points from the object:

```plaintext
startpts = randn(40,7)  % 40 seven-dimensional start points
cs = CustomStartPointSet(startpts);  % cs is an object
startpts2 = list(cs)  % startpts2 = startpts
```

See Also

CustomStartPointSet | createOptimProblem

Tutorials

• “CustomStartPointSet Object for Start Points” on page 3-18
### Purpose
Generate start points

### Syntax
```
points = list(RandSet,problem)
```

### Description
```
points = list(RandSet,problem) generates pseudorandom start points using the parameters in the RandSet RandomStartPointSet object, and information from the problem problem structure.
```

### Input Arguments
- **RandSet**
  A RandomStartPointSet object. This contains parameters for generating the points: number of points, and artificial bounds.
- **problem**
  An optimization problem structure. `list` generates points uniformly within the bounds of the problem structure. If a component is unbounded, `list` uses the artificial bounds from RandSet. `list` takes the dimension of the points from the `x0` field in problem.

### Output Arguments
- **points**
  A k-by-n matrix. The number of rows k is the number of start points that RandSet specifies. The number of columns n is the dimension of the start points. n is equal to the number of elements in the `x0` field in problem. The MultiStart algorithm uses each row of points as an initial point in an optimization.

### Examples
Create a matrix representing 40 seven-dimensional start points:
```
rs = RandomStartPointSet('NumStartPoints',40); % 40 points
problem = createOptimProblem('fminunc','x0',ones(7,1),...
   'objective',@rosenbrock);
ptmatrix = list(rs,problem); % matrix values between
   % -1000 and 1000 since those are the default bounds
   % for unconstrained dimensions
```
Algorithms

The `list` method generates a pseudorandom random matrix using the default random number stream. Each row of the matrix represents a start point to run. `list` generates points that satisfy the bounds in `problem`. If `lb` is the vector of lower bounds, `ub` is the vector of upper bounds, there are `n` dimensions in a point, and there are `k` rows in the matrix, the random matrix is

\[ \text{lb} + (\text{ub} - \text{lb}).*\text{rand}(k,n) \]

- If a component has no bounds, `RandomStartPointSet` uses a lower bound of `-ArtificialBound`, and an upper bound of `ArtificialBound`.
- If a component has a lower bound `lb`, but no upper bound, `RandomStartPointSet` uses an upper bound of `lb + 2*ArtificialBound`.
- Similarly, if a component has an upper bound `ub`, but no lower bound, `RandomStartPointSet` uses a lower bound of `ub - 2*ArtificialBound`.

The default value of `ArtificialBound` is 1000.

To obtain identical pseudorandom results, reset the default random number stream. See “Reproducing Results” on page 3-67.

See Also

`RandomStartPointSet` | `createOptimProblem` | `MultiStart`

Tutorials

- “RandomStartPointSet Object for Start Points” on page 3-17
- “Reproducing Results” on page 3-67
**Purpose**
Find multiple local minima

**Description**
A MultiStart object contains properties (options) that affect how the run method repeatedly runs a local solver, or generates a GlobalOptimSolution object.

**Construction**

\[
MS = \text{MultiStart}
\]

constructs \( MS \), a MultiStart solver with its properties set to the defaults.

\[
MS = \text{MultiStart('PropertyName',PropertyValue,...)}
\]

constructs \( MS \) using options, specified as property name and value pairs.

\[
MS = \text{MultiStart(oldMS,'PropertyName',PropertyValue,...)}
\]

creates a copy of the \( oldMS \) MultiStart solver, with the named properties changed to the specified values.

\[
MS = \text{MultiStart(GS)}
\]

constructs \( MS \), a MultiStart solver, with common parameter values from the \( GS \) GlobalSearch solver.

**Properties**

**Display**

Detail level of the output to the Command Window:

- 'final' — Report summary results after run finishes.
- 'iter' — Report results after each local solver run, in addition to the final summary.
- 'off' — No display.

**Default:** final

**MaxTime**

Tolerance on the time MultiStart runs. MultiStart and its local solvers halt when they see MaxTime seconds have passed since the beginning of the run. Time means wall clock as opposed to processor cycles.

**Default:** Inf
OutputFcns

A function handle or cell array of function handles to output functions. Output functions run after each local solver call. They also run when the global solver starts and ends. Write your output functions using the syntax described in “OutputFcns” on page 9-3. See “Example: GlobalSearch Output Function” on page 3-35.

Default: []

PlotFcns

A function handle or cell array of function handles to plot functions. Plot functions run after each local solver call. They also run when the global solver starts and ends. Write your plot functions using the syntax described in “OutputFcns” on page 9-3. There are two built-in plot functions:

- @gsplotbestf plots the best objective function value.
- @gsplotfunccount plots the number of function evaluations.

See “Example: MultiStart Plot Function” on page 3-39.

Default: []

StartPointsToRun

Directs the solver to exclude certain start points from being run:

- all — Accept all start points.
- bounds — Reject start points that do not satisfy bounds.
- bounds-ineqs — Reject start points that do not satisfy bounds or inequality constraints.

Default: all

TolFun
MultiStart

Describes how close two objective function values must be for solvers to consider them identical for creating the vector of local solutions. Solvers consider two solutions identical if they are within $\text{TolX}$ distance of each other and have objective function values within $\text{TolFun}$ of each other. If both conditions are not met, solvers report the solutions as distinct. Set $\text{TolFun}$ to 0 to obtain the results of every local solver run. Set $\text{TolFun}$ to a larger value to have fewer results.

**Default:** 1e-6

$\text{TolX}$

Describes how close two points must be for solvers to consider them identical for creating the vector of local solutions. Solvers compute the distance between a pair of points with $\text{norm}$, the Euclidean distance. Solvers consider two solutions identical if they are within $\text{TolX}$ distance of each other and have objective function values within $\text{TolFun}$ of each other. If both conditions are not met, solvers report the solutions as distinct. Set $\text{TolX}$ to 0 to obtain the results of every local solver run. Set $\text{TolX}$ to a larger value to have fewer results.

**Default:** 1e-6

$\text{UseParallel}$

Distribute local solver calls to multiple processors:

- **always** — Distribute the local solver calls to multiple processors.
- **never** — Cannot not run in parallel.

**Default:** never
Methods

run
Run local solver from multiple points

Copy Semantics
Value. To learn how value classes affect copy operations, see Copying Objects in the MATLAB Programming Fundamentals documentation.

Examples
Run MultiStart on 20 instances of a problem using the fmincon sqp algorithm:

```matlab
opts = optimset('Algorithm','sqp');
problem = createOptimProblem('fmincon','objective',... @(x) x.^2 + 4*sin(5*x),'x0',3,'lb',-5,'ub',5,'options',opts);
ms = MultiStart;
[x,f] = run(ms,problem,20)
```

Algorithms
A detailed description of the algorithm appears in “MultiStart Algorithm” on page 3-49.

See Also
GlobalSearch | createOptimProblem | RandomStartPointSet | CustomStartPointSet | GlobalOptimSolution

Tutorials
• Chapter 3, “Using GlobalSearch and MultiStart”
• “GlobalSearch and MultiStart Examples” on page 3-70
• Chapter 8, “Parallel Processing”

How To
• Class Attributes
• Property Attributes
**Purpose**
Find minimum of function using pattern search

**Syntax**

\[
x = \text{patternsearch}(@\text{fun},x0)
\]

\[
x = \text{patternsearch}(@\text{fun},x0,A,b)
\]

\[
x = \text{patternsearch}(@\text{fun},x0,A,b,Aeq,beq)
\]

\[
x = \text{patternsearch}(@\text{fun},x0,A,b,Aeq,beq,\text{LB},\text{UB})
\]

\[
x = \text{patternsearch}(@\text{fun},x0,A,b,Aeq,beq,\text{LB},\text{UB},\text{nonlcon})
\]

\[
x = \text{patternsearch}(\text{problem})
\]

\[
[x,fval] = \text{patternsearch}(@\text{fun},x0, ...)
\]

\[
[x,fval,\text{exitflag}] = \text{patternsearch}(@\text{fun},x0, ...)
\]

\[
[x,fval,\text{exitflag},\text{output}] = \text{patternsearch}(@\text{fun},x0, ...)
\]

**Description**

patternsearch finds the minimum of a function using a pattern search.

\[
x = \text{patternsearch}(@\text{fun},x0)
\]

finds the local minimum, \(x\), to the MATLAB function, \(\text{fun}\), that computes the values of the objective function \(f(x)\), and \(x0\) is an initial point for the pattern search algorithm. The function \text{patternsearch} accepts the objective function as a function handle of the form @fun. The function \text{fun} accepts a vector input and returns a scalar function value.

**Note** To write a function with additional parameters to the independent variables that can be called by \text{patternsearch}, see the section on “Passing Extra Parameters” in the Optimization Toolbox documentation.

\[
x = \text{patternsearch}(@\text{fun},x0,A,b)
\]

finds a local minimum \(x\) to the function \text{fun}, subject to the linear inequality constraints represented in matrix form by \(Ax \leq b\).

If the problem has \(m\) linear inequality constraints and \(n\) variables, then

- \(A\) is a matrix of size \(m\)-by-\(n\).
- \(b\) is a vector of length \(m\).
\[ x = \text{patternsearch}(\texttt{@fun}, x_0, A, b, Aeq, beq) \]
finds a local minimum \( x \) to the function \( \texttt{fun} \), starting at \( x_0 \), and subject to the constraints

\[
Ax \leq b
\]
\[
Aeq \times x = beq
\]

where \( Aeq \times x = beq \) represents the linear equality constraints in matrix form. If the problem has \( r \) linear equality constraints and \( n \) variables, then

- \( Aeq \) is a matrix of size \( r \)-by-\( n \).
- \( beq \) is a vector of length \( r \).

If there are no inequality constraints, pass empty matrices, \([\ ]\), for \( A \) and \( b \).

\[ x = \text{patternsearch}(\texttt{@fun}, x_0, A, b, Aeq, beq, LB, UB) \]
defines a set of lower and upper bounds on the design variables, \( x \), so that a solution is found in the range \( LB \leq x \leq UB \). If the problem has \( n \) variables, \( LB \) and \( UB \) are vectors of length \( n \). If \( LB \) or \( UB \) is empty (or not provided), it is automatically expanded to -\( \text{Inf} \) or \( \text{Inf} \), respectively. If there are no inequality or equality constraints, pass empty matrices for \( A \), \( b \), \( Aeq \) and \( beq \).

\[ x = \text{patternsearch}(\texttt{@fun}, x_0, A, b, Aeq, beq, LB, UB, nonlcon) \]
subjects the minimization to the constraints defined in \( \texttt{nonlcon} \), a nonlinear constraint function. The function \( \texttt{nonlcon} \) accepts \( x \) and returns the vectors \( C \) and \( Ceq \), representing the nonlinear inequalities and equalities respectively. \( \texttt{fmincon} \) minimizes \( \texttt{fun} \) such that \( C(x) \leq 0 \) and \( Ceq(x) = 0 \). (Set \( LB = [\ ] \) and \( UB = [\ ] \) if no bounds exist.)

\[ x = \text{patternsearch}(\texttt{@fun}, x_0, A, b, Aeq, beq, LB, UB, nonlcon, options) \]
minimizes \( \texttt{fun} \) with the default optimization parameters replaced by values in \( \texttt{options} \). The structure \( \texttt{options} \) can be created using \( \texttt{psoptimset} \).

\[ x = \text{patternsearch}(\texttt{problem}) \]
finds the minimum for \( \texttt{problem} \), where \( \texttt{problem} \) is a structure containing the following fields:
- **objective** — Objective function
- **x0** — Starting point
- **Aineq** — Matrix for linear inequality constraints
- **bineq** — Vector for linear inequality constraints
- **Aeq** — Matrix for linear equality constraints
- **beq** — Vector for linear equality constraints
- **lb** — Lower bound for \( x \)
- **ub** — Upper bound for \( x \)
- **nonlcon** — Nonlinear constraint function
- **Solver** — 'patternsearch'
- **options** — Options structure created with `psoptimset`
- **rngstate** — Optional field to reset the state of the random number generator

Create the structure `problem` by exporting a problem from the Optimization Tool, as described in “Importing and Exporting Your Work” in the Optimization Toolbox documentation.

**Note** `problem` must have all the fields as specified above.

\[
\begin{align*}
[x,fval] &= \text{patternsearch}(@\text{fun},x0, \ldots) \text{ returns the value of the objective function } \text{fun} \text{ at the solution } x. \\
[x,fval,\text{exitflag}] &= \text{patternsearch}(@\text{fun},x0, \ldots) \text{ returns } \text{exitflag}, \text{ which describes the exit condition of patternsearch. Possible values of } \text{exitflag} \text{ and the corresponding conditions are}
\end{align*}
\]
<table>
<thead>
<tr>
<th>Exit Flag</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Without nonlinear constraints — Magnitude of the mesh size is less than the specified tolerance and constraint violation is less than TolCon.</td>
</tr>
<tr>
<td></td>
<td>With nonlinear constraints — Magnitude of the complementarity measure (defined after this table) is less than sqrt(TolCon), the subproblem is solved using a mesh finer than TolMesh, and the constraint violation is less than TolCon.</td>
</tr>
<tr>
<td>2</td>
<td>Change in x is less than the specified tolerance and the constraint violation is less than TolCon.</td>
</tr>
<tr>
<td>4</td>
<td>Magnitude of step smaller than machine precision and the constraint violation is less than TolCon.</td>
</tr>
<tr>
<td>0</td>
<td>Maximum number of function evaluations or iterations reached.</td>
</tr>
<tr>
<td>-1</td>
<td>Optimization terminated by the output or plot function.</td>
</tr>
<tr>
<td>-2</td>
<td>No feasible point found.</td>
</tr>
</tbody>
</table>

In the nonlinear constraint solver, the **complementarity measure** is the norm of the vector whose elements are \( c_i \lambda_i \), where \( c_i \) is the nonlinear inequality constraint violation, and \( \lambda_i \) is the corresponding Lagrange multiplier.

\[
[x,fval,exitflag,output] = \text{patternsearch}(\text{@fun},x0, \ldots)
\]

returns a structure output containing information about the search. The output structure contains the following fields:

- **function** — Objective function
- **problemtype** — Type of problem: unconstrained, bound constrained or linear constrained
- **pollmethod** — Polling technique
- **searchmethod** — Search technique used, if any
patternsearch

- iterations — Total number of iterations
- funccount — Total number of function evaluations
- meshsize — Mesh size at x
- maxconstraint — Maximum constraint violation, if any
- message — Reason why the algorithm terminated

Note patternsearch does not accept functions whose inputs are of type complex. To solve problems involving complex data, write your functions so that they accept real vectors, by separating the real and imaginary parts.

Examples

Given the following constraints

\[
\begin{bmatrix}
1 & 1 \\
-1 & 2 \\
2 & 1 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\end{bmatrix}
\leq
\begin{bmatrix}
2 \\
2 \\
3 \\
\end{bmatrix},
\]

\[x_1 \geq 0, \quad x_2 \geq 0,\]

the following code finds the minimum of the function, lincontest6, that is provided with your software:

```matlab
A = [1 1; -1 2; 2 1];
b = [2; 2; 3];
lb = zeros(2,1);
[x,fval,exitflag] = patternsearch(@(lincontest6,[0 0],...A,b,[],[],lb)
Optimization terminated: mesh size less than
options.TolMesh.

x =
0.6667   1.3333
```
fval =
-8.2222

exitflag =
1

References


**See Also**

optimtool | psoptimget | psoptimset | ga | simulannealbnd
Purpose
Obtain values of pattern search options structure

Syntax
val = psoptimget(options,'name')
val = psoptimget(options,'name',default)

Description
val = psoptimget(options,'name') returns the value of the parameter name from the pattern search options structure options. psoptimget(options,'name') returns an empty matrix [] if the value of name is not specified in options. It is only necessary to type enough leading characters of name to uniquely identify it. psoptimget ignores case in parameter names.

val = psoptimget(options,'name',default) returns the value of the parameter name from the pattern search options structure options, but returns default if the parameter is not specified (as in []) in options.

Examples
val = psoptimget(opts,'TolX',1e-4);
returns val = 1e-4 if the TolX property is not specified in opts.

See Also
psoptimset | patternsearch

How To
• “Pattern Search Options” on page 9-9
**Purpose**
Create pattern search options structure

**Syntax**

```matlab
psoptimset
options = psoptimset
options = psoptimset('param1',value1,'param2',value2,...)
options = psoptimset(oldopts,'param1',value1,...)
options = psoptimset(oldopts,newopts)
```

**Description**

`psoptimset` with no input or output arguments displays a complete list of parameters with their valid values.

```matlab
options = psoptimset` (with no input arguments) creates a structure called options that contains the options, or parameters, for the pattern search and sets parameters to their default values.

options = psoptimset('param1',value1,'param2',value2,...) creates a structure options and sets the value of 'param1' to value1, 'param2' to value2, and so on. Any unspecified parameters are set to their default values. It is sufficient to type only enough leading characters to define the parameter name uniquely. Case is ignored for parameter names.

options = psoptimset(oldopts,'param1',value1,...) creates a copy of oldopts, modifying the specified parameters with the specified values.

options = psoptimset(oldopts,newopts) combines an existing options structure, oldopts, with a new options structure, newopts. Any parameters in newopts with nonempty values overwrite the corresponding old parameters in oldopts.

**Options**
The following table lists the options you can set with `psoptimset`. See “Pattern Search Options” on page 9-9 for a complete description of the options and their values. Values in {} denote the default value. You can also view the optimization parameters and defaults by typing `psoptimset` at the command line.
<table>
<thead>
<tr>
<th><strong>Option</strong></th>
<th><strong>Description</strong></th>
<th><strong>Values</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cache</td>
<td>With Cache set to 'on', <code>patternsearch</code> keeps a history of the mesh points it polls and does not poll points close to them again at subsequent iterations. Use this option if <code>patternsearch</code> runs slowly because it is taking a long time to compute the objective function. If the objective function is stochastic, it is advised not to use this option.</td>
<td>'on'</td>
</tr>
<tr>
<td>CacheSize</td>
<td>Size of the history</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>CacheTol</td>
<td>Positive scalar specifying how close the current mesh point must be to a point in the history in order for <code>patternsearch</code> to avoid polling it. Use if 'Cache' option is set to 'on'.</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>CompletePoll</td>
<td>Complete poll around current iterate</td>
<td>'on'</td>
</tr>
<tr>
<td>CompleteSearch</td>
<td>Complete search around current iterate when the search method is a poll method</td>
<td>'on'</td>
</tr>
<tr>
<td>Display</td>
<td>Level of display</td>
<td>'off'</td>
</tr>
<tr>
<td>InitialMeshSize</td>
<td>Initial mesh size for pattern algorithm</td>
<td>Positive scalar</td>
</tr>
<tr>
<td><strong>Option</strong></td>
<td><strong>Description</strong></td>
<td><strong>Values</strong></td>
</tr>
<tr>
<td>----------------</td>
<td>--------------------------------------------------------------------------------</td>
<td>------------------------------------------------</td>
</tr>
<tr>
<td>InitialPenalty</td>
<td>Initial value of the penalty parameter</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>MaxFunEvals</td>
<td>Maximum number of objective function evaluations</td>
<td>Positive integer</td>
</tr>
<tr>
<td>MaxIter</td>
<td>Maximum number of iterations</td>
<td>Positive integer</td>
</tr>
<tr>
<td>MaxMeshSize</td>
<td>Maximum mesh size used in a poll/search step</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>MeshAccelerator</td>
<td>Accelerate convergence near a minimum</td>
<td>'on'</td>
</tr>
<tr>
<td>MeshContraction</td>
<td>Mesh contraction factor, used when iteration is unsuccessful</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>MeshExpansion</td>
<td>Mesh expansion factor, expands mesh when iteration is successful</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>MeshRotate</td>
<td>Rotate the pattern before declaring a point to be optimum</td>
<td>'off'</td>
</tr>
<tr>
<td>OutputFcns</td>
<td>Specifies a user-defined function that an optimization function calls at each iteration</td>
<td>Function handle or cell array of function handles</td>
</tr>
<tr>
<td>PenaltyFactor</td>
<td>Penalty update parameter</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>PlotFcns</td>
<td>Specifies plots of output from the pattern search</td>
<td>@psplotbestf</td>
</tr>
<tr>
<td><strong>Option</strong></td>
<td><strong>Description</strong></td>
<td><strong>Values</strong></td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------------------------------------------------------------------</td>
<td>-------------------------------------------------</td>
</tr>
<tr>
<td>PlotInterval</td>
<td>Specifies that plot functions will be called at every interval</td>
<td>{1}</td>
</tr>
<tr>
<td>PollingOrder</td>
<td>Order of poll directions in pattern search</td>
<td>'Random'</td>
</tr>
<tr>
<td>PollMethod</td>
<td>Polling strategy used in pattern search</td>
<td>{'GPSPositiveBasis2N'}</td>
</tr>
<tr>
<td>ScaleMesh</td>
<td>Automatic scaling of variables</td>
<td>{'on'}</td>
</tr>
<tr>
<td>SearchMethod</td>
<td>Type of search used in pattern search</td>
<td>@GPSPositiveBasis2N</td>
</tr>
<tr>
<td>TimeLimit</td>
<td>Total time (in seconds) allowed for optimization</td>
<td>Positivescalar</td>
</tr>
<tr>
<td>TolBind</td>
<td>Binding tolerance</td>
<td>Positivescalar</td>
</tr>
<tr>
<td>TolCon</td>
<td>Tolerance on constraints</td>
<td>Positivescalar</td>
</tr>
<tr>
<td>TolFun</td>
<td>Tolerance on function</td>
<td>Positivescalar</td>
</tr>
<tr>
<td>TolMesh</td>
<td>Tolerance on mesh size</td>
<td>Positivescalar</td>
</tr>
<tr>
<td>TolX</td>
<td>Tolerance on variable</td>
<td>Positivescalar</td>
</tr>
</tbody>
</table>
## `psoptimset` Function

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>UseParallel</td>
<td>Compute objective functions of a poll or search in parallel.</td>
<td>'always'</td>
</tr>
<tr>
<td>Vectorized</td>
<td>Specifies whether functions are vectorized</td>
<td>'on'</td>
</tr>
</tbody>
</table>

### See Also
- `patternsearch` | `psoptimget`
**Purpose**
Find global minimum

**Syntax**

\[
x = \text{run}(gs, \text{problem})
\]

\[
[x, fval] = \text{run}(gs, \text{problem})
\]

\[
[x, fval, exitflag] = \text{run}(gs, \text{problem})
\]

\[
[x, fval, exitflag, output] = \text{run}(gs, \text{problem})
\]

\[
[x, fval, exitflag, output, solutions] = \text{run}(gs, \text{problem})
\]

**Description**

\[
x = \text{run}(gs, \text{problem})\text{ finds a point } x \text{ that solves the optimization problem described in the } \text{problem} \text{ structure.}
\]

\[
[x, fval] = \text{run}(gs, \text{problem})\text{ returns the value of the objective function in } \text{problem} \text{ at the point } x.
\]

\[
[x, fval, exitflag] = \text{run}(gs, \text{problem})\text{ returns an exit flag describing the results of the multiple local searches.}
\]

\[
[x, fval, exitflag, output] = \text{run}(gs, \text{problem})\text{ returns an output structure describing the iterations of the run.}
\]

\[
[x, fval, exitflag, output, solutions] = \text{run}(gs, \text{problem})\text{ returns a vector of solutions containing the distinct local minima found during the run.}
\]

**Input Arguments**

- **gs**
  A GlobalSearch object.

- **problem**
  Problem structure. Create problem with `createOptimProblem` or by exporting a problem structure from the Optimization Tool. problem must contain at least the following fields:
  - solver
  - objective
  - x0
• options — Both createOptimProblem and the Optimization Tool always include an options field in the problem structure.

Output Arguments

x

Minimizing point of the objective function.

fval

Objective function value at the minimizer x.

exitflag

Describes the results of the multiple local searches. Values are:

2  At least one local minimum found. Some runs of the local solver converged.
1  At least one local minimum found. All runs of the local solver converged.
0  No local minimum found. Local solver called at least once, and at least one local solver exceeded the MaxIter or MaxFunEvals tolerances.
-1  One or more local solver runs stopped by the local solver output or plot function.
-2  No feasible local minimum found.
-5  MaxTime limit exceeded.
-8  No solution found. All runs had local solver exit flag -2 or smaller, not all equal -2.
-10 Failures encountered in user-provided functions.

output

A structure describing the iterations of the run. Fields in the structure:
funcCount
Number of function evaluations.

localSolverIncomplete
Number of local solver runs with 0 exit flag.

localSolverNoSolution
Number of local solver runs with negative exit flag.

localSolverSuccess
Number of local solver runs with positive exit flag.

localSolverTotal
Total number of local solver runs.

message
Exit message.

solutions
A vector of GlobalOptimSolution objects containing the distinct local solutions found during the run. The vector is sorted by objective function value; the first element is best (smallest value). The object contains:

- **X** Solution point returned by the local solver.
- **Fval** Objective function value at the solution.
- **Exitflag** Integer describing the result of the local solver run.
- **Output** Output structure returned by the local solver.
- **X0** Cell array of start points that led to the solution.

**Examples**
Use a default GlobalSearch object to solve the six-hump camel back problem (see “Run the Solver” on page 3-19):

```matlab
gs = GlobalSearch;
sixmin = @(x)(4*x(1)^2 - 2.1*x(1)^4 + x(1)^6/3 ... + x(1)*x(2) - 4*x(2)^2 + 4*x(2)^4);
problem = createOptimProblem('fmincon','x0',[-1,2],... 'objective',sixmin,'lb',[-3,-3],'ub',[3,3]);
[xmin,fmin,flag,outpt,allmins] = run(gs,problem);
```
**Algorithms**
A detailed description of the algorithm appears in “GlobalSearch Algorithm” on page 3-45. Ugray et al. [1] describes both the algorithm and the scatter-search method of generating trial points.

**References**

**See Also**
GlobalSearch | createOptimProblem | GlobalOptimSolution

**Tutorials**
- “Run the Solver” on page 3-19
- “How to Optimize with GlobalSearch and MultiStart” on page 3-2
- “GlobalSearch and MultiStart Examples” on page 3-70
Purpose
Run local solver from multiple points

Syntax

\[
x = \text{run}(ms,\text{problem},k)
\]

\[
x = \text{run}(ms,\text{problem, startpts})
\]

\[
[x, fval] = \text{run}(...)
\]

\[
[x, fval, exitflag] = \text{run}(...)
\]

\[
[x, fval, exitflag, output] = \text{run}(...)
\]

\[
[x, fval, exitflag, output, solutions] = \text{run}(...)
\]

Description

\(x = \text{run}(ms,\text{problem},k)\) runs the \(ms\) MultiStart object on the optimization problem specified in \text{problem} for a total of \(k\) runs. \(x\) is the point where the lowest function value was found. For the \text{lsqcurvefit} and \text{lsqnonlin} solvers, MultiStart minimizes the sum of squares at \(x\), also known as the residual.

\(x = \text{run}(ms,\text{problem, startpts})\) runs the \(ms\) MultiStart object on the optimization problem specified in \text{problem} using the start points described in \text{startpts}.

\([x, fval] = \text{run}(...)\) returns the lowest objective function value \(fval\) at \(x\). For the \text{lsqcurvefit} and \text{lsqnonlin} solvers, \(fval\) contains the residual.

\([x, fval, exitflag] = \text{run}(...)\) returns an exit flag describing the return condition.

\([x, fval, exitflag, output] = \text{run}(...)\) returns an output structure containing statistics of the run.

\([x, fval, exitflag, output, solutions] = \text{run}(...)\) returns a vector of solutions containing the distinct local minima found during the run.

Input Arguments

\(ms\)
MultiStart object.

\(\text{problem}\)
Problem structure. Create problem with createOptimProblem or by exporting a problem structure from the Optimization Tool. problem must contain the following fields:

- solver
- objective
- x0
- options — Both createOptimProblem and the Optimization Tool always include an options field in the problem structure.

k

Number of start points to run. MultiStart generates k - 1 start points using the same algorithm as list for a RandomStartPointSet object. MultiStart also uses the x0 point from the problem structure.

startpts

A CustomStartPointSet or RandomStartPointSet object. startpts can also be a cell array of these objects.

Output Arguments

x

Point at which the objective function attained the lowest value during the run. For lsqcurvefit and lsqnonlin, the objective function is the sum of squares, also known as the residual.

fval

Smallest objective function value attained during the run. For lsqcurvefit and lsqnonlin, the objective function is the sum of squares, also known as the residual.

exitflag

Integer describing the return condition:
2 At least one local minimum found. Some runs of the local solver converged.
1 At least one local minimum found. All runs of the local solver converged.
0 No local minimum found. Local solver called at least once, and at least one local solver exceeded the MaxIter or MaxFunEvals tolerances.
-1 One or more local solver runs stopped by the local solver output or plot function.
-2 No feasible local minimum found.
-5 MaxTime limit exceeded.
-8 No solution found. All runs had local solver exit flag -2 or smaller, not all equal -2.
-10 Failures encountered in user-provided functions.

output
Structure containing statistics of the optimization. Fields in the structure:

- `funcCount` Number of function evaluations.
- `localSolverIncomplete` Number of local solver runs with 0 exit flag.
- `localSolverNoSolution` Number of local solver runs with negative exit flag.
- `localSolverSuccess` Number of local solver runs with positive exit flag.
- `localSolverTotal` Total number of local solver runs.
- `message` Exit message.

solutions
A vector of GlobalOptimSolution objects containing the distinct local solutions found during the run. The vector is sorted by
objective function value; the first element is best (smallest value). The object contains:

- \( X \) Solution point returned by the local solver.
- \( Fval \) Objective function value at the solution.
- \( Exitflag \) Integer describing the result of the local solver run.
- \( Output \) Output structure returned by the local solver.
- \( X0 \) Cell array of start points that led to the solution.

### Examples

Use a default `MultiStart` object to solve the six-hump camel back problem (see “Run the Solver” on page 3-19):

```matlab
ms = MultiStart;
sixmin = @(x)(4*x(1)^2 - 2.1*x(1)^4 + x(1)^6/3 ... + x(1)*x(2) - 4*x(2)^2 + 4*x(2)^4);
problem = createOptimProblem('fmincon','x0',[-1,2],... 'objective',sixmin,'lb',[-3,-3],'ub',[3,3]);
[xmin,fmin,flag,outpt,allmins] = run(ms,problem,30);
```

### Algorithms

A detailed description of the algorithm appears in “MultiStart Algorithm” on page 3-49.

### See Also

- `MultiStart`
- `createOptimProblem`
- `CustomStartPointSet`
- `RandomStartPointSet`
- `GlobalOptimSolution`

### Tutorials

- “Run the Solver” on page 3-19
- “How to Optimize with GlobalSearch and MultiStart” on page 3-2
- “GlobalSearch and MultiStart Examples” on page 3-70
Purpose Random start points

Description Describes how to generate a set of pseudorandom points for use with MultiStart. A RandomStartPointSet object does not contain points. It contains parameters used in generating the points when MultiStart runs, or by the list method.

Construction $RS = \text{RandomStartPointSet}$ constructs a default RandomStartPointSet object.

$RS = \text{RandomStartPointSet}(\text{'PropertyName'},\text{PropertyValue},\ldots)$ constructs the object using options, specified as property name and value pairs.

$RS = \text{RandomStartPointSet}(\text{OLDRS},\text{'PropertyName'},\text{PropertyValue},\ldots)$ creates a copy of the OLDRS RandomStartPointSet object, with the named properties altered with the specified values.

Properties

ArtificialBound

Absolute value of default bounds to use for unbounded problems (positive scalar).

If a component has no bounds, list uses a lower bound of $\text{-ArtificialBound}$, and an upper bound of $\text{ArtificialBound}$.

If a component has a lower bound $lb$, but no upper bound, list uses an upper bound of $lb + 2\times\text{ArtificialBound}$. Similarly, if a component has an upper bound $ub$, but no lower bound, list uses a lower bound of $ub - 2\times\text{ArtificialBound}$.

Default: 1000

NumStartPoints

Number of start points to generate (positive integer)

Default: 10
RandomStartPointSet

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>list</td>
<td>Generate start points</td>
</tr>
</tbody>
</table>

**Copy Semantics**

Value. To learn how value classes affect copy operations, see Copying Objects in the MATLAB Programming Fundamentals documentation.

**Examples**

Create a RandomStartPointSet object for 40 points, and use list to generate a point matrix for a seven-dimensional problem:

```matlab
r = RandomStartPointSet('NumStartPoints',40); % 40 points
problem = createOptimProblem('fminunc','x0',ones(7,1),...
    'objective',@rosenbrock);
ptmatrix = list(r,problem); % 'list' generates the matrix
```

**See Also**

MultiStart | CustomStartPointSet | list

**Tutorials**

- “RandomStartPointSet Object for Start Points” on page 3-17

**How To**

- Class Attributes
- Property Attributes
Purpose

Values of simulated annealing options structure

Syntax

val = saoptimget(options, 'name')
val = saoptimget(options, 'name', default)

Description

val = saoptimget(options, 'name') returns the value of the parameter name from the simulated annealing options structure options. saoptimget(options, 'name') returns an empty matrix [] if the value of name is not specified in options. It is only necessary to type enough leading characters of name to uniquely identify the parameter. saoptimget ignores case in parameter names.

val = saoptimget(options, 'name', default) returns the 'name' parameter, but returns the default value if the 'name' parameter is not specified (or is []) in options.

Examples

opts = saoptimset('TolFun',1e-4);
val = saoptimget(opts,'TolFun');

returns val = 1e-4 for TolFun.

See Also

saoptimset | simulannealbnd

How To

• “Simulated Annealing Options” on page 9-56
**Purpose**
Create simulated annealing options structure

**Syntax**

```
saoptimset
options = saoptimset
options = saoptimset('param1',value1,'param2',value2,...)
options = saoptimset(oldopts,'param1',value1,...)
options = saoptimset(oldopts,newopts)
options = saoptimset('simulannealbnd')
```

**Description**

`saoptimset` with no input or output arguments displays a complete list of parameters with their valid values.

```
options = saoptimset (with no input arguments) creates a structure called options that contains the options, or parameters, for the simulated annealing algorithm, with all parameters set to [].
```

```
options = saoptimset('param1',value1,'param2',value2,...)
creates a structure options and sets the value of 'param1' to value1, 'param2' to value2, and so on. Any unspecified parameters are set to []. It is sufficient to type only enough leading characters to define the parameter name uniquely. Case is ignored for parameter names. Note that for string values, correct case and the complete string are required.
```

```
options = saoptimset(oldopts,'param1',value1,...) creates a copy of oldopts, modifying the specified parameters with the specified values.
```

```
options = saoptimset(oldopts,newopts) combines an existing options structure, oldopts, with a new options structure, newopts. Any parameters in newopts with nonempty values overwrite the corresponding old parameters in oldopts.
```

```
options = saoptimset('simulannealbnd') creates an options structure with all the parameter names and default values relevant to 'simulannealbnd'. For example,
```

```
saoptimset('simulannealbnd')
```

```
ans =
```
AnnealingFcnc: @annealingfast
TemperatureFcnc: @temperatureexp
AcceptanceFcnc: @acceptancesa
TolFun: 1.0000e-006
StallIterLimit: '500*numberofvariables'
MaxFunEvals: '3000*numberofvariables'
TimeLimit: Inf
MaxIter: Inf
ObjectiveLimit: -Inf
Display: 'final'
DisplayInterval: 10
HybridFcnc: []
HybridInterval: 'end'
PlotFcns: []
PlotInterval: 1
OutputFcns: []
InitialTemperature: 100
ReannealInterval: 100
DataType: 'double'

Options

The following table lists the options you can set with saoptimset. See Chapter 9, “Options Reference” for a complete description of these options and their values. Values in {} denote the default value. You can also view the options parameters by typing saoptimset at the command line.

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>AcceptanceFcnc</td>
<td>Handle to the function the algorithm uses to determine if a new point is accepted</td>
<td>Function handle</td>
</tr>
<tr>
<td>AnnealingFcnc</td>
<td>Handle to the function the algorithm uses to generate new points</td>
<td>Function handle</td>
</tr>
<tr>
<td>DataType</td>
<td>Type of decision variable</td>
<td>'custom'</td>
</tr>
<tr>
<td><strong>Option</strong></td>
<td><strong>Description</strong></td>
<td><strong>Values</strong></td>
</tr>
<tr>
<td>-------------------------</td>
<td>------------------------------------------</td>
<td>----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Display</td>
<td>Level of display</td>
<td>'off'</td>
</tr>
<tr>
<td>DisplayInterval</td>
<td>Interval for iterative display</td>
<td>Positive integer</td>
</tr>
<tr>
<td>HybridFcn</td>
<td>Automatically run HybridFcn (another optimization function) during or at the end of iterations of the solver</td>
<td>Function handle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>or</td>
</tr>
<tr>
<td>HybridInterval</td>
<td>Interval (if not 'end' or 'never') at which HybridFcn is called</td>
<td>Positive integer</td>
</tr>
<tr>
<td>InitialTemperature</td>
<td>Initial value of temperature</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>MaxFunEvals</td>
<td>Maximum number of objective function evaluations allowed</td>
<td>Positive integer</td>
</tr>
<tr>
<td>MaxIter</td>
<td>Maximum number of iterations allowed</td>
<td>Positive integer</td>
</tr>
<tr>
<td>ObjectiveLimit</td>
<td>Minimum objective function value desired</td>
<td>Scalar</td>
</tr>
<tr>
<td>OutputFcns</td>
<td>Function(s) get(s) iterative data and can change options at run time</td>
<td>Function handle or cell array of function handles</td>
</tr>
<tr>
<td>PlotFcns</td>
<td>Plot function(s) called during iterations</td>
<td>Function handle or cell array of function handles</td>
</tr>
<tr>
<td><strong>Option</strong></td>
<td><strong>Description</strong></td>
<td><strong>Values</strong></td>
</tr>
<tr>
<td>------------------</td>
<td>--------------------------------------------------------------------------------</td>
<td>-------------------------------------</td>
</tr>
<tr>
<td>PlotInterval</td>
<td>Plot functions are called at every interval</td>
<td>Positive integer</td>
</tr>
<tr>
<td>ReannealInterval</td>
<td>Reannealing interval</td>
<td>Positive integer</td>
</tr>
<tr>
<td>StallIterLimit</td>
<td>Number of iterations over which average change in fitness function value at current point is less than options.TolFun</td>
<td>Positive integer</td>
</tr>
<tr>
<td>TemperatureFcn</td>
<td>Function used to update temperature schedule</td>
<td>Function handle</td>
</tr>
<tr>
<td>TimeLimit</td>
<td>The algorithm stops after running for TimeLimit seconds</td>
<td>Positive scalar</td>
</tr>
<tr>
<td>TolFun</td>
<td>Termination tolerance on function value</td>
<td>Positive scalar</td>
</tr>
</tbody>
</table>

**See Also** saoptimget | simulannealbnd

**How To** • “Simulated Annealing Options” on page 9-56
simulannealbnd

Purpose
Find unconstrained or bound-constrained minimum of function of several variables using simulated annealing algorithm.

Syntax
\[
x = \text{simulannealbnd}(\text{fun},x0)
\]
\[
x = \text{simulannealbnd}(\text{fun},x0,lb,ub)
\]
\[
x = \text{simulannealbnd}(\text{fun},x0,lb,ub,\text{options})
\]
\[
x = \text{simulannealbnd}(\text{problem})
\]
\[
[x,fval] = \text{simulannealbnd}(...)
\]
\[
[x,fval,\text{exitflag}] = \text{simulannealbnd}(...)
\]
\[
[x,fval,\text{exitflag},\text{output}] = \text{simulannealbnd}(\text{fun},...)
\]

Description
\[
x = \text{simulannealbnd}(\text{fun},x0)\] starts at \(x_0\) and finds a local minimum \(x\) to the objective function specified by the function handle \(\text{fun}\). The objective function accepts input \(x\) and returns a scalar function value evaluated at \(x\). \(x_0\) may be a scalar or a vector.

\[
x = \text{simulannealbnd}(\text{fun},x0,lb,ub)\] defines a set of lower and upper bounds on the design variables, \(x\), so that a solution is found in the range \(lb \leq x \leq ub\). Use empty matrices for \(lb\) and \(ub\) if no bounds exist. Set \(lb(i)\) to \(-\infty\) if \(x(i)\) is unbounded below; set \(ub(i)\) to \(\infty\) if \(x(i)\) is unbounded above.

\[
x = \text{simulannealbnd}(\text{fun},x0,lb,ub,\text{options})\] minimizes with the default optimization parameters replaced by values in the structure \(\text{options}\), which can be created using the \text{saoptimset} function. See the \text{saoptimset} reference page for details.

\[
x = \text{simulannealbnd}(\text{problem})\] finds the minimum for \(\text{problem}\), where \(\text{problem}\) is a structure containing the following fields:

<table>
<thead>
<tr>
<th>Field</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{objective}</td>
<td>Objective function</td>
</tr>
<tr>
<td>\text{x0}</td>
<td>Initial point of the search</td>
</tr>
<tr>
<td>\text{lb}</td>
<td>Lower bound on (x)</td>
</tr>
<tr>
<td>\text{ub}</td>
<td>Upper bound on (x)</td>
</tr>
<tr>
<td>\text{rngstate}</td>
<td>Optional field to reset the state of the random number generator</td>
</tr>
</tbody>
</table>
**simulannealbnd**

<table>
<thead>
<tr>
<th>solver</th>
<th>'simulannealbnd'</th>
</tr>
</thead>
<tbody>
<tr>
<td>options</td>
<td>Options structure created using saoptimset</td>
</tr>
</tbody>
</table>

Create the structure problem by exporting a problem from Optimization Tool, as described in “Importing and Exporting Your Work” in the Optimization Toolbox documentation.

\[ [x,fval] = \text{simulannealbnd}(...) \] returns \( fval \), the value of the objective function at \( x \).

\[ [x,fval,exitflag] = \text{simulannealbnd}(...) \] returns \( exitflag \), an integer identifying the reason the algorithm terminated. The following lists the values of \( exitflag \) and the corresponding reasons the algorithm terminated:

- 1 — Average change in the value of the objective function over \( \text{options.StallIterLimit} \) iterations is less than \( \text{options.TolFun} \).
- 5 — \( \text{options.ObjectiveLimit} \) limit reached.
- 0 — Maximum number of function evaluations or iterations exceeded.
- -1 — Optimization terminated by the output or plot function.
- -2 — No feasible point found.
- -5 — Time limit exceeded.

\[ [x,fval,exitflag,output] = \text{simulannealbnd}(fun,...) \] returns \( output \), a structure that contains information about the problem and the performance of the algorithm. The \( output \) structure contains the following fields:

- **problemtype** — Type of problem: unconstrained or bound constrained.
- **iterations** — The number of iterations computed.
- **funccount** — The number of evaluations of the objective function.
- **message** — The reason the algorithm terminated.
simulannealbnd

- `temperature` — Temperature when the solver terminated.
- `totaltime` — Total time for the solver to run.
- `rngstate` — State of the MATLAB random number generator, just before the algorithm started. You can use the values in `rngstate` to reproduce the output of `simulannealbnd`. See “Reproducing Your Results” on page 6-20.

**Examples**

Minimization of De Jong’s fifth function, a two-dimensional function with many local minima. Enter the command `dejong5fcn` to generate the following plot.

```matlab
x0 = [0 0];
[x,fval] = simulannealbnd(@dejong5fcn,x0)
```
Optimization terminated: change in best function value less than options.TolFun.

\[ x = \]
\[
0.0392 \quad -31.9700
\]

\[ \text{fval} = \]
\[
2.9821
\]

Minimization of De Jong’s fifth function subject to lower and upper bounds:

\[
x0 = [0 \ 0];
\]
\[
\text{lb} = [-64 \ -64];
\]
\[
\text{ub} = [64 \ 64];
\]
\[
[x, \text{fval}] = \text{simulannealbnd} (@\text{dejong5fcn}, x0, \text{lb}, \text{ub})
\]

Optimization terminated: change in best function value less than options.TolFun.

\[ x = \]
\[
-31.9652 \quad -32.0286
\]

\[ \text{fval} = \]
\[
0.9980
\]

The objective can also be an anonymous function:

\[
\text{fun} = @(x) 3*\sin(x(1))+\exp(x(2));
\]
\[
x = \text{simulannealbnd} (\text{fun}, [1;1], [0 \ 0])
\]

Optimization terminated: change in best function value less than options.TolFun.

\[ x = \]
\[
457.1045
\]
Minimization of De Jong’s fifth function while displaying plots:

```matlab
x0 = [0 0];
options = saoptimset('PlotFcns',{@saplotbestx,...
    @saplotbestf,@saplotx,@saplotf});
simulannealbnd(@dejong5fcn,x0,[],[],options)
```

Optimization terminated: change in best function value
less than options.TolFun.

```matlab
ans =
    0.0230   -31.9806
```

The plots displayed are shown below.
Simulated Annealing

See Also: ga, patternsearch, saoptimget, saoptimset
simulannealbnd
Examples

Use this list to find examples in the documentation.
**GlobalSearch and MultiStart**

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“Example: Creating a Problem Structure with the Optimization Tool” on page 3-11
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“Example: Creating a Nondefault MultiStart Object” on page 3-16
“CustomStartPointSet Object for Start Points” on page 3-18
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