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The Four Purposes of Multidimensional Scaling

Multidimensional scaling (MDS) is a method that represents measurements of similarity (or dissimilarity) among pairs of objects as distances between points of a low-dimensional multidimensional space. The data, for example, may be correlations among intelligence tests, and the MDS representation is a plane that shows the tests as points that are closer together the more positively the tests are correlated. The graphical display of the correlations provided by MDS enables the data analyst to literally "look" at the data and to explore their structure visually. This often shows regularities that remain hidden when studying arrays of numbers. Another application of MDS is to use some of its mathematics as models for dissimilarity judgments. For example, given two objects of interest, one may explain their perceived dissimilarity as the result of a mental arithmetic that mimics the distance formula. According to this model, the mind generates an impression of dissimilarity by adding up the perceived differences of the two objects over their properties.

In the following, we describe four purposes of MDS: (a) MDS as a method that represents (dis)similarity data as distances in a low-dimensional space in order to make these data accessible to visual inspection and exploration; (b) MDS as a technique that allows one to test if and how certain criteria by which one can distinguish among different objects of interest are mirrored in corresponding empirical differences of these objects; (c) MDS as a data-analytic approach that allows one to discover the dimensions that underlie judgments of (dis)similarity; (d) MDS as a psychological model that explains judgments of dissimilarity in terms of a rule that mimics a particular type of distance function.

TABLE 1.1. Correlations of crime rates over 50 U.S. states.

Crime	No.	1	2	3	4	5	6	7
Murder	1	1.00	0.52	0.34	0.81	0.28	0.06	0.11
Rape	2	0.52	1.00	0.55	0.70	0.68	0.60	0.44
Robbery	3	0.34	0.55	1.00	0.56	0.62	0.44	0.62
Assault	4	0.81	0.70	0.56	1.00	0.52	0.32	0.33
Burglary	5	0.28	0.68	0.62	0.52	1.00	0.80	0.70
Larceny	6	0.06	0.60	0.44	0.32	0.80	1.00	0.55
Auto theft	7	0.11	0.44	0.62	0.33	0.70	0.55	1.00

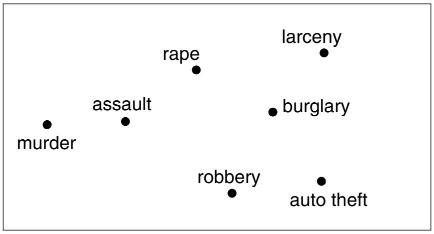

FIGURE 1.1. A two-dimensional MDS representation of the correlations in Table 1.1.

1.1 MDS as an Exploratory Technique

Exploratory data analysis is used for studying theoretically amorphous data, that is, data that are not linked to an explicit theory that predicts their magnitudes or patterns. The purpose of such explorations is to help the researcher to *see* structure in the data. MDS, too, can be used for such data explorations.

Consider an example. The U.S. Statistical Abstract 1970 issued by the Bureau of the Census provides statistics on the rate of different crimes in the 50 U.S. states (Wilkinson, 1990). One question that can be asked about these data is to what extent can one predict a high crime rate of murder, say, by knowing that the crime rate of burglary is high. A partial answer to this question is provided by computing the correlations of the crime rates over the 50 U.S. states (Table 1.1). But even in such a fairly small correlation matrix, it is not easy to understand the structure of these coefficients. This task is made much simpler by representing the correlations in the form of a "picture" (Figure 1.1). The picture is a two-dimensional MDS representation where each crime is shown as a point. The points are arranged in such a way that their distances correspond to the correlations. That is, two points are close together (such as murder and assault) if their corresponding crime rates are highly correlated. Conversely, two points are far apart if their crime rates are not correlated that highly (such as assault and larceny). The correspondence of data and distances is tight in this example: the product-moment correlation between the coefficients in Table 1.1 and the distances in Figure 1.1 is $r = -.98$.

The reader need not be concerned, at this point, with the question of how such an MDS representation, $\mathbf{X}$, is found. We return to this issue in considerable detail in later chapters. For now, it suffices to assume that the data are fed to an MDS computer program and that this program provides a best-possible solution in a space with a dimensionality selected in advance by the user. The quality of this solution can be checked without knowing how it was found. All one has to do is measure the distances between the points of $\mathbf{X}$ and compare them with the data.[1] If distances and data are highly correlated in the sense of the usual product-moment correlation, say, then the distances represent the data well in a linear sense.[2] This is obviously true in the given case, and so the distances in Figure 1.1 represent the correlations in Table 1.1 very precisely.

What does the MDS picture in Figure 1.1 tell us? It shows that the crimes are primarily distributed along a horizontal dimension that could be interpreted as "violence vs. property" crimes. Moreover, the "property crimes" are less homogeneous, exhibiting some spread along the vertical axis, a dimension that could be interpreted as "hidden vs. street" crimes.

Although here we looked at dimensions, it is important to keep in mind that *any* property of the MDS representation that appears unlikely to result from chance can be interesting. The points may, for example, form certain groupings or clusters. Or, they may fall into different *regions* such as a center region surrounded with bands. The points may also lie on certain *manifolds* such as curved lines (a circle, for example) or on some surface in a higher-dimensional space. Looking for particular directions that would explain the points' distribution is just one possibility to search for structure. Later on in this book, we explore a variety of geometric regularities that have been found useful in practical research.

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[1] Consider an analogy. Anyone can check the proposition that the number 1.414 approximates $\sqrt{2}$ simply by multiplying 1.414 by itself. The result shows that the proposition is nearly correct. For checking it, it is irrelevant how the number 1.414 was found. Indeed, few would know how to actually compute such a solution, except by trial and error, or by pushing a button on a calculator.

[2] With few points, one can even do (two-dimensional) MDS by hand. To find an MDS solution for the data in Table 1.1, first cut out seven small pieces of paper and write onto each of them one of the labels of the variables in Table 1.1, i.e., "murder", "rape", ..., "auto theft", respectively. Place these pieces of paper arbitrarily in a plane and then move them around in small steps so that higher correlations tend to correspond to smaller distances. Repeat these corrective point movements a few times until the match of distances and data is satisfactory or until it cannot be improved anymore. Such a manual approach is typically quite easy to perform as long as the number of variables is small. With many variables, computer algorithms are needed for doing the work. Good algorithms also make it more likely that one ends up with an optimal MDS solution, that is, a configuration whose distances represent the given data "best" (in some well-defined sense).

Such insights into the data structure are aided by the visual access made possible by the simple MDS picture. Of course, as it is true for exploratory data analysis in general, it is left to further studies to test whether the patterns thus detected are stable ones. Moreover, it is desirable to also develop a theory that provides a rationale for the findings and enables one to predict such structures.

1.2 MDS for Testing Structural Hypotheses

When more is known about a field of interest, exploratory methods become less important. The research items, then, are well designed and the general interest lies in studying effect hypotheses. That is, in particular, what one wants to know is if and how the facets (dimensions, factors, features, etc.) by which the items are conceptually distinguished are reflected in corresponding differences among observations on these items. MDS may be useful for studying such questions. Consider a case.

Levy (1983) reports a study on attitudes towards political protest behavior. She distinguished 18 types of attitudes towards political protest acts. These types correspond to the $3 \cdot 3 \cdot 2 = 18$ different ways of reading the following design scheme (mapping sentence):

$$\text{The} \left\{ \begin{array}{l} \underline{A\text{: modality of attitude}} \\ a_1 = \text{evaluation} \\ a_2 = \text{approval} \\ a_3 = \text{likelihood of own overt action} \end{array} \right\} \text{behavior of respondent } x$$

$$\text{with respect to} \left\{ \begin{array}{l} \underline{B\text{: strength of execution}} \\ b_1 = \text{demanding} \\ b_2 = \text{obstructive} \\ b_3 = \text{physically damaging} \end{array} \right\} \text{protest acts of}$$

$$\begin{array}{c} \underline{C\text{: way to carry out}} \\ \left\{ \begin{array}{l} c_1 = \text{omission} \\ c_2 = \text{commission} \end{array} \right\} \end{array} \rightarrow \left\{ \begin{array}{c} \underline{R\text{: direction}} \\ \text{very positive} \\ \text{to} \\ \text{very negative} \end{array} \right\} \text{behavior towards acts.}$$

Thirty items were selected from a study by Barnes et al. (1979), using this mapping sentence as a culling rule. Short verbal labels and the codings for the selected items with respect to the three facets of the mapping sentence are given in Table 1.2. For example, item no. 6 effectively asked: "To what extent is 'painting slogans on walls' effective when people use this act in pressing for change?" The respondent's answer was, for this item, recorded on a scale from "very effective" to "not effective". (This scale is the "range" R of the observational mapping.) According to Levy, this item asks about

TABLE 1.2. A classification of protest acts by three facets; numbers in table refer to item numbers.

Item	a_1	a_2	a_3		
Petitions	1	11	21	b_1	c_2
Boycotts	2	12	22	b_2	c_1
Lawful demonstrations	3	13	23	b_1	c_2
Refusing to pay rent	4	14	24	b_2	c_1
Wildcat strikes	5	15	25	b_2	c_1
Painting slogans on walls	6	16	26	b_3	c_2
Occupying buildings	7	17	27	b_2	c_2
Blocking traffic	8	18	28	b_2	c_2
Damaging property	9	19	29	b_3	c_2
Personal violence	10	20	30	b_3	c_2

an effectiveness evaluation ($= a_1$) of a physically damaging act ($= b_3$) of commission ($= c_2$).

How are these 18 different forms of attitudes towards protest behavior related to each other? Will the facets used by Levy for *conceptually* classifying the items show up in the survey data? The distinction "omission vs. commission", for example, is, after all, an organizing principle that comes from Levy. It may be clear enough and even useful to other researchers in the field of political behavior. However, that does not mean that the uninitiated respondent would use similar notions, especially not *implicitly* when making his or her ratings. In fact, it is not even guaranteed that evaluating protest acts in terms of "effectiveness", "approval", and "likelihood of own overt action" will lead to different ratings.

Levy (1983) approached these questions by MDS. The intercorrelations of the items from surveys taken in five different countries were first "scaled" by MDS. It turned out that three-dimensional spaces were needed in each case to adequately represent the correlations of the 30 items by corresponding distances. Figure 1.2 shows the MDS space for the German data.

One could inspect this space in an exploratory manner, as above. However, three-dimensional MDS configurations are hard to understand, in particular when projected onto paper or onto the computer screen. What we want here is, in any case, not exploration. Rather, we want to link the MDS configuration to the item design. For that purpose, it is easier not to look at the complete three-dimensional space at once, but only at certain projection planes. Such planes are, for example, the planes spanned by the three coordinate axes, that is, the plane spanned by axes X and Y, or by X and Z. Inspecting the X–Y plane or the "bottom" plane of Figure 1.2, one finds that Figure 1.3 can be split in two ways that clearly reflect the distinctions $a_1, \ldots, a_3$ and $b_1, \ldots, b_3$, respectively, made by the first two facets of the mapping sentence. The solid vertical lines show, for example, that all "demanding" items lie on the left-hand side, all "obstruction" items lie

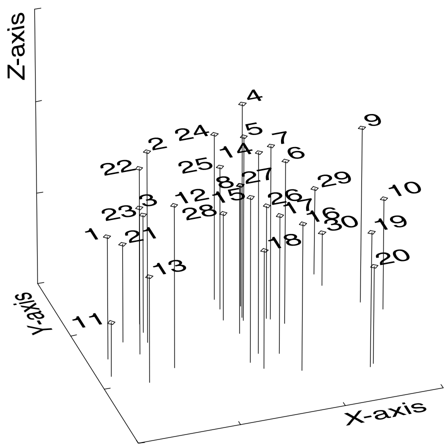

FIGURE 1.2. Three-dimensional MDS representation of protest acts.

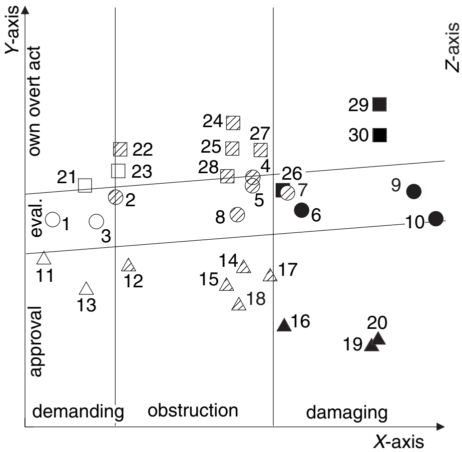

FIGURE 1.3. Plane spanned by X- and Y-axes of Fig. 1.2; drawn-in lines represent the first two facets (A and B) from mapping sentence. The filling of the markers reflects the levels of facet A, the shape the level of facet B.

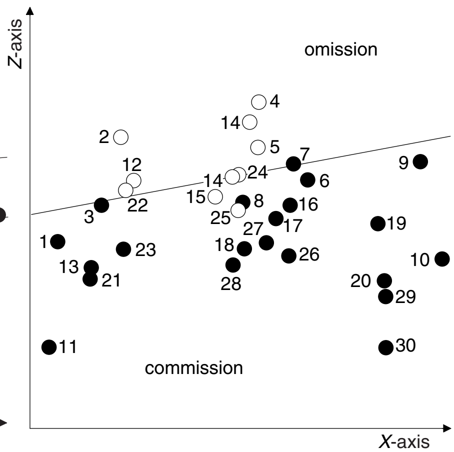

FIGURE 1.4. Plane spanned by X- and Z-axes of Fig. 1.2; drawn-in line represents third facet (C). The filling of the markers reflects the levels of facet C.

in the middle, and all "damaging" items lie on the right-hand side of the space. Figure 1.4 makes clear that the "omission" points are placed above the "commission" items along the Z-axis. Putting these findings together, one notes that the three-dimensional MDS space is thus cut into box-like regions that result from projecting the conceptual codings of the items onto the MDS configuration. Hence, Levy's distinctions on protest acts are not only conceptually possible, but they are also useful for explaining data variance.

1.3 MDS for Exploring Psychological Structures

MDS has been used primarily in psychology. Psychologists usually have psychological questions in mind. Even when used in an exploratory manner, MDS thus typically carried with it, as an implicit purpose, the search for "underlying dimensions" that would explain observed similarities or dissimilarities. In the exploratory MDS application on crime rates considered above, such notions were absent or had, at least, a much lower priority. The purpose of MDS, in the above crime context, was simply to enable the data analyst to look at the data structure in order to find rules that would help to *describe* the distribution of the points. One could thus say that in pure data-analytic MDS, one attempts to find rules of formation that allow one to describe the data structure in as simple terms as possible, whereas in the kind of exploratory MDS that is typical for psychologists the researcher is interested in discovering psychological dimensions that would meaningfully explain the data.

In psychology, the data used for MDS are often based on direct similarity judgments by the respondents. Wish (1971), for example, asked 18 students to rate the global similarity of different pairs of nations such as France and China on a 9-point rating scale ranging from 1 = very different to 9 = very similar. Table 1.3 shows the mean similarity ratings.

The similarity data of Table 1.3 are, roughly, represented by the distances of the two-dimensional MDS configuration in Figure 1.5. It thus holds that the higher the similarity measures, the smaller the corresponding distance. The dashed lines in this figure were not generated by MDS. Rather, they are an interpretation by Kruskal and Wish (1978) that can help to explain the distribution of the points. Interpreting an MDS representation means linking some of its geometric properties to substantive knowledge about the objects represented by the points. One such geometric property is the scatter of the points along a straight line or *dimension*. The lines are chosen by first identifying points that are far apart and about which one already knows something. Based on this prior knowledge, one attempts to formulate a substantive criterion that could have led *the subjects* to distinguish so

TABLE 1.3. Matrix of average similarity ratings for 12 nations (Wish, 1971).

Nation		1	2	3	4	5	6	7	8	9	10	11	12
Brazil	1	–											
Congo	2	4.83	–										
Cuba	3	5.28	4.56	–									
Egypt	4	3.44	5.00	5.17	–								
France	5	4.72	4.00	4.11	4.78	–							
India	6	4.50	4.83	4.00	5.83	3.44	–						
Israel	7	3.83	3.33	3.61	4.67	4.00	4.11	–					
Japan	8	3.50	3.39	2.94	3.83	4.22	4.50	4.83	–				
China	9	2.39	4.00	5.50	4.39	3.67	4.11	3.00	4.17	–			
USSR	10	3.06	3.39	5.44	4.39	5.06	4.50	4.17	4.61	5.72	–		
U.S.A.	11	5.39	2.39	3.17	3.33	5.94	4.28	5.94	6.06	2.56	5.00	–	
Yugoslavia	12	3.17	3.50	5.11	4.28	4.72	4.00	4.44	4.28	5.06	6.67	3.56	–

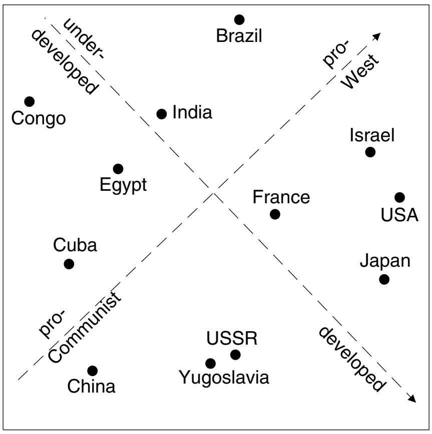

FIGURE 1.5. MDS for data in Table 1.3; dashed lines are an interpretation of the point scatter.

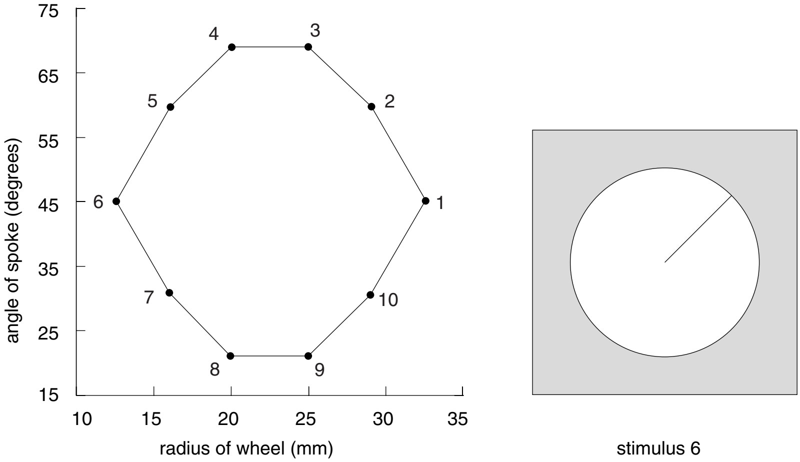

FIGURE 1.6. Design configuration for Broderson's one-spoked wheels; a specimen for such a stimulus is shown in the insert on the right-hand side.

clearly between these objects, placing them at opposite ends of a dimension. This is known as interpreting a dimension.

Interpreting an MDS space, therefore, involves data-guided speculations about the psychology of those who generated the similarity data. Testing the validity of the conclusions is left to further studies.

1.4 MDS as a Model of Similarity Judgments

Finally, the mathematics of MDS can serve as a model of similarity judgments. The most common approach is to hypothesize that a person, when asked about the dissimilarity of pairs of objects from a set of objects, acts *as if* he or she computes a distance in his or her "psychological space" of these objects.

Questions of this sort are studied mostly in the context of well-designed stimuli. One such example is the following. Broderson (1968) studied the dissimilarity of stimuli that looked like one-spoked wheels. That is, his stimuli were circles varying in diameter from 12.5 mm to 32.5 mm; they also had a drawn-in radius line at angles varying from 21° to 69°. Figure 1.6 shows an example of such a stimulus, together with a geometric description of the 10 stimuli selected for experimentation. (The line connecting the points in this figure has no particular meaning. It only helps to better understand the structure of the point configuration.)

Each of the 45 pairs of the one-spoked wheels $1, \ldots, 10$ from Figure 1.6 was drawn on a card and presented to subjects with the instruction to rate this pair's global similarity on a scale from $1 =$ minimal similarity to

TABLE 1.4. Mean similarity scores for one-spoked wheels described in Figure 1.6.

Item	1	2	3	4	5	6	7	8	9	10
1	–									
2	5.10	–								
3	3.86	5.42	–							
4	3.24	4.74	5.30	–						
5	3.52	4.98	4.56	5.06	–					
6	4.60	3.76	3.06	3.68	4.86	–				
7	4.02	3.08	2.88	3.26	4.82	5.06	–			
8	3.42	3.42	2.94	4.44	3.34	3.44	4.90	–		
9	3.98	3.36	4.30	3.26	2.92	3.06	4.64	5.48	–	
10	5.30	4.78	3.70	3.36	3.12	4.36	4.68	4.40	5.06	–

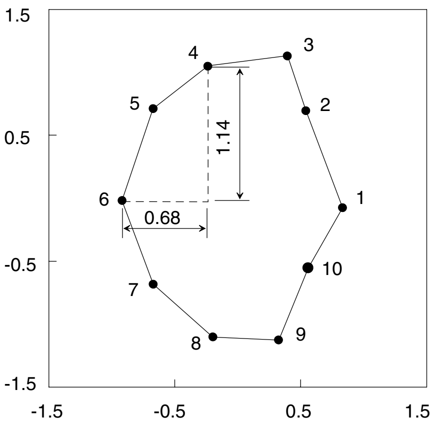

FIGURE 1.7. MDS representation of similarity data in Table 1.4; the combined lengths of the dashed line segments is the city-block distance of points 4 and 6.

7 = maximal similarity. This led to a 10×10 matrix of similarity scores for each subject. The mean scores for all 50 subjects are shown in Table 1.4.

It was hypothesized that a subject arrives at a similarity judgment by computing a particular distance in his or her psychological space. This space should essentially correspond to the physical design space in Figure 1.6. Given two points in this space, their *city-block distance* is the sum of their distances along the X- and Y-axes, respectively.

Figure 1.7 shows an MDS representation of the values in Table 1.4. One notes immediately that this spatial representation of the subjects' similarity scores is very similar to the design configuration in Figure 1.6.

The MDS representation has been computed so that its city-block distances correspond to the similarity scores in Table 1.4. In Figure 1.7, it is shown how such a city-block distance is computed. For points 4 and 6, it is equal to the sum of the lengths of the dashed line segments connecting points 4 and 6: $0.68 + 1.14 = 1.82$. Broderson claims that his subjects arrived at their similarity ratings by comparing each pair of one-spoked wheels dimension by dimension, adding the perceived dimensional differences, and converting the resulting global dissimilarity impressions into the format of the response scale.

Do the similarity values in Table 1.4 support this theory? The answer is quite positive, because the (city-block) distances between any two points i and j in Figure 1.7 are highly correlated ($r = -.92$) with the similarity values in Table 1.4. Hence, this particular two-dimensional distance geometry is indeed a possible model of judgment of similarity for the given stimuli.

Such psychological model building goes considerably beyond a mere searching for structure in the data. It also differs from testing an abstract structural hypothesis. Rather, it involves a particular distance function that is defined on particular dimensions and is interpreted quite literally as a psychological *composition rule*.[3]

1.5 The Different Roots of MDS

The different purposes of MDS, and the existence of an enormous variety of related geometric models, have led to unnecessary confusion over the question of how MDS should be used. Social scientists such as sociologists, political scientists, or social psychologists, for example, are often interested in using MDS to test hypotheses on correlations in a way similar to what we saw above in Section 1.2. Consequently, they often do not even use the term multidimensional scaling but rather speak of *smallest space analysis* (Guttman, 1968) or of *multidimensional similarity structure analysis* (Borg & Lingoes, 1987).

Psychophysicists, on the other hand, are usually concerned not with correlations but with models that relate stimuli with well-known physical properties to their perceptual or cognitive representations. For them, the notion of multidimensional scaling has a very direct meaning in the sense that they study how *known* physical dimensions are represented psychologically. Because psychophysics is the domain where MDS came from [see De Leeuw

[3]There are theories closely related to MDS modeling that do not concentrate very much on the distance function, but instead concentrate on other properties of multidimensional geometry such as "incidence", "perpendicularity", or "inclusion". Often, geometries are chosen that appear very strange to the nonmathematician, such as curved spaces, bounded spaces, or finite geometries [see, for example, Drösler (1979) and Müller (1984)]. Such models are, however, typically highly specialized and thoroughly bound to a particular substantive field of interest (such as "monocular space perception" or "color vision"). There is usually no reason to use them for general data-analytic purposes, and so very little attention is given to them in this book.

and Heiser (1982) on the history of MDS], it is enlightening to read what Torgerson (1952) thought about MDS:

> *The traditional methods of psychophysical scaling presuppose knowledge of the dimensions of the area being investigated. The methods require judgments along a particular defined dimension, i.e., A is brighter, twice as loud, more conservative, or heavier than B. The observer, of course, must know what the experimenter means by brightness, loudness, etc. In many stimulus domains, however, the dimensions themselves, or even the number of relevant dimensions, are not known. What might appear intuitively to be a single dimension may in fact be a complex of several. Some of the intuitively given dimensions may not be necessary... Other dimensions of importance may be completely overlooked. In such areas the traditional approach in inadequate.*
>
> *Richardson, in 1938 (see also Gulliksen, 1946) proposed a model for multidimensional scaling that would appear to be applicable to a number of these more complex areas. This model differs from the traditional scaling methods in two important respects. First, it does not require judgments along a given dimension, but utilizes, instead, judgments of similarity between the stimuli. Second, the dimensionality, as well as the scale values, of the stimuli is determined from the data themselves.*

This clearly shows that early MDS was strongly dominated by notions of dimensional modeling of similarity judgments. Later consumers of MDS, even when they used MDS for purely exploratory purposes, were apparently so much influenced by this dimensional thinking that they often almost automatically looked for interpretable dimensions even though they set out to generally explore the data structure.

Data analysts, in contrast to psychophysicists, are generally not interested in building models for a particular substantive domain. Rather, they want to provide general-purpose tools for empirical scientists that will help the substantive researchers to better understand the structure of their data. For this purpose, of course, it would make no sense to employ a distance function such as the city-block distance used in Section 1.4 above, because the relations among the points of such geometries often are *not* what they appear to be. For example, the city-block distance between points 4 and 6 in Figure 1.7 is about the same as the city-block distance between points 1 and 6. The natural (Euclidean) distance between 4 and 6 is, in contrast, considerably shorter than the distance between 1 and 6. Hence, MDS representations that employ distance functions other than the Euclidean tend to be misleading when inspected intuitively. Therefore, they are useless for exploratory purposes.

1.6 Exercises

Exercise 1.1 Consider the following correlation matrix of eight intelligence test items (Guttman, 1965).

Item	1	2	3	4	5	6	7	8
1	1.00	.40	.25	.12	.67	.39	.26	.19
2	.40	1.00	.31	.39	.50	.24	.18	.52
3	.25	.31	1.00	.46	.28	.38	.42	.49
4	.12	.39	.46	1.00	.20	.14	.29	.55
5	.67	.50	.28	.20	1.00	.38	.26	.26
6	.39	.24	.38	.14	.38	1.00	.40	.22
7	.26	.18	.42	.29	.26	.40	1.00	.25
8	.19	.52	.49	.55	.26	.22	.25	1.00

(a) Use the procedure outlined in Footnote 2 on page 5 to find an MDS representation of these data in the plane by hand. That is, items should be represented as points, and the distances between any two points should be smaller the higher the corresponding items are correlated.

(b) The MDS representation will exhibit a particularly simple structure among the items. Use this structure to reorder the above correlation matrix. What pattern does this matrix exhibit?

(c) A typical beginner's mistake when using MDS is to incorrectly specify how the MDS distances should be related to the data. Correlations are indices of similarity, not of dissimilarity, and so correlations should be *inversely* related to MDS distances. Check what happens when you tell your MDS program that you want larger correlations represented by larger distances. (Hint: Depending on the MDS computer program, you may have to request something like "Regression=ascending" or you may have to specify that the correlations are "similarities." For a description of MDS programs, see Appendix A.)

Exercise 1.2 Consider the following correlation matrix of seven vocational interest scales (Beuhring & Cudeck, 1985).

Scale	Health	Science	Techn.	Trades	Bus.O.	Bus.C.	Social
Health	1.00						
Science	.65	1.00					
Technology	.45	.64	1.00				
Trades	.25	.44	.76	1.00			
Business Operations	.12	.16	.55	.49	1.00		
Business Contact	.22	.21	.57	.46	.75	1.00	
Social	.50	.26	.37	.20	.47	.65	1.00

(a) Use the procedure outlined in Footnote 2 on page 5 to find an MDS representation of these data in the plane by hand.

(b) Interpret the resulting MDS representation: What does it tell you about interests?

Exercise 1.3 Consider the data in Table 1.4 on page 12. They were scaled in Figure 1.7 by using the city-block distance, not the "usual" (that is, Euclidean) distance. What happens to city-block distances if the coordinate system is rotated by, say, 30 degrees? What happens to Euclidean distances in the same case? Based on your answers to these two questions above, what can you say about the coordinate system when dealing with city-block distances?

Exercise 1.4 Representing proximity data such as correlations in an MDS plane is often useful for an exploratory investigation of the data structure. Yet, the MDS configuration can also be misleading. When?

Exercise 1.5 Replicate the experiment of Section 1.3 with 10 U.S. States or countries of your choice.

(a) Prepare a list of all possible pairs of states. Rate the similarity of the states in each pair on a scale from 0=not different to 10=very different. (You may want to begin by first picking the two states that appear most different and by setting their similarity equal to 10. This establishes a frame of reference for your judgments.)

(b) Scale the resulting similarity ratings by hand or by an MDS computer program.

(c) Study the MDS solution and search for a dimensional interpretation.

Exercise 1.6 Consider the matrix below (Lawler, 1967). It shows the correlations among nine items. The items assess three performance criteria (T_1 = quality of job performance, T_2 = ability to perform the job, T_3 = effort put forth on the job) by three different methods (M_1 = superior ratings, M_2 = peer ratings, M_3 = self ratings). Such a matrix is called a multitrait-multimethod matrix.

Item	No.	1	2	3	4	5	6	7	8	9
T_1M_1	1	1.00								
T_2M_1	2	.53	1.00							
T_3M_1	3	.56	.44	1.00						
T_1M_2	4	.65	.38	.40	1.00					
T_2M_2	5	.42	.52	.30	.56	1.00				
T_3M_2	6	.40	.31	.53	.56	.40	1.00			
T_1M_3	7	.01	.01	.09	.01	.17	.10	1.00		
T_2M_3	8	.03	.13	.03	.04	.09	.02	.43	1.00	
T_3M_3	9	.06	.01	.30	.02	.01	.30	.40	.40	1.00

(a) Check whether the facets trait and method are reflected as regions in an MDS representation of the correlations.

(b) What substantive conclusions can you derive with respect to the facets trait and method? Is there, for example, reason to conclude that the facets may be ordered rather than just categorical?

(c) What other insights can you derive from the MDS solution concerning performance appraisals? How do the different kinds of appraisals differ?

Exercise 1.7 Consider Table 1.5 on page 18. It shows data from an experiment where 10 experienced psychiatrists each fabricated archetypal psychiatric patients by characterizing them on the 17 variables of the Brief Psychiatric Rating Scale (Mezzich, 1978). The variables are A = somatic concern, B = anxiety, C = emotional withdrawal, D = conceptual disorganization, E = guilt feelings, F = tension, G = mannerism and posturing, H = grandiosity, I = depressive mood, J = hostility, K = suspiciousness, L = hallucinatory behavior, M = motor retardation, N = uncooperativeness, O = unusual thought content, P = blunted affect, Q = excitement.

(a) Correlate the rows of this data matrix to get similarity coefficients for the 40 patients. Then use MDS to explore the structure of the correlations.

(b) Does a 2D MDS representation allow you to distinguish the four psychiatric types?

(c) The MDS representation indicates that the four types are ordered in certain ways. Describe and explain that order.

Exercise 1.8 Consider the data in Table 1.5 on page 18.

(a) Compute the Euclidean distance of any two rows. Use these distances as proximities and do a two-dimensional MDS with them. Compare the resulting solution to an MDS solution that uses correlations as proximity measures.

(b) Repeat the above for city-block distances as proximity measures.

(c) Are the MDS solutions very different? Discuss why this is so.

TABLE 1.5. Severity ratings (on 0..6 scale) of four prototypical psychiatric patients on 17 symptoms by 10 psychiatrists (Mezzich, 1978).

Type	No.	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Depressive	1	4	3	3	0	4	3	0	0	6	3	2	0	5	2	2	2	1
	2	5	5	6	2	6	1	0	0	6	1	0	1	6	4	1	4	0
	3	6	5	6	5	6	3	2	0	6	0	5	3	6	5	5	0	0
	4	5	5	1	0	6	1	0	0	6	0	1	2	6	0	3	0	2
	5	6	6	5	0	6	0	0	0	6	0	4	3	5	3	2	0	0
	6	3	3	5	1	4	2	1	0	6	2	1	1	5	2	2	1	1
	7	5	5	5	2	5	4	1	1	6	2	3	0	6	3	5	2	3
	8	4	5	5	1	6	1	1	0	6	1	1	0	5	2	1	1	0
	9	5	3	5	1	6	3	1	0	6	2	1	1	6	2	5	5	0
	10	3	5	5	3	2	4	2	0	6	3	2	0	6	1	4	5	1
Manic	11	2	2	1	2	0	3	1	6	2	3	3	2	1	4	4	0	6
	12	0	0	0	4	1	5	0	6	0	5	4	4	0	5	5	0	6
	13	0	3	0	5	0	6	0	6	0	3	2	0	0	3	4	0	6
	14	0	0	0	3	0	6	0	6	1	3	1	1	0	2	3	0	6
	15	3	4	0	0	0	5	0	6	0	6	0	0	0	5	0	0	6
	16	2	4	0	3	1	5	1	6	2	5	3	0	0	5	3	0	6
	17	1	2	0	2	1	4	1	5	1	5	1	1	0	4	1	0	6
	18	0	2	0	2	1	5	1	5	0	2	1	1	0	3	1	0	6
	19	0	0	0	6	0	5	1	6	0	5	5	4	0	5	6	0	6
	20	5	5	1	4	0	5	5	6	0	4	4	3	0	5	5	0	6
Schizophrenic	21	3	2	5	2	0	2	2	1	2	1	2	0	1	2	2	4	0
	22	4	4	5	4	3	3	1	0	4	2	3	0	3	2	4	5	0
	23	2	0	6	3	0	0	5	0	0	3	3	2	3	5	3	6	0
	24	1	1	6	2	0	0	1	0	0	3	0	1	0	1	1	6	0
	25	3	3	5	6	3	2	5	0	3	0	2	5	3	3	5	6	2
	26	3	0	5	4	0	0	3	0	2	1	1	1	2	3	3	6	0
	27	3	3	5	4	2	4	2	1	3	1	1	1	4	2	2	5	2
	28	3	2	5	2	2	2	2	1	2	2	3	1	2	2	3	5	0
	29	3	3	6	6	1	3	5	1	3	2	2	5	3	3	6	6	1
	30	1	1	5	3	1	1	3	0	1	1	1	0	5	1	2	6	0
Paranoid	31	2	4	3	5	0	3	1	4	2	5	6	5	0	5	6	3	3
	32	2	4	1	1	0	3	1	6	0	6	6	4	0	6	5	0	4
	33	5	5	5	6	0	5	5	6	2	5	6	6	0	5	6	0	2
	34	1	4	2	1	1	1	0	5	1	5	6	5	0	6	6	0	1
	35	4	5	6	3	1	6	3	5	2	6	6	4	0	5	6	0	5
	36	4	5	4	6	2	4	2	4	1	5	6	5	1	5	6	2	4
	37	3	4	3	4	1	5	2	5	2	5	5	3	1	5	5	1	5
	38	2	5	4	3	1	4	3	4	2	5	5	4	0	5	4	1	4
	39	3	3	4	4	1	5	5	5	0	5	6	5	1	5	5	3	4
	40	4	4	2	6	1	4	1	5	3	5	6	5	1	5	6	2	4