Psychometric Methods in Marketing Research: Part II, Multidimensional Scaling
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*Guest Editorial*

# Psychometric Methods in Marketing Research: Part II, Multidimensional Scaling

Recently, we presented some views about the history, growth, and future of psychometric techniques in marketing research (Carroll and Green 1995). Our Part I commentary focuses on conjoint analysis and related methods. In this concluding editorial, we discuss multidimensional scaling (MDS) in marketing, which goes back even earlier than conjoint analysis—to at least the early 1960s, following Shepard's pioneering papers on the nonmetric analysis of proximity data (Shepard 1962a, b).

Space does not permit a discussion of the many classes of methods for the analysis of proximity and preference data often included in a "broad" definition of MDS. In particular, our purview does not cover correspondence analysis and cluster analysis.[1] Excellent reviews of these two areas have been prepared by Hoffman, De Leeuw, and Arjunji (1994) and Arabie and Hubert (1994), respectively. In addition to these publications, we recommend reading DeSarbo, Manrai, and Manrai's (1994) review of latent class MDS and their (1993) review of nonspatial tree models. Each review presents an insightful and comprehensive coverage of these specialized areas.

Subsequent sections of this editorial discuss the history and maturation of MDS in marketing, including models and applications of individual differences models, constrained MDS, stochastic MDS modeling, normatively based MDS models for optimal product design, and scaling models developed for analysis of scanner data. We conclude with an appraisal of the state of practice of MDS in marketing, some of the problems associated with the gap between models and applications, and some suggestions for increasing the practical utility of MDS in marketing research.

## THE EARLY DAYS

Multidimensional scaling dates from the pioneering paper by Young and Householder (1938) and the first (unidimensional) application by Richardson (1938). It then lay fallow and essentially unused until "revived" and modernized in the 1950s by Torgerson (1958) and others, stimulated in large part by the development of modern digital computers—which made the complex methodology computationally feasible, especially in the multidimensional as well as nonmetric cases. The early history of MDS in marketing research is described in three review articles: Green (1975) discusses several issues (e.g., computer program differences, the metric versus nonmetric controversy, multidimensional psychophysics) and problems facing the future of MDS methodology in designing new products; Green and Rao (1977) describe the major types of nonmetric scaling techniques and illustrate solution recovery; and Cooper (1983) provides a comprehensive review of marketing applications and also discusses trends in the use of this methodology in the future.

The earliest application of MDS in marketing research appears to have been conducted by a psychometrician. Torgerson (personal communication) applied MDS in the late 1950s to a practical problem involving consumers' perceptions of a new set of patterns designed by a New England silverware manufacturer. Stefflre (1969) is probably the earliest marketing researcher to use MDS systematically—in his case mostly as a graphic device to present consumers' perceptions of brand similarities in a spatially powerful manner to businesspeople. His three-dimensional representations of MDS results, which he called "tinkertoys," provide effective devices for communicating the findings of his company's studies. The tinkertoys show interrelationships among real and/or hypothesized brands of coffee, paper products, soaps, and so on, as defined in terms of important perceptual dimensions.

Stefflre's work emphasizes one of the main advantages of MDS, namely, the easy presentation of data, as filtered through sophisticated data analysis techniques designed to highlight critical features related to buyers' judged similarities of stimuli (e.g., brands or products). He did not generally put a great deal of emphasis on the interpretation of dimensions but used the representations primarily for their visual power in displaying interrelationships among products. To emphasize the graphic aspect, Stefflre almost always dealt exclusively with three-dimensional representations.

Green and colleagues (Green and Carmone 1970; Green and Rao 1972; Green and Wind 1973) introduce MDS on a wider scale to marketing and describe many varieties of MDS, beyond the classical "two-way metric MDS," as used

[1]However, in the context of hybrid (i.e., spatial/discrete) modeling, we do provide some limited discussion of tree structures. Still, we recommend reading DeSarbo, Manrai, and Manrai (1993) for a comprehensive treatment of this topic. De Soete and Carroll (1996) provide another more psychometrically oriented review of this topic.

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by Torgerson in his pioneering application described previously. These varieties include metric and nonmetric scaling (Coombs 1964; Guttman 1968; Kruskal 1964a, b; Shepard 1962a, b; Torgerson 1958), the newly developed individual differences MDS (initially the "Points of View" approach of Tucker and Messick 1963, which was never actually used by Green or others in marketing applications), and then the dominant Carroll and Chang (1970) INDSCAL and IDIOSCAL models (see also Chang and Carroll 1969, 1972a, 1989b). Applications also include internal and external multidimensional preference analysis using MDPREF and PREFMAP procedures (Carroll 1972, 1980; Chang and Carroll 1968, 1972b, 1989a, c; Meulman, Heiser, and Carroll 1986) and multidimensional unfolding analysis using one of the later versions of Kruskal's MDSCAL program (e.g., Kruskal and Carmone 1972).

In addition, Green and colleagues put considerable emphasis on the use of various techniques for dimensional interpretation, such as PROFIT (Carroll and Chang 1964; Chang and Carroll 1972c, 1989d). Hierarchical clustering methods (Hartigan 1967; Johnson 1967) also are employed as an adjunct to MDS. This early work, including the important contributions by Shocker and Srinivasan (1974, 1979) integrates many of the advantages of MDS and clustering for marketing applications, which include effective graphic presentation of data on proximities (similarities, dissimilarities, or other measures of stimulus "closeness"), preferences, and combinations of the two types of data using joint space representations. Proximities typically are represented in terms of distances between pairs of products/stimuli. Preferences are represented by projections onto consumer vectors, distances from consumers' ideal points, or other relationships among product/consumer points, vectors, or other geometric entities.

Researchers in the marketing area are also quick to utilize and highlight the special advantages of three-way, individual differences MDS methods for proximity data, as exemplified by INDSCAL—most particularly the property of "dimensional uniqueness." This property leads to solutions uniquely identified, up to at most a permutation (and possible reflection or rescaling) of dimensions. Hence, this feature obviates the need to seek an "interpretable" rotation of the coordinate system, which is characteristic of two-way MDS and other two-way multivariate data analytic tools, such as factor/components analysis. *The need to seek an interpretable rotation of axes makes two-way MDS largely impractical in more than two or three dimensions.* In principle, the dimensional uniqueness property of INDSCAL makes it feasible to obtain interpretable and comprehensible MDS solutions in arbitrarily high dimensionalities.

For example, Wish (see Carroll and Wish 1974a; Wish 1970; Wish and Carroll 1974; Wish, Deutsch, and Biener 1970, 1972) is able to extract interpretable structures in dimensionalities as high as nine. Although a nine-dimensional structure cannot be plotted so as to be comprehended (by ordinary mortals, at least) in a unitary fashion, it is often possible to find subsets of dimensions that "cohere" in a fairly integrated manner. One then can make plots of two- or three-dimensional subspaces corresponding to subsets of dimensions. For example, in a study of perceptions of nations, Wish finds nine dimensions that could be partitioned naturally into three subsets—each, as it happens, containing three of the nine dimensions: political aspects, economics/power, and geography/culture.

Graphically, this nine-dimensional structure then could be presented in terms of three separate but interrelated three-dimensional subspaces: a "politics" subspace, an "economics/power" subspace, and a "geography/culture" subspace, each of which is both graphically tractable and substantively coherent. Moreover, each subspace is defined in terms of statistically and psychologically unique (and interpretable) dimensions. Arabie and Soli (1982) are able to produce a similarly interpretable five-dimensional structure for speech perception data (due to Miller and Nicely 1955); other INDSCAL examples abound in the psychological literature.

Much of this now "classical" methodology for MDS and related data analyses is synopsized in the technical appendix to the Green and Wind (1973) book by Carroll (1973). (Also see Carroll and Arabie 1980; Carroll and Pruzansky 1980, 1984; Carroll and Wish 1974a, b; Wish and Carroll 1974.) Fairly nontechnical summaries of the theory and methodology for two-way and three-way MDS, respectively, can be found in Kruskal and Wish (1978) and Arabie, Carroll, and De Sarbo (1987). The latter publication discusses the most widely used methodology for fitting the weighted Euclidean INDSCAL model: namely, Pruzansky's (1975) SINDSCAL procedure. (Also see Young and Lewyckyj's 1981 ALSCAL [as well as Takane, Young, and De Leeuw 1977], and Ramsay's 1977, 1980, 1982, and 1991 maximum likelihood–based MULTISCALE.) Kruskal and Wish (1978) focus primarily on KYST, which is currently the most widely used method of two-way nonmetric MDS. The most up-to-date version of KYST is KYST2A (Kruskal, Young, and Seery 1977).

However, marketing researchers became increasingly skeptical about the value of MDS in marketing research because of the problem of controllability/manipulability of the dimensions arising from this primarily exploratory data analytic tool. Many researchers moved in the direction of conjoint analysis, regression methodology, and other techniques utilizing predefined dimensions, variables, or attributes rather than dimensions derived through exploratory MDS. The problem with the latter is twofold:

1. Before being even potentially controllable or manipulable, such dimensions first must be clearly identified and "interpreted." Identification, particularly in the case of two-way MDS, often involves the so-called rotation of axes problem: rotating the coordinate system arbitrarily produced by whatever algorithm was used initially to an orientation in which the coordinates correspond to "interpretable dimensions." In some cases, no meaningful interpretation can be found, even after rotation or other transformation of dimensions. In others, the "interpretations" are questionable—this process being, at best, highly subjective.
2. Successfully interpreting dimensions in terms of meaningful constructs does not necessarily solve the important practical problem of operationalizing the definition of these constructs in terms of manipulable physical or other variables, which can be controlled through concrete operations. The objective is to produce prototypes of products or other entities that correspond to specific vectors of values on several dimensions, that is, to produce a new product or other entity corresponding to a specified point in the perceptual space.

For example, a researcher might wish to estimate the position or positions of "optimal" products that would maximize some well-defined objective function (e.g., market share or profitability). However, these procedures do not solve the problem of making MDS a useful tool for new product development without a practical methodology for producing a prototype actually corresponding to the optimal point. This prototype then could be used in product testing and the eventual development of actual products to be introduced into the market.

Conjoint analysis has the notable advantage that the product dimensions or attributes are defined so as to be explicitly manipulable in terms of physical or other controllable variables. Each variable has only a small finite number of discrete values, so that it is comparatively straightforward to generate a prototype corresponding to any specified set of values along the dimensions or attributes. (We return to these issues in the concluding section.)

## *RECENT DEVELOPMENTS IN MDS METHODOLOGY*

In recent years the sophistication and power of MDS and tree structure models has increased enormously. Carroll and colleagues and DeSarbo and colleagues play leading roles in these newer developments. We first discuss several conceptual and algorithmic extensions to MDS, such as the following:

- Three-way unfolding models,
- Stochastic MDS models,
- Nonsymmetric matrix models, and
- MDS/clustering combinations (i.e., hybrid models).

We then describe further new developments in the following:

- Scanner data applications of MDS and
- Prescriptive or normative product design and competitive action/reaction modeling.

### *Three-Way Unfolding Models*

Several new developments (or new "wrinkles" on old ones) in MDS methodology have occurred over the past several years, including three-way generalizations of unfolding/ideal point analysis (DeSarbo and Carroll 1979, 1981, 1985; Jedidi and DeSarbo 1991), which generalize unfolding to the three-way case in a manner analogous to the generalization of "standard" MDS (of symmetric proximity data) from the two-way to the INDSCAL-based, three-way case. Three-way unfolding applies to three-way nonsymmetric proximities (e.g., among two sets of products) as well as to three-way preference or choice data.

Improved algorithms for fitting three-way unfolding models definitely would be an important contribution to the field. However, users should be warned that the popular ALSCAL program, even though its documentation implies that it allows both two- and three-way unfolding analysis, should not be used for this purpose because of an inappropriate fit measure. There are also inappropriate options for treatment of the data. The general problems with unfolding, in the two-way case, are discussed in Kruskal and Carroll (1969) and Carroll (1972, 1980).

### *Stochastic MDS Models*

Another development of considerable research interest is stochastic MDS—particularly for paired comparisons preference data. Exemplifying this approach are such models and methods as the wandering vector and ideal point models (Carroll, De Soete, and DeSarbo 1990; De Soete and Carroll 1983; De Soete, Carroll, and DeSarbo 1986) and a family of stochastic unfolding models proposed by MacKay and Zinnes (1986; see also Zinnes and MacKay 1989). These are moderate stochastic choice models, leading to the important feature that the multidimensional structure of both stimuli and subjects, at least in principle, can be derived from a single matrix of paired comparisons data. Many other models for different types of preferential choice data entail strong stochastic choice models, which require more than a single set of data (e.g., from different subjects) to produce multidimensional structure, such as contributions by Schönemann and Wang (1972) and Zinnes and Griggs (1974).

Böckenholt (1992), Carroll and De Soete (1991), and De Soete and Carroll (1992) present overviews of geometrically based stochastic choice models for paired comparisons data. Although spatial or nonspatial geometrically based stochastic models of more general choice data are possible, few have been developed to date. One notable exception is a stochastic model for "pick any/n" data, which is based on a vector model combined with a threshold principle proposed and implemented by DeSarbo and Cho (1989). This approach could be extended easily to an unfolding or ideal point model and/or to various generalized ideal point models.

### *Models for Nonsymmetric Proximity Matrices*

Another area in which there has been considerable research activity is that of models and methods for nonsymmetric proximity data. One of the principal and most straightforward approaches to the analysis of such data already has been discussed: the use of unfolding analysis, with the row and column stimuli (albeit objectively the same) being treated as two different sets of objects. Specifically, we treat the data as unconditional off-diagonal proximity data and, to avoid degeneracies, we do the analysis metrically. This accounts for nonsymmetries by representing each stimulus twice—once as a row point and once as a column point. If the analysis is done "correctly," then lack of symmetry will be indicated by the two "copies" of a given stimulus being positioned differently in the space. The distance between the two copies is a measure of the degree of nonsymmetry in the data for that object and the direction, or more precisely, the pattern of directions, which indicates the nature of the nonsymmetries.

However, it is critical that the analysis be done correctly. Specifically, if the diagonal entries ("self proximities") are given, they should be included. If the diagonals are missing, they should be filled in with numbers lower (if the data are "dissimilarities") or higher (if the data are "similarities") than all other entries in the matrix.

Then, the unfolding analysis must be done metrically. This assumes that the data either are or can be converted by preprocessing into a set of dissimilarities that can be treated as ratio scale distance estimates, using a linear regression without constant (i.e., homogeneous linear regression). Other "weaker" analyses are susceptible, as pointed out by Gower (1978) and Gower and Greenacre (1996), to serious indeterminacies. Even if the proximities are in fact perfectly symmetric, there will be an infinite family of "equally good" solutions—most of which are not characterized by having the two points representing the same stimulus coincide. Even worse, as pointed out by Kruskal and Carroll (1969), if inappropriate options are used, we could obtain degenerate solutions that preserve none of the essential structure in the data. Therefore, use of unfolding combined with the "two sets of points" approach to analyze nonsymmetric proximities is fraught with danger. Most important, diagonal entries must be provided and the unfolding analysis must be done unconditionally and metrically.

Many models attempt to account for nonsymmetric proximity data using geometric models in which each stimulus object is represented only once, but with additional parameters accounting for nonsymmetries. One is Tversky's (1977) "Features of Similarity" model, which treats proximities (similarities) by a model that can be reformulated (as shown by Carroll and Arabie, in press) as the Shepard and Arabie (1979) ADCLUS model. This includes a pair of additive constants, one for the row stimulus and one for the column stimulus. DeSarbo and colleagues (1992) describe a method called TSCALE for fitting a version of the Tversky model.

Holman (1979) proposes a model formally equivalent to Tversky's but with the additional assumption that the observed proximities are monotone functions of quantities of the same general form. Krumhansl's (1978) distance-density model represents each proximity as a monotonic function of a distance plus a pair of constants for rows and for columns. There is a constraint that the row and column constants for the same stimulus be proportional to each other, with the same proportionality constant for all stimuli. DeSarbo and colleagues (see DeSarbo and Manrai 1992) implement MDS algorithms for fitting versions of both Tversky's "Features of Similarity" and Krumhansl's distance-density model to nonsymmetric proximities. Other models have been proposed that model proximities using linear or monotonic functions of distances (or, in some cases, squared distances) plus different row and column constants.

Another class of models generically can be called "drift models," in which the row object "drifts" (e.g., because of memory factors, if it is the first presented) in some direction before being compared with a column stimulus. This is implemented by either adding a fixed vector to the row stimulus point or assuming the row object drifts toward some fixed point (e.g., a "stereotype" toward which all stimulus percepts or memory traces tend over time) by an amount proportional to its distance from that fixed point. Various versions of drift models have been proposed by several researchers. A specific model has been implemented by Zielman and Heiser (1993, 1994), involving what they call a "slide vector," which corresponds generally to the first version of the general drift model.

### *MDS/Clustering Combinations*

A final class of models/methods is that of hybrid MDS/discrete models and associated methods for fitting these. "Discrete" geometric models include ultrametric tree structures, closely associated with hierarchical clustering (Hartigan 1967; Johnson 1967) as well as path length or additive trees (Carroll and Chang 1973; De Soete 1983a, b, 1984, a, b, c, d, 1988; Sattath and Tversky 1977). Carroll (1976) and Carroll and Pruzansky (1975, 1980, 1983, 1986) pioneer penalty function approaches to fitting either ultrametric or additive trees, as well as hybrid models combining the continuous spatial structure that are so closely associated with MDS and discrete tree structures (as well as such alternate discrete structures as multiple tree structures). Other discrete models that can be combined with an MDS structure are various kinds of nonhierarchical clustering models, including various partitioning approaches, as well as overlapping clustering models such as that in Shepard and Arabie (1979). See also Arabie and Carroll's (1980) ADCLUS, INDCLUS (Carroll and Arabie 1983; Chaturvedi and Carroll 1994), and GENNCLUS (DeSarbo 1982), a two-way generalization of ADCLUS that has been further generalized to the three-way case called MUMCLUS by Carroll and Chaturvedi (1995). Carroll and Chaturvedi (1995) describe an explicit algorithm for fitting a hybrid extension of ADCLUS/INDCLUS as well as other clustering models (e.g., MUMCLUS) combined with an MDS spatial structure. The general approach, called CANDCLUS, fits two-way or multiway models incorporating discrete, continuous, or combinations of continuous (spatial) and discrete dimensions. Discrete dimensions are generally binary—only taking on values of 1 or 0, indicating membership or nonmembership in an associated class or cluster.

Closely related to discrete models are spatial models that are based on the "city block" rather than the Euclidean distance model. As discussed by Arabie (1991) and Hubert, Arabie, and Hesson-McInnis (1992), fitting these models can be shown to be basically a combinatorial optimization problem. Heiser (1989) proposes a method for fitting city block metrics to three-way data using the majorization approach proposed by De Leeuw (1988) and De Leeuw and Heiser (1980).

## *NEW TYPES OF MARKETING APPLICATIONS*

### *MDS Applied to Scanner Data*

Consumer diary panels long have been a useful source of data for analyzing competitive structures and consumer brand switching. With the advent of scanner equipment, the opportunities have become even greater for developing market structure maps from revealed choice behavior. The general study of market structures and competitive product positioning has been of major interest among marketing scholars (Blattberg and Sen 1974; Day, Shocker, and Srivastava 1980; Lehmann 1972; Srivastava, Leone, and Shocker 1981; Urban, Johnson, and Hauser 1984) for more than two decades.

It seemed only a matter of time for researchers to recognize the potential for expanding MDS applications from their usual reliance on data from surveys and/or experiments to the use of panel data, as obtained from either consumer diaries or scanners. Rao and Sabavala (1981) pioneer this type of research with their examination of hierarchical structures in the soft drink market, using the *Chicago Tribune* Consumer Panel data. They illustrate, using hierarchical trees, the major components of market structure (e.g., national versus regional brands, diet versus regular, and cola versus non-cola flavors).

Other researchers (e.g., Chintagunta 1994; Cooper and Nakanishi 1988; Elrod 1988; Moore and Winer 1987; Ramaswamy and DeSarbo 1990; Shugan 1987) also make significant contributions to the general idea of constructing market structure maps from panel/scanner data. Elrod's Choice Map modeling typifies the approach.

Elrod (1988, Table 1) provides a useful taxonomy of various competitive mapping models, on the basis of the descriptors:

- •Aggregate versus panel data;
- •Consumer preference differences: idiosyncratic, explained, stochastic, ignored;
- •Brand choice: deterministic, stochastic; and
- •Brand positions: known, inferred, omitted.

Within this taxonomy, Elrod's model is classified as panel data based, with stochastic brand choice. Brand positions are inferred and the distribution of consumer preference differences is stochastic.

Elrod employs SAMI scanner panel data to develop a map of three major coffee brands: Folgers, Maxwell House, and Butternut regular ground coffee. As he points out, the only data required to fit his choice map model are the number of times each household in the panel bought each brand. As in the case of survey-based applications, Elrod relies on average consumer ratings of each brand on subjective attributes to suggest appropriate dimension labels.

Elrod views choice maps in general as a means for testing hypotheses in market structures over time. He cautions that the approach works best for mature markets involving frequently purchased categories (e.g., packaged goods). Furthermore, the structure should be capable of being represented by a low dimensional space (e.g., two-dimensional). The continued development and extension of choice maps from panel/scanner data is still going strong (Chintagunta 1994). The prospects for continued research are high, particularly given the potential insights that such market structure analyses can provide for managerial strategy.

*Prescriptive Models for Product Design and Competitive Strategy*

A particularly exciting area of MDS modeling in marketing entails the use of MDS maps in the design of optimal products. Although the general idea of relocating brands in perceptual/preference space goes back at least to Stefflre (1969) and Morgan and Purnell (1969), Shocker and Srinivasan (1974, 1979) are the first to formalize the optimal product location problem and present a programmatic approach toward solving it. (Also see Srinivasan and Shocker 1973.)

Several MDS-based models for optimal product design have appeared since the Shocker and Srinivasan articles, such as those by Albers (1979, 1982), Albers and Brockhoff (1977), Bachem and Simon (1981), DeSarbo and Rao (1984, 1986), Eliashberg and Manrai (1989), Gavish, Horsky, and Srikanth (1983), Hauser and Simmie (1981), Sudharshan, May, and Shocker (1987), and Zufryden (1979).

More recently, several dynamic models using MDS methodology (Choi, DeSarbo, and Harker 1990, 1992; Sudharshan, Kumar, and Gruca 1995) have been proposed. Green and Krieger (1989) review early work in the field of prescriptive model building using MDS. Kaul and Rao (1994) present a more up-to-date and comprehensive coverage. They emphasize the distinction between product characteristics (i.e., physical or otherwise manipulable product properties) and product attributes, which represent *perceived* dimensions in a multidimensional space. As Kaul and Rao (1994, p. 304) state:

> In order to make an optimal decision we should (1) determine the product position in attribute space that maximizes the given objective, and (2) determine the product characteristic levels and marketing mix that would lead to the realizations of this position.

Conversely, as Green (1975, p. 28) states (in the context of new products):

> [T]he problem is exacerbated by the presence of *two* sources of uncertainty: (a) the consumer's uncertainty about what the real object, whose verbalized description s/he is being asked to evaluate, will be like; and (b) the designer's uncertainty about translating a verbalized description into a physical prototype (for those concepts that receive high consumer evaluations).

This problem illustrates the classical case of reverse engineering, in which back translation from perceived attributes to product characteristics is typically not a one-to-one mapping. This means that there will generally be not just one, but many (often, in theory, an infinite number of) sets of values of the physical variables corresponding to a single (presumably "optimal") profile of values of the psychological variables (dimensions). Although seemingly problematic at first glance, one aspect of this situation is, in fact, an advantage: This many-one mapping from psychological dimensions to physical variables leads to much greater flexibility by potentially enabling actual physical realizations that satisfy other specified constraints or are actually (as opposed to only theoretically) implementable. In practice, however, actual implementation of optimal products is difficult in this MDS-based approach; we suggest some remedies in the concluding section.

The difficulty of implementing the reverse engineering step has led to product design models that emphasize conjoint analysis methods, in which the manipulable characteristics are assumed to be known in advance. Although these characteristics also could serve as arguments of functions whose dependent variables are (perceived) attributes, the task of developing such functions has been daunting for several reasons:

- •The difficulty of defining the transformation,
- •The usual inability (with rotationally indeterminant, two-way MDS) to interpret attribute dimensions, and
- •The general tendency in MDS research to restrict representations to two (or possibly three) dimensions.

In addition to these problems, both MDS and conjoint-based optimal product design models need variable cost estimates at the product characteristic level. In practice, these costs are difficult to obtain from firms' accounting groups. Although conjoint-based optimal design models seem to have enjoyed more real-world applications than have those that are based on MDS, the growth in both research domains has not been rapid, despite their obvious appeal.

## *THE WIDENING GAP BETWEEN MDS MODELING AND APPLICATIONS IN MARKETING*

Since the mid-1960s, MDS methods have been applied primarily to marketing problems involving the following:

- •Brand positioning maps for single-mode, two-way proximities data and

- Brand/attribute maps based on:
  - —Point-point configurations (i.e., unfolding models) and
  - —Point-vector configurations for portraying two-mode, two-way data.

In addition, weighted distance models, such as INDSCAL and SINDSCAL, have received extensive applications.

Computer programs include KYST, MDPREF, PREFMAP, and INDSCAL or SINDSCAL from the Bell Laboratories program suite (now available by e-mail or the World Wide Web through netlib) as well as ALSCAL and MULTISCALE. Smith (1990) has distributed PC versions of many of the Bell Laboratories MDS and clustering programs. SPSS and SAS provide versions of ALSCAL. In addition, Johnson (1987) has distributed a PC-based mapping package utilizing a metric multiple discriminant analysis program.

All these programs have been around since the mid-1970s. It is surprising (and unfortunate) that few, if any, industry applications have been made of the new MDS models. Nor do we know of any commercially available computer packages of three-way unfolding, stochastic MDS, nonsymmetric matrix mapping, or hybrid (MDS/discrete) models.

Why does the gap between the new models and business application remain? The problem is primarily the lack of entrepreneurial interest in demonstrating that the new methods

- Offer significant and practical advantages either in mapping new kinds of data structures or in providing more insightful configuration interpretations and
- Are easy to use and "guaranteed" to avoid degeneracies or other kinds of problems in solution recovery and configuration interpretation.

With the advent of conjoint analysis techniques, many industry researchers have relegated perceptual/preference mapping to ancillary roles in portraying conjoint findings (Green, Krieger, and Carroll 1987) or relationships among existing products. This situation is unfortunate because MDS and hybrid spatial/discrete models (so it seems to us) also have potential as *predictive* methods in marketing research.

## *COMBINING THE STRENGTHS OF MDS AND CONJOINT ANALYSIS*

As suggested previously, MDS approaches to product design, when applied in an exploratory manner, often lead to ambiguity in dimension interpretation. Moreover, if external rating scales (followed by use of PROFIT or other types of regression methods) are employed, it is not clear that these subjective judgments are any more manipulable than are the MDS dimensions themselves. This situation suggests that to combine the strength of MDS as an exploratory data analysis approach with the confirmatory analytic advantages of conjoint analysis, we should move toward the use of various forms of *constrained* MDS (Bentler and Weeks 1978; Carroll, Green, and Carmone 1976; Carroll, Pruzansky, and Kruksal 1980; DeSarbo et al. 1982). In this case, the perceptual dimensions are constrained to be linear or nonlinear functions of a set of fixed external variables, which can be restricted to be manipulable variables. Takane and Shibayama (1991) introduce a constrained approach to principal components analysis, which could be extended easily to MDS models. De Leeuw and Heiser (1980) discuss a general approach to constrained MDS, and Winsberg and De Soete (1996) present a recent approach to constrained MDS.

We should point out that in some of these techniques, such as those that are based on the CANDELINC (Carroll, Pruzansky, and Kruskal 1980) approach, relationships between these variables and dimensions need not be either one-to-one or linear. Hence, one of the principal advantages of MDS—its exploratory nature—can still be maintained to a large extent. In principle, a particular dimension could be defined as a *complex nonlinear* (e.g., polynomial) function of two or more manipulable external variables, for example. Care would have to be taken to avoid overparameterizing such constrained models and thus inviting undue capitalization on error. However, judicious use of constrained MDS methods could enable the user to avoid this pitfall, while providing an effective synthesis of the major strengths of MDS and conjoint analysis.

We also can envision an application of a constrained MDS approach that is based on the INDSCAL weighted Euclidean individual differences MDS model. This could provide a marketing analogue of some of the higher dimensional (but interpretable) MDS structures in the psychological literature. The combined advantages of the dimensional uniqueness property of INDSCAL could lead to identifiable (nonrotatable) dimensions, and the constraints requiring these dimensions to be predictable and controllable by external physical or other manipulable variables. This approach could produce a true "multidimensional psychophysics," the name given to the earliest form of what might be called *proto-multidimensional scaling* (Richardson 1938). The approach could be applicable to not only marketing, but also psychology and other social and behavioral sciences. We can imagine, for example, a nine-dimensional space of food products, with each dimension describable as a well-defined, but possibly nonlinear, function of biochemical, chemical, physical, or other variables in terms of which the food products are defined (i.e., their "recipes").

### *Further Research and Practical Marketing Work*

We view the principal goal of MDS as applied in marketing to be accounting for preferential choice data in terms of underlying perceptual dimensions and, ultimately, *manipulable* variables. Proximity data, such as judged similarities or dissimilarities, are especially appropriate in defining these perceptual dimensions, but the bottom line remains that of using the results of MDS for the prediction of consumer choice. Current methods such as PREFMAP or LINMAP (Srinivasan and Shocker 1973) can be used to map vectors, ideal points, or other geometrically related parameters into a space defined in terms of the fixed perceptual dimensions estimated using an MDS analysis of proximity data, so as to account optimally for such preference data in a so-called external analysis approach.

A more integrated and possibly more useful approach for marketing applications, however, might be one in which a perceptual space is derived that simultaneously accounts for perceptual (e.g., proximity) and preferences data, and possibly other data (such as rating scale judgments), in terms of a single geometric representation. Such an approach—which might have escaped the attention of many marketers—is proposed and implemented by Ramsay (1980, 1991); also see

MacKay and Zinnes (1986), whose PROSCAL approach allows fitting (two-way) MDS models through a maximum likelihood criterion (Hefner 1958) to either proximity or preference data, but not to both simultaneously. Ramsay's approach allows the inclusion of direct rating scale judgments or other variables as well as both proximity and preferential choice data in a unified analysis, utilizing a maximum likelihood criterion of fit. (Some of the direct rating scale judgments could be other types of preference data, allowing, for example, representation of both revealed and stated preference data along with proximity data in a single configuration.) MacKay and Zinnes (1995) discuss another MDS approach that allows fitting of both proximity and preference data in a joint spatial representation. Ramsay's approach is limited to the two-way case and so does not include the advantages of INDSCAL analysis (nor does that of MacKay and Zinnes). INDSCAL enables researchers to represent individual differences among consumers in terms of the salience of perceptual dimensions and, perhaps more important, to make the unique identification of dimensions that is so useful in the practical interpretation and use of MDS results.

What we argue is needed to make MDS a central component of the "tool kit" of research procedures for practical marketing research is a methodology that combines the following features:

- The ability to develop perceptual dimensions that are broadly constrained to well-defined functions of a priori physical or other manipulable variables.
- The ability to account for proximity data (direct or derived) and preferential choice data simultaneously, as well as rating scale data on attributes that could be used as aids in interpretation of dimensions in an individual differences (or three-way) MDS framework. This would utilize the INDSCAL weighted Euclidean model or other more general models incorporating individual differences parameters. (These latter parameters can be utilized in turn for market segmentation and/or related statistically to demographic, psychographic, or other characteristics of consumers.) Deterministic or stochastic choice models, or combinations of the two, could be used to account for the preference data, with relevant individual differences parameters, if appropriate.
- Ideally, all this should be done within the framework of maximum likelihood fitting, as exemplified by Ramsay's (1977, 1982, 1991) MULTISCALE and, more recently, by more general assumptions about error structure. These extensions involve the partitioning of dimensions into common dimensions relevant to all products or other entities and dimensions specific to individual stimuli, products, or entities (Carroll and Winsberg 1995; Winsberg and Carroll 1989) or to subsets of products (De Soete, Carroll, and Chaturvedi 1993). Maximum likelihood fitting with realistic distributional assumptions for error enables researchers to test hypotheses about dimensionality and other aspects of the complex structures being fit. Researchers also can determine confidence regions for parameter estimates and utilize the rest of the statistical armamentarium necessary to move MDS from the domain of purely exploratory methodology to that of the more confirmatory methodology needed for a maturing marketing science. This is not to say that MDS as an exploratory methodology will not continue to have its place in marketing and other social and behavioral services. We believe it definitely will—especially when a new domain of hitherto unexplored products is being studied in a marketing context. But in the final analysis, confirmatory tools—broadly defined—will be needed increasingly in the context of real-world applications.

### *Latent Class Modeling*

In addition to the flexibility provided by the combination of INDSCAL modeling and constrained model fitting, we believe that latent class methodology also can provide practical benefits for marketing researchers. For example, Winsberg and De Soete (1993) assume a latent class structure for weights in an INDSCAL-type, weighted Euclidean individual differences MDS model. This approach retains virtually all the advantages of INDSCAL while segmenting subjects or consumers using the latent class structure imposed by the analysis. De Soete and Winsberg (1993) propose a latent class vector model for preferential choice data from several people, and De Soete and Heiser (1993) propose an analogous approach that is based on an unfolding (or ideal point) model. De Sarbo and colleagues (1992) impose a latent class structure in conjoint analysis, and DeSarbo, Howard, and Jedidi (1991) propose a latent class approach that implements a form of hybrid model entailing a mixture of cluster and spatial structure.

### *A Research Program*

Putting these various strands together, we can envision an ambitious research and development program, which leads to a class of models that

- accommodates similarities and dissimilarities, deterministic or stochastic preferences, subjective rating scales, and data on physical or other kinds of manipulable variables;
- enables constrained or unconstrained MDS with individual differences parameters; and
- can impose a latent class structure on some (or all) of the individual differences parameters.

Ideally, maximum likelihood, with appropriate distributional assumptions, would be used for fitting model(s) to data, making available all the confirmatory statistical tools associated with that approach. Clearly, not all these features are likely to be included in a single analysis, but appropriate combinations could be included in specific applications.

These steps then could be combined with appropriate response surface methodology for seeking products that optimize such marketing-relevant objective functions as market share or profitability. Researchers then could find optimal values on dimensions that are, in turn, manipulable through variation in the controllable variables used to define the constrained multidimensional representations. We believe that this could result in a powerful methodology for practical marketing work. Much of the methodology already exists, and we know how to produce much of what does not. We could use gradient-based nonlinear optimization techniques, alternating least squares, and/or other alternating optimization procedures for fitting complex measurement models. Fitting could be based on gradient methods, alternating least squares, weighted least squares, Fisher scoring procedures, EM algorithms, penalty function approaches, combinatorial optimization, mathematical programming techniques, and so on—all of which have been well tested for fitting a wide variety of MDS and related models in the general class of models articulated here.

Basically, we believe the marketing research field needs a well-focused and properly targeted research and development program to bring this vision to fruition. If other MDS researchers agree, we hope that such a program could be initiated soon, perhaps sponsored by the Marketing Science Institute or some other industry-supported consortium. A principal objective would be to devise user-friendly, well-documented software that can be made widely available for implementing the approaches described here. In summary, we envision a research path that augments the current emphasis on MDS/clustering as exploratory tools to an operational and prescriptive approach for product/service design and consumer response prediction.

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