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Some Current Models for the Perception and Judgment of Risk

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We survey some of the models more recently used to portray panelists' perceptions of risk, viewed as a complex psychological response. These models are compared (a) as continuous versus discrete, (b) with regard to type of data and tasks required of panelists, and (c) by facility for portraying different patterns of judgments among panelists. Substantive results from applying different models to the same risks and/or data are presented. Finally, we consider possible future directions for research in the perception of risk, oriented toward use of the models presented here.

INTRODUCTION

Pioneering studies of utility that used preference for gambles (e.g., Coombs & Komorita, 1958; Mosteller & Nogee, 1951) implicitly viewed risk as a unidimensional psychological response varying systematically as a function of factors in an experimental design. This tradition emphasized the (stimulus) factors and attempted to identify a preferred set of factors to characterize gambles. Slovic and Lichtenstein (1968), for example, sought to determine the relative importance of the "four basic risk di-

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mensions": (a) probability of winning, (b) amount to be won, (c) probability of losing, and (d) amount to be lost.

While the approach just described led to some interesting results and models, and is appropriate enough for laboratory studies of utility, the tradition is not well suited for current problems of risk and hazard assessment.[1] The hallmarks of these problems are (a) severely limited ability to manipulate a "real world" stimulus such as a catastrophe (in contrast to adjusting the probability of winning a bet, à la Slovic & Lichtenstein, 1968) and (b) diverse, multidimensional, and often unpredictable responses to such hazards by individuals of the relevant public. When expert opinion and rational (i.e., a priori based) expectations are used to supplant knowledge about (b), the consequences can be counterproductive. Covello (1983, pp. 290–291) provides a telling example:

> "In several countries, including France, proposals are currently being considered to compensate those who live in the vicinity of nuclear power plants. If the intention is to win wider public acceptance, then the policy is misdirected. Those living nearest to the power plant are already supportive and little would be gained by compensating them. By comparison, compensating those who are least supportive (i.e., those living in areas where power plants are under construction or being planned) might have a major impact. Such a policy of course, could also backfire by providing support for the belief that the risks of nuclear power are indeed substantial.
>
> "What this specific case and others similar to it teach us is that analysts and decision-makers need a better understanding of how people think and make decisions about technological risks. Public risk acceptance and the success of risk management policies are likely to hinge on such understanding. Stated more forcefully, without such understanding well-intended policies may be ineffective or even counterproductive."

Sage and White (1980, p. 426) have noted that "there appears to be no commonly accepted definition of 'risk' in the area of risk and hazard assignment." *If we are to gain an understanding of psychological reaction to risks and hazards over diverse contexts, then more versatile models than a unidimensional continuum are needed to represent the perception of risk.* For example, Fischhoff, Watson, and Hope (1984) proffered multiattribute utility theory (Keeney & Raiffa, 1976) as a framework for defining a given risk as a function of attributes characterizing possible consequences of risky decisions. In this framework, the crucial tasks are (a) determining an agreed upon set of attributes and (b) finding an acceptable set of numerical weights for them. While Fischhoff *et al.* (1984) demonstrated the utility of this approach, the current paper emphasizes empirical rather than a priori methods for approaching problem (a).

[1] Gooding (1978, p. 401) draws a parallel similar to the view of risk as a unidimensional concept in modern finance versus a multidimensional concept in the theory of stock valuation.

As such, the present paper (a) seeks to review some models other than multiattribute utility theory and contrasts such models as *continuous* versus *discrete*, (b) discusses some applications, and (c) considers possible future directions of research using such models.

CONTINUOUS (SPATIAL) MODELS

The continuous (and generally multidimensional) models most commonly used for the study of risk, whether emphasizing "risk perception" (e.g., Slovic, Fischhoff, & Lichtenstein, 1982) or "risk judgment" (Borcherding, Brehmer, Vlek, & Wagenaar, 1984b; Kuyper & Vlek, 1984), have been such spatial models as principal components analysis, factor analysis, and multidimensional scaling. The history of their applications to the study of risk in some ways mirrors their development and usage in the psychometric literature, where they originated. A study by Fischhoff, Slovic, and Lichtenstein (1981, 1982) had panelists rate 90 hazards (stimuli) on 18 qualitative and often subjective aspects (scales) of risk compiled from various sources. A factor analysis of correlations from the resulting data resulted in a two-dimensional representation of the 90 hazards. The dimensions were interpreted as "some combination of novelty and mystery" and "possibility of uncontrollable consequences" (Fischhoff *et al.*, 1982, p. 249; also see further discussion in Slovic, Fischhoff, & Lichtenstein, 1984, pp. 188–190). Similarly, Hohenemser, Kates, and Slovic (1983) had subjects rate 93 technological hazards (e.g., dynamite blasts, elevator falls, hexachlorophine-toxic effects, oil tanker spills) on twelve investigator-stipulated scales that emphasized physical, biological, and social characteristics. A principal components analysis yielded a five-dimensional solution.

Both of these studies simply used spatial models as a method for reducing the nominal dimensionality of the panelists' rating data. That is, the scales supplied by the investigator are replaced by a smaller set of factors or dimensions. (In the case of principal components analysis, the factors will be linear combinations of the scales.) Hohenemser *et al.* obtained a solution in a space of dimensionality too high for visualization, did not present any lower-dimensional projections, and pursued the construction of a taxonomy for which discrete methods (see below) would probably have been more appropriate. Fischhoff *et al.* (1981, 1982) offered a spatial representation that, although readily visualizable, revealed nothing about possible differences among panelists because of limitations inherent in the psychometric models used.[2] As noted below, these two deficiencies are often related.

[2] Fischhoff *et al.* did, however, succeed in imbuing their results with considerable generality by obtaining similar spatial representations from a variety of subject groups.

The Vector Model

We have already observed that factor analysis and principal components analysis require panelists' ratings of a set of hazards or events on a set of scales (attributes) specified by the investigator. Thus, the resulting dimensions or factors of the spatial representation can easily bear the stamp of the investigator's judgment as well as the imprint of the panelists' ratings. This disadvantage is highly regrettable, since it makes results from these spatial models vulnerable to a major drawback of many decision-analytic approaches: the investigator's role in the procedure can unduly influence the results.

A highly innovative paper by Vlek and Stallen (1981) used a related spatial model, the vector model (described below), and thus allowed their panelists to rate twenty-six stimulus events on a 7-point category scale, using the "Q-sort" technique, with respect to such attributes as "the seriousness of risks and dangers" (see Vlek & Stallen, 1981, pp. 242–244 for details). That is, each panelist i produced a row vector $\mathbf{e}_i$ for the matrix $\mathrm{E} = \{e_{ij}\}$, where $i = 1, \ldots, m$ (the number of panelists) and $j = 1, \ldots, n$ (the number of stimuli). Panelists' data constitute row vectors of ratings that can be viewed as dominance judgments. To elaborate, if $e_{3,7} = 2$ and $e_{2,4} = 2$, then the third panelist put event seven in the second least risky category, while the second panelist placed the fourth event in the second least risky category. First, note that although the two entries just described have identical numerical values, those values are not comparable, since they came from different subjects who may have very different underlying scales of "riskiness." The data are thus conditional by respective panelists, who are indexed as rows in matrix E, which is thus labeled (in the terminology of Coombs, 1964) a "row conditional" matrix. Second, note that the entries within each row are treated as "dominance" data (cf. Carroll, 1980), as would arise, for example, from a tournament. Although such data are more commonly associated with the task of paired comparisons than with straightforward ranking, the vector model described below can be fitted to data from either paradigm. (In spite of the classic arguments (Kendall & Babington Smith, 1940, p. 324) advocating paired comparisons, both Vlek and Stallen (1981) and the present authors avoided that technique, in order to save panelists time and effort. A derivation of the vector model for paired comparisons data is given in Carroll (1973, pp. 326–328).)

Given the row conditional matrix E of dominance data, the vector model (first proposed independently by Slater, 1960; Tucker, 1960) seeks to represent the n events as points in a multidimensional Euclidean space, and the m panelists as vectors in the same space. A given panelist's dominance ordering of the set of risky events is reconstructed from the projections of events onto the vector for that panelist. The earliest application we have found of the vector model to the study of risk was Green and Maheshwari's (1969, pp. 452–454) investigations of preference for common stock.

Algebraically, the model can be written

$$\hat{e}_{ij} = f\left(\sum_{t=1}^{r} b_{it}x_{jt}\right), \tag{1}$$

where $\hat{e}_{ij}$ is the predicted or reconstructed rank given by panelist i to event j, f is a linear function, r is the dimensionality (with index t) of the Euclidean space, x_{jt} is the coordinate in this Euclidean space of the jth event on the tth dimension, and b_{it} is proportional to the direction cosine of the angle between panelist i's vector and the axis for the tth dimension. The value of b_{it} can be interpreted as a measure of the importance of dimension t for panelist i.

The computer program used most often for fitting the vector model (viz., solving for both variables in the summation on the right side of Eq. (1)) is Carroll and Chang's (1964; Chang & Carroll, 1969) MDPREF. Further discussions of the model and algorithms for fitting it are given by Carroll (1972, 1973, 1980) and Carroll and Arabie (1980).

Vlek and Stallen (1981) used an alternative to MDPREF, PRINCALS (de Leeuw & van Rijckevorsel, 1980; Gifi, 1985; van Rijckevorsel & de Leeuw, 1979) to fit data from 456 panelists on 26 events (e.g., transporting chlorine gas by freight train, smoking in bed, etc.). Vlek and Stallen (1981) gave dimensional interpretations of "size of a potential accident" and "degree of organized safety" for the two-dimensional solution they presented (1981, Fig. 4), which accounted for 51% of the variance. The authors grouped their panelists according to age, sex, and other variables, and then looked for and interpreted differences in directions of panelists' average vectors according to such background variables. Vlek and Stallen (1981, p. 269) emphasized the importance of being able to look at individuals' representations, in contrast to aggregate spatial solutions (e.g., as in principal components). Another advantage those authors (1981, p. 239) claimed for their approach, of deriving (from data) rather than selecting (for panelists' questionnaires) the dimensions of perceived risk, was that the newer practice allowed "a wider perspective for judging risky activities than a rational decision analysis (presumably using fewer, if any, psychological variables) is capable of accommodating" (1981, p. 239).

Following the tradition of Vlek and Stallen (1981), we used the fourteen events listed in Table 1 for a study in which panelists were asked to rank these events according to risk posed for the U.S. economy. Selection was based on relative prominence given these and related topics by the media during the summer of 1983. The panelists were eight stock brokers, eight

TABLE 1
FOURTEEN ECONOMIC EVENTS USED BY ARABIE, MASCHMEYER, AND CARROLL (1986)

Code used for plotting	Economic event
PR +	An increase of at least 1% in the prime interest rate occurs
PR −	A decrease of at least 1% in the prime interest rate occurs
TX +	There is an increase in United States income tax
TX −	There is a cut in United States income tax
IM +	The United States makes more money available to International Monetary Fund
IM −	The United States makes less money available to International Monetary Fund
O −	An oil embargo occurs (causing a shortage in the United States)
O +	An oil glut occurs
NUC	A breakthrough in safety and economy for nuclear electric power is achieved
TDR	There is further deregulation of the airline and the trucking industry
MMT	Much more money is made available for mass transit
OSH	OSHA guideines are relaxed
IMQ	United States imposes import quotas on a large variety of manufactured products
EBT	A new electronic breakthrough, comparable to the invention of the transistor, is announced

Ph.D. students, and thirteen MBA students. The latter two groups were specializing in finance at the Business School of the University of Illinois at Urbana-Champaign. (Further details of the study are available in Arabie, Maschmeyer, & Carroll (1986).) Like Gooding (1975), Slovic, Fischhoff, and Lichtenstein (1981), and others, we were interested in how professionals and students (two groups of the latter, in our study) might differ in the perception of risk.

A four-dimensional MDPREF solution based on the twenty-nine panelists' rankings accounted for 75.0% of the variance. (Corresponding values for each of the four dimensions were 40.2, 13.6, 12.6, and 8.7%. Mean product-moment correlations between input data and predicted values were .88 for MBA and .83 for Ph.D. students, and .87 for brokers.) Figure 1 presents the first two dimensions, accounting for the highest percentages of variance. Dimension 1 clearly reflects a concern for transportation and energy, in pitting an oil shortage (O −) against more money for mass transit and a breakthrough in safety for nuclear power. Dimension 2, contrasting such events as changes in the prime lending rate versus relaxation of OSHA rules, can be interpreted as monetary versus nonmonetary governmental actions.

Figure 2 plots the third and fourth dimensions. The third contrasts government versus private initiatives to boost the economy. Specifically, fur-

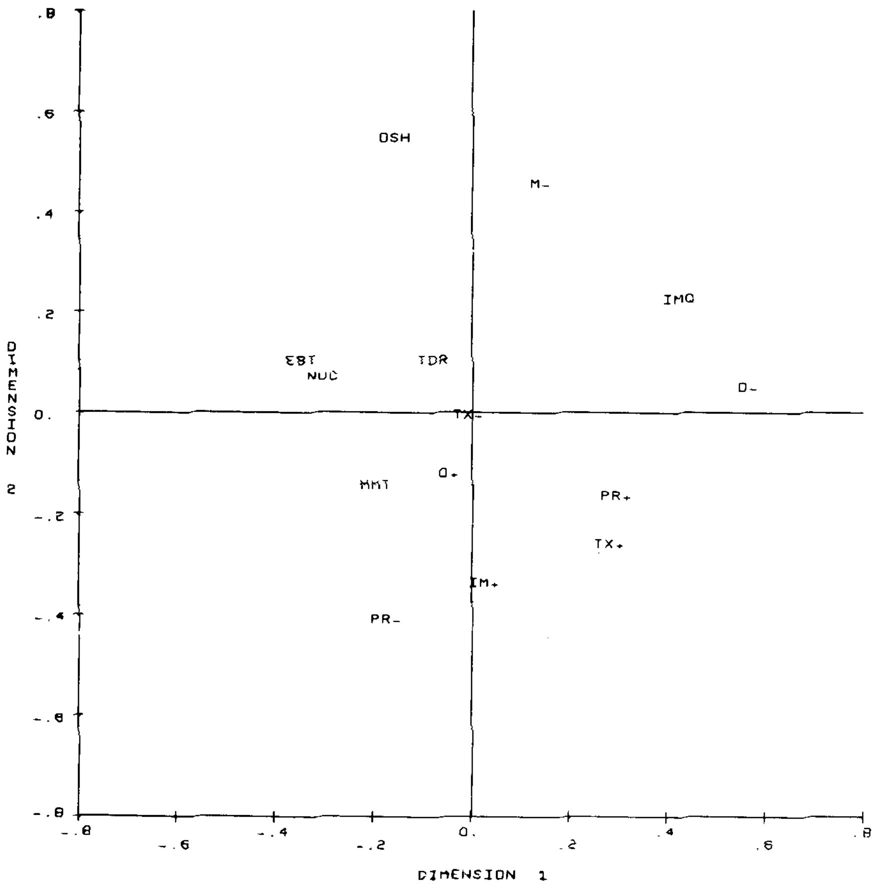

FIG. 1. Dimensions 1 and 2 of four-dimensional MDPREF solution, with events plotted, using codes given in Table 1.

ther deregulation of airline and trucking industries, more money for mass transit, etc., are contrasted with a tax break, an electronic breakthrough, etc. The fourth dimension of the MDPREF solution, accounting for the least variance, is predictably also the least interpretable. That dimension's extreme events, an oil surplus and more money for the International Monetary Fund (IMF), suggest a stance of concern for the Third World. Concretely, an oil surplus often is associated with less concern for Third World suppliers of oil (with the notable exception of their ability to pay international bank debts), in contrast to the involvement shown by making more money available to the IMF.

Figure 3 depicts the panelists' judgments of risk, with each panelist represented as a vector in the same space depicted in Fig. 1 (Dimensions

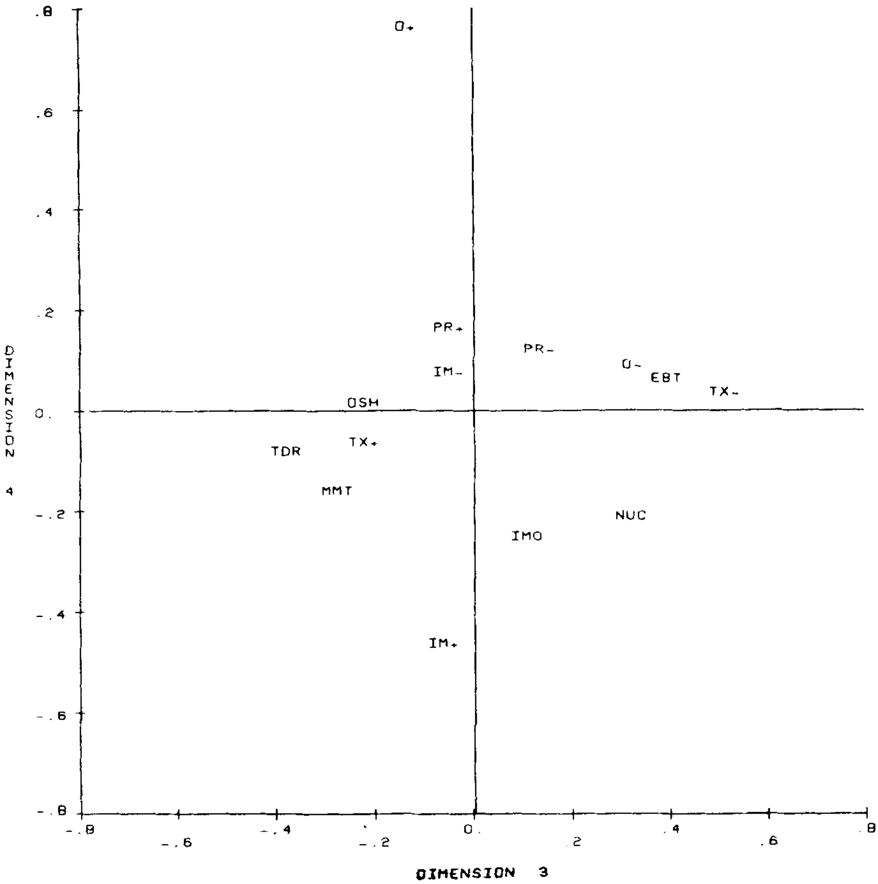

FIG. 2. Dimensions 3 and 4 of four-dimensional MDPREF solution, with events plotted, using codes given in Table 1.

1 and 2 from the four-dimensional MDPREF solution). The length of a vector is proportional to the correlation (R) between the corresponding panelist's original rankings of the events and those reconstructed from the projections of the events onto the MDPREF-positioned vector. The fourteen events from Fig. 1 have not been included in Fig. 3 simply because they make it too cluttered. In Fig. 3, all vectors except the one for a doctoral student (22P6) aim toward quadrants one and four. Interesting differences emerge between groups. For example six of the eight brokers' vectors (denoted by "BRK" as the first three characters in the labels) lie in quadrant four, indicating that increases in the prime rate and in taxes were judged most risky for the U.S. economy. In contrast, ten of the thirteen MBA candidates (whose labels have an "M" as the third char-

acter and year of study as the fourth character) point to quadrant one, in the direction of oil shortage and imposition of import quotas, as greatest risks for the U.S. economy. The vectors of the doctoral students (having "P" as the third and year of study as the fourth character in the labels) show more variability in direction, although there is some concentration in the first quadrant, especially in the direction lying between the imposition of import quotas and less money for the IMF. Another noteworthy point is that the more idiosyncratic panelists are readily noticed. For example, MBA student "19M2" ostensibly views more funding for the IMF as a risk. That panelist's vector is diametrically opposed by doctoral student "22P6" who regards the further deregulation of trucking and airlines and the relaxation of OSHA rules as most risky for the U.S. economy. If

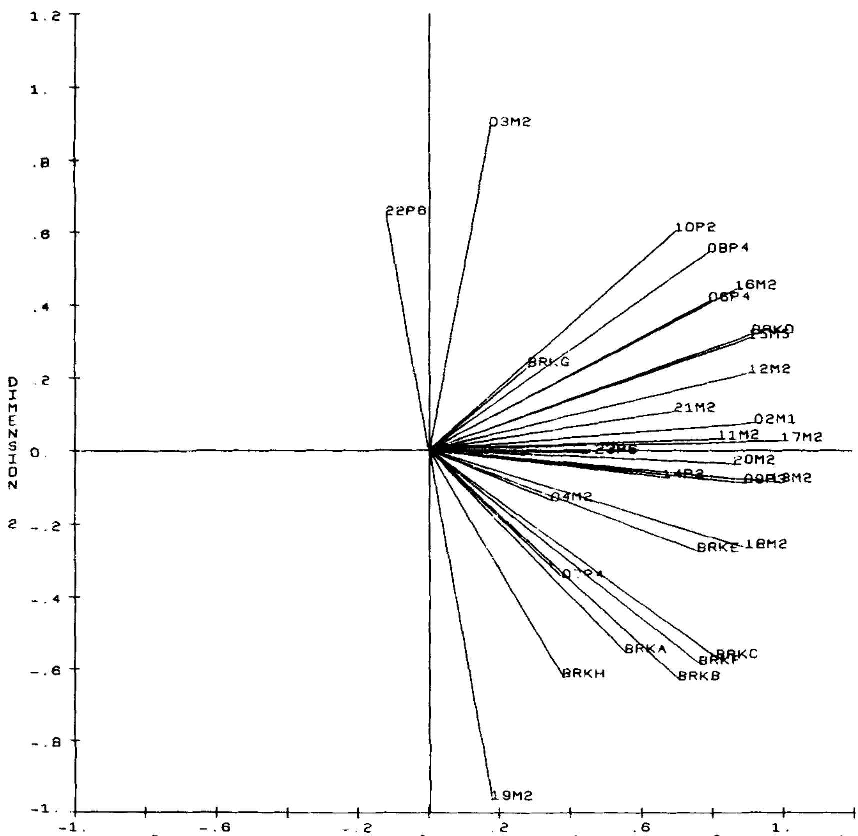

FIG. 3. Dimensions 1 and 2 of four-dimensional MDPREF solution (as in Fig. 1), with panelists represented as vectors. BRK denotes brokers, a third character of M denotes MBA students, and P denotes doctoral students.

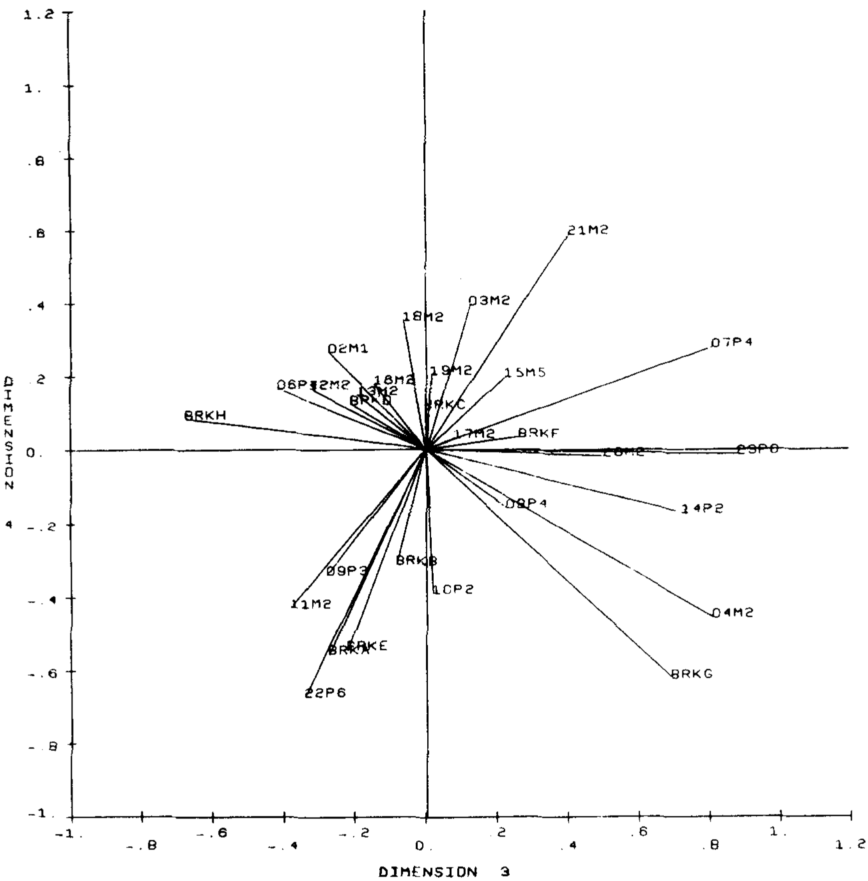

FIG. 4. Dimensions 3 and 4 of four-dimensional MDPREF solution (as in Fig. 2), with panelists represented as vectors. BRK denotes brokers, a third character of M denotes MBA students, and P denotes doctoral students.

one wants to evaluate the representativeness of a panelist's rankings vis-à-vis the other panelists, Fig. 3 presents a graphic answer.

Figure 4 shows the panelists' vectors in the same space depicted in Fig. 2 (Dimensions 3 and 4 from the four-dimensional MDPREF solution). Again, the fourteen events from Fig. 2 are not included in Fig. 4 because of clutter. Clearly, there is much greater variability in the directions of the vectors in Fig. 4 than was the case in Fig. 3. As noted earlier, the dimensions in Figs. 2 and 4 accounted for considerably less variance than did the dimensions in Figs. 1 and 3, so that it is fair to say that the pictures in Figs. 2 and 4 are more noisy. The MBA students' vectors show the greatest uniformity in Fig. 4, with concentration in changes in the prime rate as risky for the U.S. economy. As a group, the stock brokers show less coherence (i.e., their vectors point in more diverse directions) than they did in Fig. 3, and the doctoral students are again highly variable in the directions of their vectors.

In addition to studying MDPREF representations of the panelists' vectors to find patterns of agreement and individual differences, one can also go directly to the raw data (viz., the panelists' orderings of the events vis-à-vis risk to the U.S. economy). Using Kendall's (1948) coefficient of concordance, $W = .340$ ($p < .001$) for all 29 panelists, .316 ($p < .005$) for the eight doctoral students, .456 ($p < .001$) for the thirteen MBA students, and .446 ($p < .001$) for the stock brokers. Vlek and Stallen (1981, p. 253) reported a corresponding statistic of .20 for their 238 panelists' orderings of a subset of 12 of the 26 events according to the criterion "size of a possible accident following a serious and credible error." The values of this statistic, computed for different studies and groups of panelists, suggest that there is enough variability among panelists to justify caution in the use of aggregate representations.

Other Spatial Models

Of course, principal components, factor analysis, and the vector model do not exhaust the possibilities for (continuous) spatial models that can be used to represent judged and perceived risk. (A taxonomy of such models is presented by Carroll & Arabie (1980).) Another especially useful spatial model for such purposes is the INDSCAL model of Carroll and Chang (1970) for three-way (panelists × events × events) scaling. In fact this model was the original basis for the development of Impact Scaling (Carroll, 1977; Carroll & Sen, 1976), designed as a forecasting and planning technique for managers faced with risky decisions. In addition to early applications of INDSCAL to the perception of investment risk (Gooding, 1975, 1978), we refer the reader to Arabie *et al.* (1986) for a detailed description of the methodology of Impact Scaling as well as recent extensions. That paper includes an INDSCAL solution for the same fourteen events represented above using MDPREF. INDSCAL requires dyadic measures of proximity between all pairs of events from each panelist as input, in contrast to the (monadic) orderings of risk required for the MDPREF analyses. In the INDSCAL analysis by Arabie *et al.* (1986), the data were derived from modified subjective conditional probabilities between all pairs of the fourteen events listed in Table 1. Thus, although the same 29 panelists gave (different types of) data for both the MDPREF and INDSCAL analyses, the resulting dimensional interpretations for the two solutions were neither expected nor found to be identical. To elaborate, because MDPREF was applied to dominance data and the INDSCAL solution came from proximities data, there is no a priori psychological reason the two solutions should be symmetrically related. Empirical evidence to date has left much room for argument over whether any predictable relationship in general is to be expected. (See, for example, the exchange of views between J. D. Carroll and C. H. Coombs in the Discussion chapter of Lantermann & Feger (1980, pp. 368–369).)

Comparative Relevance of Various Spatial Models to the Study of Risk

We give a brief overview of differences and similarities among the spatial models just considered, with respect to (a) their utility for studying the perception and judgment of risk, (b) types of input data and requisite paradigms for collection, and (c) scale types assumed for the input data. We hope that a fourth important consideration, types of representation afforded by fitting the models, has already been covered in the preceding summaries of the analyses discussed earlier.

A useful distinction in this discussion stems from Tucker's (1964) nomenclature of "modes." Although all the approaches considered here assume as basic data a two-way matrix (or a collection of them for such "three-way" models as INDSCAL and INDCLUS), the vector, principal components, and factor analytic models are generally fitted to a (single two-way) matrix where rows and columns represent such different sets of entities as panelists and events. Thus such data are said to be two-mode as well as two-way. In contrast, the two-way matrices used in nonmetric multidimensional scaling (MDS) are one-mode, with both rows and columns corresponding to the same set of entities (e.g., events).

When a researcher seeks a representation of *both* panelists and events (for example) in the same space, then it is not surprising that a two-mode data matrix is required. In such studies as those reported above employing the vector model, each panelist is typically giving ratings or rankings for a specified criterion according to some task yielding at least ordinal information on the events, according to that panelist's judgment (see discussion above of row conditional matrices). The computer program used most often for fitting this model (MDPREF, written by Chang & Carroll, 1969) treats the numerical entries in the data matrix as interval scale data. That is, $\hat{e}_{ij}$ in Eq. (1) is intended to reconstruct (approximately) the numerical ratings given by a panelist, according to the stipulation that those values have interval rather than ordinal scale information. (This interpretation of reconstructing or predicting the panelists' data, or some transformation of them, according to a specific type of scale, will be central to the discussion of models for the judgment and perception of risk.)

Kruskal and Shepard (1974) devised a "nonmetric" or ordinally based technique for fitting the same vector model of Eq. (1), but concluded (1974, p. 153) that the robustness of the linear approach (as taken by

Carroll & Chang, 1964) enabled it to do a better job of recovering a known structure, even when the data had been subjected to nonlinear (but monotone) distortion. This result suggests that there may be little advantage to using the monotone option available in the PRINCALS program (de Leeuw & van Rijckevorsel, 1980; Gifi, 1985; van Rijckevorsel & de Leeuw, 1979) employed by Vlek and Stallen (1981) for fitting the vector model.

The general principle for comparing different assumptions about underlying scale types is straightforward: if the user believes the input data (e.g., ratings, rankings, or measures derived from subjective estimates of probabilities) are not linearly related to the parameters fitted to them in constructing a spatial representation (i.e., distances between points depicting objects in MDS, cosines of angles between vectors depicting variables in principal components analysis), then assuming a functional relationship that is only required to be monotone (for ordinal scales) rather than linear (for interval scales) is preferable. When subjective probabilities are involved, assuming monotonicity seems highly advantageous, given the well-known distortions that are especially prevalent for extreme values. However, the arguments based solely on measurement characteristics of the data must sometimes be tempered by limitations inherent in the actual models or their associated algorithms. We have already cited Kruskal and Shepard's surprising results in fitting a nonmetric (ordinal) model for principal components analysis. Similarly, Carroll and Arabie (1983, p. 167) provided technical arguments precluding the development of a (conventional) ordinal approach to fitting the ADCLUS model (described below in the discussion of discrete models).

Returning to the discussion of various spatial models appropriate for the perception and judgment of risk, both principal components analysis (PCA) and common-factor analysis (as well as other approaches to the latter technique; see McDonald, 1985) assume a two-mode matrix as input. PCA and factor analysis can also be regarded as vector models, even though their typical input data are not preference (or dominance) data, as discussed here for studies using MDPREF or PRINCALS. In the studies cited earlier, Fischhoff *et al.* (1982) had panelists rate 90 hazards on 18 qualitative attributes of risk, and Hohenemser *et al.* (1983) had subjects consider 93 technological hazards on twelve scales. We have noted earlier that the requirement of two-mode data for PCA and factor analysis entails the selection by the researcher of relevant attributes or scales. In contrast, MDS assumes one-mode matrices (as discussed below) and thus allows the experimenter a less obtrusive role.

A reviewer of the present paper called attention to the fact that the experimenter also selects the other mode of a two-mode matrix—namely the entities such as technological hazards under study. This observation necessarily applies to all the scaling and clustering models considered in this paper and underscores the context-dependency of both the resulting representations and this general approach to the perception and judgment of risk. The limitation here parallels the fixed versus random effects models of analysis of variance. There simply are no scaling and clustering models presently available embodying the random effects philosophy. Since many of the major projects studying perception and judgment of risk in recent years have been specific to such domains as energy (see examples in Borcherding *et al.*, 1984a), there does not seem to be a mismatch in generality between models and context-specific domains of investigation.

Both PCA and most approaches to factor analysis begin by computing product–moment correlations (or occasionally covariances; see Tiemann & Tiemann, 1985) between all pairs of attributes, variables, or "scales" selected by the experimenter, and thus assume interval scale data. PCA places ones in the principal diagonal of this correlation matrix whereas common-factor analysis uses estimates of commonalities (e.g., squared multiple correlations). Rather than trying to represent pairwise relationships between hazards by the distances between points representing hazards in a space as in MDS, PCA and factor analysis approximate the input correlations by the cosines of the angles between vectors. (These vectors are formed by the points representing the variables and the nonarbitrary origin of the "factor space." The coordinates of these vectors' endpoints are given as "factor loadings." Traditionally, these points are not included in the graphic display of the hazards, whose coordinates are given as "factor scores.") PCA finds a linearly independent set of "component" variables accounting for the larger number of variables originally selected by the experimenter. Thus, the pioneering study in this area, by Fischhoff, Slovic, Lichtenstein, Read, and Combs (1978; also see Slovic *et al.*, 1981, 1984) had panelists rate 30 potential hazards on each of nine characteristics. Those authors then used PCA to obtain two interpretable factors (accounting for 80% of the variance) to represent ratings on the original nine scales.

Several decades of psychometric literature have failed to resolve debates on whether PCA or common-factor analysis is the better technique, but recent attention that is relevant to the study of risk has focused on the former technique's lack of robustness to outlying data values (Critchley, 1985; Krzanowski, 1984).

Before considering nonmetric MDS, we first return to the question of context dependence resulting from the experimenter's selection of stimuli (e.g., hazards). While describing the vector model, we noted that such computer programs as MDPREF and PRINCALS attempt to reconstruct the panelists' ratings, subject to various assumptions about scale type.

This approach to fitting the vector model has been designated the "direct" approach by Kruskal (1978). PCA and factor analysis, however, pursue a less ambitious task of "indirect" fitting of the same model, namely reconstructing the *correlations* among the variables, rather than the numerical values used to compute the correlations. If one considers the *generality* of a representation (obtained as output from an analysis) based on the two different approaches, it is clear that equality of correlation matrices (resulting from sampling over different hazards) is a more easily satisfied and more general condition that is equality of the actual numerical values (up to a monotone or positive linear transformation, depending on the scale type assumed). Thus, although we view PCA and factor analysis as inferior to MDS for representing the perceived judgmental structure of hazards, the former methods are advantageous when considered from the perspective (raised by a reviewer) of generality over hazards sampled.

Nonmetric MDS (Kruskal, 1964a, 1964b; Shepard, 1962a, 1962b) typically assumes a single one-mode two-way data matrix as input. The data are assumed to be systematic measures of the relatedness of each distinct pair of objects (e.g., hazards) under study. Paradigms commonly used (e.g., by Johnson & Tversky, 1984) include giving "direct" judgments on (say) a nine-point scale from least to most similar, for all $n(n - 1)/2$ distinct pairs of n hazards. In general, any consistent measure of pairwise similarity, dissimilarity, confusability, substitutability, cooccurrence, or association can serve in principle as input data. Because the data required for MDS are one-mode, the researcher is spared the intrusive role of selecting the crucial attributes or "scales" for the requisite second mode of an input matrix for PCA or factor analysis.

Nonmetric MDS assumes only a monotone function relating the input proximities to the (reconstructed) interpoint distance between all pairs of hazards in the spatial representation. As noted earlier, because the input data are only assumed to be ordinal, nonmetric MDS can better accommodate such data as subjective probabilities that are rarely viewed as interval scale. A second advantage of nonmetric MDS is that its weaker assumptions about the data often allow fitting nonlinear data in fewer dimensions than are required by such linear techniques as PCA and factor analysis (Shepard, 1962a, 1962b). Thus, MDS may offer a more parsimonious description of risk spaces.

The techniques discussed thus far in this section have all assumed a single input matrix. (For MDS, such a proximities matrix is necessarily aggregated over panelists and/or replications.) A consequence is that unless one of the modes for two-mode approaches is for panelists (as in the MDPREF example given earlier), none of the techniques just described has any facility for depicting individual differences among panelists' perceptions and judgments. Realizing this limitation, Slovic *et al.* (1981) did separate analyses of data from four different groups of panelists varying in expertise. However, this approach can lead to difficulties in comparing groups' solutions and offers little information concerning within-group differences.

Beginning with the advent of the INDSCAL (for INdividual Differences SCALing) model and computer program (Carroll & Chang, 1970), three-way (or "individual differences") MDS has seen greatly increasing usage, especially in the behavioral sciences. However, with the notable exception of work by Vlek and his colleagues (cited earlier), we believe that three-way MDS has been underemployed in studies of the perception and judgment of risks. For example, Cvetkovich and Earle's (1985, p. 19) review listed the lack of information concerning individual judgments as a major weakness of psychometric approaches to the study of hazardous events. The description offered here of three-way techniques that remedy this deficiency will be fairly terse so as not to be redundant with Arabie *et al.* (1986).

The INDSCAL technique assumes a symmetric proximities matrix from each panelist or other source of data. The INDSCAL method places n stimuli (events) in a Euclidean space of specified dimensionality, so that events perceived to be closely related are positioned close together, whereas relatively unrelated events are distantly placed from each other. This Euclidean space has a mathematically preferred orientation, and only reflections and permutations (but *not* other rotations) leave the variance accounted for unchanged, as a measure of goodness-of-fit. The axes defining the preferred orientation have, in most published applications, been substantively interpretable. Differences among subjects (panelists) are depicted by stretching or shrinking (i.e., weighting) axes (dimensions) according to the salience attributed to those axes by the individual panelists' data. As a result, the INDSCAL model is sometimes referred to as the "weighted Euclidean model."

Formally, the estimated distance between a pair of events E_j and E_k in the weighted Euclidean space can be written as

$$D_{jk}^{(i)} = \sqrt{\sum_{t=1}^{r} w_{it}(x_{jt} - x_{kt})^2}, \tag{2}$$

where $D_{jk}^{(i)}$ is the distance between the events E_j and E_k for the ith panelist, w_{it} is the weight for panelist i in the tth dimension, and x_{jt} is the coordinate of event E_j on the tth dimension.

Graphically, the events are represented in a space of r dimensions, and the panelists are positioned in a separate space of the same dimensionality r. The coordinates for the panelists (represented as points in the space) are given by the weights w_{it}. Thus, both graphically and numerically, the experimenter has a representation of potential differences among panelists' judgments.

A noteworthy aspect of the INDSCAL model is that, unlike two-way nonmetric multidimensional scaling in Euclidean space and also unlike most approaches to factor analysis, the axes in an INDSCAL event space have a mathematically preferred orientation. As such, the goodness-of-fit (variance accounted for) of a spatial solution is only invariant under reflections and permutations of the axes. (The data analyst, of course, is responsible for the substantive interpretation of these axes or dimensions.) The preferred orientation is conferred upon the event space by the presence of the weights, w_{it}, in Eq. (2). As noted above, these weights are the coordinates of the panelists' space. Although not constrained to be positive, the weights empirically are nearly always in the positive orthant of the space. A subject (or other source of data) giving judgments inconsistent with the model for any of the r dimensions will generally have very small weights for those dimensions. Psychologically, the weights w_{it} gauge the salience of dimension t for panelist i by differentially stretching or shrinking that dimension. Statistically, the size of the weights gives an indication of the variance accounted for in the data from a given panelist. Thus, the weights provide an opportunity for studying within- as well as between-group differences in patterns of judgments and are thus potentially quite useful for policy-making decisions (see Arabie *et al.*, 1986). Another important feature of the INDSCAL approach is that in fitting the weights of the model, one is simultaneously fitting the dimensions of the space *and* giving them a preferred orientation. In contrast, a multiattribute utility approach (see Fischhoff *et al.*, 1984) requires that the investigator first select the dimensions *before* panelists can give data suitable for estimating weights for these a priori dimensions.

Considering assumptions of scale type for INDSCAL analyses, both the original INDSCAL program and Pruzansky's (1975) SINDSCAL assumed interval scale proximities data. ALSCAL, an alternative program devised by Takane, Young, and de Leeuw (1977), has options for fitting the INDSCAL model while assuming only ordinal data. However, Hahn, Widaman, and MacCallum (1978) concluded that the INDSCAL program, even though it assumes interval scale data, generally recovered *ordinally* defined structure better than ALSCAL did, using the latter's option for ordinal data.

DISCRETE MODELS

Vlek and Stallen (1980) sought to provide a "psychological categorization" (1980, p. 273) or taxonomy of risk and therefore offered (1980, Fig. 4) a "rational ordering of the various aspects of risk." This ordering was based on a priori considerations rather than analysis of empirical data.

(Also see the review of such schemes of classification of hazardous events in Cvetkovich & Earle (1985, pp. 16–24).) As noted earlier, Hohenemser *et al.* (1983) were also attempting to classify hazards. For such purposes, discrete models (in which a risky event either is or is not (i.e., all-or-none) relevant to a given cluster, subset, or feature of the model) can offer advantages of additional information over the continuous spatial models considered in the preceding section. Various types of clustering (see Hartigan, 1975; Hubert, 1974; Morrison, 1967) have long been available as discrete alternatives to spatial models. For example, Cooley (1977) used hierarchical clustering to depict judgmental differences in perceived risk among professional investment managers. However, more recent developments offer as promising candidates discrete models for representing the perception of risky events. Reviews of these models are given by Carroll (1976), Carroll and Arabie (1980), and Shepard (1980).

Johnson and Tversky (1984) provided a useful data set, concerning perceived relations among prevalent causes of death, for illustrating the use of these models. The eighteen risks (stimuli) covered in the study were accidental falls, airplane accidents, electrocution, fire, flood, heart disease, homicide, leukemia, lightning, lung cancer, nuclear accident, stomach cancer, stroke terrorism, tornado, toxic chemical spill, traffic accidents, and war. Among the panelists' tasks were (a) giving judgments of similarity on a nine-point scale for all distinct pairs of the eighteen risks, and (b) "conditional[3] predictions" which referred to panelists' willingness to increase the estimated incidence rate for risk i given information that their earlier estimates of incidence for risk j were conservative. The "conditional predictions" task is the more novel of the two and was regarded by Johnson and Tversky (1984, p. 57) as a measure of judged covariation. Both tasks were described as comparative and holistic, in contrast to an evaluative task not considered here since our analyses will not use data from the third task. The authors predicted that data from paradigms (a) and (b) to be fitted better by discrete "feature" models than by either metric (interval scale) or nonmetric (ordinal) MDS, and this prediction was confirmed. (Further details are available in the source article.)

Johnson and Tversky (1984, Table A1) helpfully published the aggregate proximity matrices resulting from each of these two judgmental tasks. Among other analyses, those authors provided discrete representations of the perceived structure for the eighteen risks, via an additive tree (1984, p. 61) solution for the conditional prediction data, using the ADD-

[3] The three senses of "conditional" used in this paper (row conditional matrix, conditional prediction, and matrix conditional analyses) are unrelated and should not be confused.

TREE program of Sattath and Tversky (1977). This solution, which accounted for 74% of the variance, was described as a hierarchy of clusters interpreted as hazards, accidents, violent acts, technological disasters, and diseases. The authors (1984, p. 61) concluded that "Nevertheless, the results indicate that the clustering of the risks is not entirely hierarchical, and there is some evidence for overlapping clusters." (For further details of the analysis and the model involved, the reader should consult Johnson & Tversky (1984) and Sattath & Tversky (1977).) This conclusion is consonant with the view of Vlek and Stallen (1980, p. 294) that various aspects of risk are "naturally" viewed as clusters and that these aspects overlap or mutually influence each other. Thus, to pursue a more complete representation allowing for overlap,[4] Johnson and Tversky obtained an "extended tree representation" for the conditional prediction data, using the EXTREE method of Corter and Tversky (1986). A solution (Johnson & Tversky, 1984, Fig. 7) having more than twenty clusters[5] for the eighteen risks accounted for 94% of the variance.

Since the representation just described has more clusters than the number of risks being studied, it seems natural to seek a more parsimonious representation for the eighteen risks. As an alternative to the Corter and Tversky (1986) EXTREE model, we used a closely related discrete model, ADCLUS (for ADditive CLUStering), devised by Shepard and Arabie (1979) and described below. J. Douglas Carroll (personal communication) has observed that the EXTREE model differs from the ADCLUS model only in that the former constrains the hierarchical tree's component of the clustering solution to conform to the path length metric (see Sattath & Tversky, 1977), when EXTREE solutions are presented in the format of Johnson and Tversky's solution in their Fig. 7 (1984, p. 62).

Like EXTREE, the ADCLUS model assumes as input a single $n \times n$ symmetric proximities matrix gauging pairwise proximity between all distinct pairs of the n stimuli (viz., the eighteen risks in the present analysis). The number of clusters, m, to be interpreted, is defined by the user, and the model is written

$$\hat{\mathbf{S}} = \mathbf{PWP}' + \mathbf{C}, \tag{3}$$

where $\hat{\mathbf{S}}$ is an $n \times n$ symmetric matrix of reconstructed similarities (with ones in the principal diagonal), W is an $m \times m$ diagonal matrix with

[4] It should perhaps be added that hierarchical clustering allows clusters to overlap only if one cluster is a proper subset of another. That is, one cluster is "nested" in the other. Thus, for any two distinct clusters X and Y, the following conditions are mutually exclusive and exhaustive: $X \subset Y$, $Y \subset X$, or $X \cap Y = \emptyset$. The methods of overlapping clustering considered below allow for intermediate degrees of overlap among clusters.

[5] The clusters constituting the hierarchical part of an EXTREE solution are constrained to satisfy the path length metric (Sattath & Tversky, 1977).

weights w_k $(k = 1, \ldots, m)$ in the principal diagonal (and zeroes elsewhere), and **P** is the $n \times m$ rectangular matrix of *binary* values p_{ik}. Here **P**′ is the $m \times n$ matrix transpose of matrix **P**. **C** is an $n \times n$ matrix having zeroes in principal diagonal, and the (fitted) additive constant in all the remaining entries. Two further points should be noted. First, for the **P** matrix, note that each column represents one of the m subsets (clusters), with the ones of that column defining constituency of stimuli (risks) within the respective subsets. Second, the additive constant c is required to allow variance (rather than just sums of squares) accounted for as a measure of goodness-of-fit. The constant can also be viewed as the weight corresponding to the complete set of stimuli, included implicitly as an $(m + 1)$-st subset. It should perhaps be added that in the analyses presented below, both the clusters (**P**) and their weights (**W**) are being fitted simultaneously by the program MAPCLUS (for MAthematical Programming CLUStering) for fitting the ADCLUS model; neither matrix was supplied by the data analysts. Further discussions of the ADCLUS model and generalizations of it can be found in Arabie and Carroll (1980b), DeSarbo (1982), Carroll and Arabie (1983), and Shepard and Arabie (1979).

We used the MAPCLUS algorithm of Arabie and Carroll (1980a, 1980b) for fitting the ADCLUS model in order to analyze Johnson and Tversky's (1984) mean conditional predictions data. The MAPCLUS program requires the user to assume interval scale proximity values as input. Table 2 presents a seven-cluster solution, obtained using a random initial configuration for the **P** matrix, and accounting for 76.5% of the variance. The representation shares with the solution of Johnson and Tversky (1984, Fig. 7) three clusters (here numbered 1, 2, and 4) and a high degree of interpretability. The most heavily weighted cluster (heart disease,

TABLE 2
MAPCLUS SOLUTION FOR MEAN CONDITIONAL PREDICTION DATA FROM JOHNSON AND TVERSKY (1984)

Subset	Weight	Risks contained in subset
(1)	.768	Heart disease, stroke
(2)	.636	Nuclear accident, toxic chemical spill
(3)	.532	Heart disease, leukemia, lung cancer, stomach cancer
(4)	.519	Homicide, terrorism, war
(5)	.427	Fire, flood, lightning, tornado
(6)	.231	Leukemia, lung cancer, stomach cancer, stroke, toxic chemical spill
(7)	.226	Accidental falls, airplane accidents, electrocution, fire, lightning, traffic accidents

Note. These seven clusters plus an additive constant of .135 accounted for 76.5% of the variance. See text for interpretation of clusters.

stroke) comprises problems and failure of the circulatory system. The second cluster concerns environmental disasters that can also have genetic consequences. The third most heavily weighted cluster consists of heart disease and all three forms of cancer included in the set of the eighteen risks. Violent acts (homicide, terrorism, and war) constitute the fourth cluster. The natural hazards of tornado, flood, lightning and its frequent concomitant, fire, are the fifth most heavily weighted cluster. Stroke, the three forms of cancer considered here, and a class of events often regarded as carcinogenic (toxic chemial spills) constitute the sixth most heavily weighted cluster. Finally, the seventh cluster consists of accidents attributable either to nature or human error.

It is of interest to note that only the fourth cluster (acts of violence) fails to have partial overlap with another cluster and that no cluster is nested within another (as in hierarchical clustering). This observation corroborates the remark quoted earlier from Johnson and Tversky (1984, p. 61) concerning overlapping clusters for these data.

Although Johnson and Tversky reported 94% of the variance accounted for in their second similarity data matrix by an EXTREE (interval scale) solution, neither the solution nor the number of clusters entailed is reported in their article. Table 3 presents a MAPCLUS representation fitting the ADCLUS model to these similarity data. A seven-cluster solution, obtained from a random initial configuration, accounted for 72.7% of the variance. The five most heavily weighted clusters from this solution also appeared in Johnson and Tversky's ADDTREE (Sattath & Tversky, 1977) representation of these data.

For the various models over which Johnson and Tversky (1984, Table

TABLE 3
MAPCLUS SOLUTION FOR SIMILARITY RATINGS OF RISKS DATA FROM JOHNSON AND TVERSKY (1984)

Subset	Weight	Risks contained in subset
(1)	.724	Flood, lightning, tornado
(2)	.613	Electrocution, fire, lightning
(3)	.606	Nuclear accident, toxic chemical spill
(4)	.542	Homicide, terrorism, war
(5)	.412	Heart disease, leukemia, lung cancer, stomach cancer, stroke
(6)	.208	Accidental falls, heart disease, lightning, stroke, traffic accidents
(7)	.170	Accidental falls, airplane accidents, electrocution, fire, flood, homicide, nuclear accident, terrorism, tornado, toxic chemical spill, traffic accidents, war

Note. These seven clusters plus an additive constant of .161 accounted for 72.7% of the variance. See text for interpretation of clusters.

2) compared goodness-of-fit[6] between the two matrices (conditional predictions vs similarities), the fit to the similarities data was always inferior. We found the same result and also that our solution for the similarities data was somewhat less interpretable than was the solution for conditional prediction data in Table 2. Clusters 3 and 4 in Table 3 are identical respectively to Clusters 2 and 4 in Table 2. The first cluster consists of natural hazards arising from bad weather. Lightning and its lethal henchmen, fire and electrocution, comprise the second cluster. Circulatory problems and the three types of cancer form the fifth cluster. The sixth cluster can be construed as causes of death that prudence and caution may enable one to avoid, and whose hazards are thus somewhat under an individual's control, unlike, for example, a nuclear accident.[7] The least weighted cluster is also the most problematic for interpretation, although it is readily described as all the risks except lightning and the health-related risks constituting the fifth cluster. Note that the last cluster also subsumes the third and fourth so that there is some (rather weak) evidence of nesting to indicate hierarchical structure. Otherwise, there are many instances of partial overlap among clusters in the MAPCLUS solution shown in Table 3. This aspect of structure cannot be represented by an ADDTREE analysis such as Johnson and Tversky's (1984, Fig. 6), which yields a (constrained) hierarchical solution.

Before presenting a discrete model that allows seeking a representation of *both* the similarities and the predictions data simultaneously, we first return to the question of parsimony. Johnson and Tversky reported 83 and 94% variances accounted for, using EXTREE (linear option) with a *twenty-one* cluster solution for the latter data set. Our corresponding seven-cluster MAPCLUS solution in Tables 2 and 3 accounted for 72.7 and 76.5% of the variances, respectively. Pursuing an even more parsimonious representation, we also succeeded in obtaining five-cluster solutions accounting for 63.2 and 74.3% of the variances, respectively. However, we found these latter solutions considerably less interpretable and therefore presented the seven-cluster solutions.

INDCLUS

The Shepard–Arabie ADCLUS model of Eq. (3) can be generalized to the (three-way) case of multiple input proximities matrices. The generalization, called INDCLUS (Carroll & Arabie, 1982, 1983) allows for ma-

[6] Those authors also provided spatial representations based on multidimensional scaling and principal components analysis for the 18 risks.

[7] Although an established literature surrounds this distinction, it still gives rise to strongly divergent views: see the exchange on this topic between Perrow (1986) and Wildavsky (1986).

trices from different individuals (hence, the acronym for INdividual Differences CLUStering), or other sources of data, such as the two experimental conditions Johnson and Tversky employed to obtain the data sets used above.

We mentioned earlier that INDSCAL (Carroll & Chang, 1970) allows a continuous spatial representation for a collection of input proximities matrices (all based on the same set of n entities). The INDSCAL model assumes a Euclidean space whose *dimensions* are differentially relevant or salient to all subjects or other sources of data. The differences between such sources are depicted in differential weights fitted to each dimension for each input matrix. The INDCLUS (Carroll & Arabie, 1983) model offers a discrete counterpart to the INDSCAL model, in assuming that a set of *clusters* potentially interpretable as features can be found that are relevant to all the input matrices. However, the weights for these clusters vary both as a function of which cluster and which input matrix is being considered. Thus, the INDCLUS model substitutes cluster (viz., discrete subsets of the n entities being studied) for the spatial dimensions assumed by INDSCAL.

The INDCLUS model can be written

$$\hat{\mathbf{S}}^{(i)} = \mathbf{P}\mathbf{W}^{(i)}\mathbf{P}' + \mathbf{C}^{(i)}, \tag{4}$$

where $\hat{\mathbf{S}}^{(i)}$ is the symmetric $n \times n$ similarities matrix estimated for the ith input matrix; $\mathbf{P}$ is the $n \times m$ *binary* matrix whose unities within a column define the constituency of the column corresponding to the kth cluster ($k = 1, \ldots, m$); $\mathbf{W}^{(i)}$ is an $m \times m$ diagonal matrix having weights for the $m \times n$ matrix transpose of $\mathbf{P}$, and $\mathbf{C}^{(i)}$ is the constant matrix supplying the additive constant required by linear regression for the ith input matrix. Thus, to the input similarities data $\mathbf{S}^{(i)}$, INDCLUS simultaneously fits the m clusters ($\mathbf{P}$) and their weights ($\mathbf{W}^{(i)}$). If it were not for the binary constraint on $\mathbf{P}$ in Eq. (4), then this model would simply be a generalization of principal components analysis, with $\mathbf{W}$ corresponding to eigenvalues and $\mathbf{P}$ to eigenvectors, and with the attendant rotational invariance leading to the "rotation problem." However, the discrete, binary constraint on $\mathbf{P}$ requires much more elaborate procedures (such as mathematical programming, alternating least squares, and combinatorial optimization, described in Carroll & Arabie, 1983) for fitting the INDCLUS model.

As with MAPCLUS analyses presented in Tables 2 and 3, the INDCLUS analysis below fitted both the clusters ($\mathbf{P}$) and the weights ($\mathbf{W}^{(i)}$) simultaneously. A random initial configuration was used so as not to bias the final solution toward either matrix, as might be maintained if the solution in either Table 2 or 3 had been used for an initial configuration. A final detail to note is that the two matrices are incommensurable in several details. First, the similarities matrix, as given by Johnson and Tversky (1984, p. 69), is the mean over subjects' ratings on a nine-point scale, whereas the predictions data are tallies of subjects. Second, the mean and standard deviation are (3.39, 1.44) for the similarities data and (22.07, 17.94) for the conditional predictions data. In our INDCLUS analysis of these two data matrices, we therefore specified a "matrix conditional" analysis (terminology of Takane *et al.*, 1977), which causes the matrices to be separately standardized so that each will have unit variance. Entries are thus rendered more comparable across the matrices as a result of this preprocessing. Selecting this option typically causes the weights to be larger than from a MAPCLUS analysis or a matrix unconditional analysis using INDCLUS. The reader is therefore cautioned not to attribute any significance to the weights generally being larger in Table 4 than in Tables 2 and 3.

The seven clusters in Table 4 account for 74.0% of the variance computed over the two separately normalized matrices and, once again, the fit is better for the predictions data. The two clusters common to both Tables 2 and 3 also reassuringly appear as Clusters 3 (homicide, terrorism, war) and 6 (nuclear accident, toxic chemical spill) in Table 4. The

TABLE 4
INDCLUS SOLUTION FOR RISKS DATA FROM JOHNSON AND TVERSKY (1984) (MATRIX UNCONDITIONAL ANALYSIS)

	Weights for different sources of data		
Subset	Similarities data	Conditional predictions data	Risks contained in subset
(1)	1.933	2.897	Heart disease, leukemia, lung cancer, stomach cancer, stroke
(2)	1.713	1.568	Electrocution, fire, flood, lightning
(3)	2.519	2.081	Homicide, terrorism, war
(4)	0.634	0.573	Accidental falls, airplane accidents, electrocution, fire, homicide, lightning, nuclear accident, terrorism, toxic chemical spill, traffic accidents, war
(5)	0.566	0.540	Accidental falls, airplane accidents, fire, flood, lightning, stroke, tornado, toxic chemical spill, traffic accidents
(6)	2.797	2.620	Nuclear accident, toxic chemical spill
(7)	1.129	0.145	Flood, homicide, nuclear accident, tornado
	−0.713	−0.688	Additive constants
	69.5%	78.6%	Variance accounted for within condition

Note. Overall variance accounted for = 74.0%.

first cluster in Table 4 had appeared earlier as Cluster 5 in Table 3 (from predictions data), so that it is not surprising to see a larger weight for this cluster for the predictions data (2.897) than for the similarities data (1.933). The second cluster consists of the effects and hazards of adverse weather. Cluster 4 subsumes Clusters 3 and 6, and also the three types of accidents as well as lightning, fire, and electrocution (Cluster 2 from Table 3). The fifth cluster overlaps considerably with the preceding cluster but is neither as interpretable nor as heavily weighted. The last cluster could be given a causal interpretation of a nuclear accident and some events likely to follow it if an explosion is involved (flood, tornado, homicide). As with the first cluster in Table 4, there is a considerable difference in the sizes of the weights for the last cluster.

While the overall goodness-of-fit and general interpretability of the INDCLUS solution in Table 4 seems satisfactory, we were disappointed to see insufficient differences in the weights for the two conditions to allow for an interpretable pattern of contrasts. Johnson and Tversky (1984, p. 58) noted that the product-moment correlation between the two matrices is .76, and we suggest that this value and the fact that the same subjects provided both matrices account for the generally similar pattern of weights across the two conditions.

FUTURE PROSPECTS

We have posed continuous spatial versus discrete models as portraying complementary aspects of the same data (cf. Kruskal, 1977); each can capitalize on aspects of structure inherently not emphasized in the other's representation. Quite recently, there has been interest in trying to offer a taxonomy of which types of substantive domains are more appropriate for spatial versus discrete models. Obtaining a general answer to this question is impeded by the practical necessity of somewhat arbitrary choices for various decisions: (a) which model(s) shall be selected to champion the respective classes, (b) how are the numbers of fitted parameters to be matched across the two models selected, when the parameters are subject to different constraints across the models, and (c) how is a goodness-of-fit measure to be selected that is fair to both classes? A seminal paper by Pruzansky, Tversky, and Carroll (1982) produced early results as well as some diagnostic measures for deciding which of the two specific exemplars of the two classes was preferred, subject to certain assumptions. Those authors concluded (1982, p. 18), however, that "Ultimately, the choice of a representation depends, in addition to goodness-of-fit, on the interpretability and the theoretical interest of the proposed solution." It is quite possible that stronger results will be forthcoming in the next several years, although such tests may not be tailored to the types of data collected in studies of risk.

Returning to the distinction between two- and three-way models, we note that the intended advantage of three-way (as opposed to aggregate two-way) models for scaling (e.g., INDSCAL) and clustering (e.g., INDCLUS) is their facility for depicting differences among panelists, experimental conditions, or other sources of data. Given the marked differences among individuals' judgments and perceptions of risk, this particular substantive domain seems especially well suited for applying such three-way analyses. We note, however, that the Carroll and Chang (1970) INDSCAL method had been available for several years before many publications appeared offering protracted analyses of the "subject space" (based on the weights) produced in an INDSCAL analysis (e.g., Bisanz, LaPorte, Vesonder, & Voss, 1978; LaPorte & Voss, 1979; Wish, Deutsch, & Biener, 1972). Even many current applications fail to realize the full potential the INDSCAL model offers for depicting individual (or other) differences, but one might hope for a more optimistic pattern of usage for the newer INDCLUS (Carroll & Arabie, 1983) method.

The obvious remedy for this problem requires investigators planning studies of judged and perceived risk to place greater emphasis on the experimental designs for observing different types of subjects and for groups of panelists. Such comparisons among panelists would be facilitated by more background and demographic data than are typically requested of panelists. We reported one and cited several other studies earlier that were not looking at data aggregated over subjects, but those studies appear to be in the minority. The models and associated computer programs for three-way representations are available, but data to exploit such models fully are scarce. We believe that the examples discussed earlier demonstrate the advantages of three-way versus (aggregate) two-way analyses (e.g., Hohenemser *et al.*, 1983). Thus, we strongly urge that future studies be designed to exploit the full capabilities of three-way analyses and representations.

A second development to note is the advent of "hybrid" models (Carroll, 1976) that simultaneously fit a (continuous) spatial and a (discrete) clustering component to the same proximity data. Examples of fitting such models (but not for stimuli relevant to risk) are given by Carroll (1976) and Carroll and Pruzansky (1980). Although development of these models is still in progress, they offer the possibility of trying to disentangle discrete versus continuous bases of judgment for such stimuli as the types of risk considered in the studies surveyed here.

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