Title:
Shepard Diagram

Author:
Jan de Leeuw

Publication Date:
08-22-2011

Series:
Department of Statistics Papers

Publication Info:
Department of Statistics Papers, Department of Statistics, UCLA, UC Los Angeles

Permalink:
http://escholarship.org/uc/item/5z86v5ws

Abstract:
This is an entry for The Encyclopedia of Statistics in Behavioral Science, to be published by Wiley in 2005.

SHEPARD DIAGRAM

JAN DE LEEUW

ABSTRACT. This is an entry for The Encyclopedia of Statistics in Behavioral Science, to be published by Wiley in 2005.

In a general nonmetric scaling situation, using the Shepard-Kruskal approach, we have data $y_i, \cdots, y_n$ and a model $f_i(\theta)$ with a number of free parameters θ. Often this is a nonmetric multidimensional scaling model, in which the model values are distances, but linear models and inner product models can be and have been treated in the same way.. We want to choose the parameters in such a way that the rank order of the model approximates the rank order of the data as well as possible.

In order to do this, we construct a loss function of the form

$$\sigma(\theta, \hat{y}) = \sum_{i=1}^{n} w_i(\hat{y}_i - f_i(\theta))^2,$$

where the w_i are known weights. We then minimize σ over all $\hat{y}$ that are monotone with the data y and over the parameters θ.

Date: April 2, 2004.

Key words and phrases. fitting distances, multidimensional scaling, unfolding, choice models.

After we have found the minimum we can make a scatterplot with the data y on the horizontal axis and the model values f on the vertical axis. This is what we would also do in linear or nonlinear regression analysis. In nonmetric scaling, however, we also have the $\hat{y}$, which are computed by **monotone regression**. We can add the $\hat{y}$ to vertical axis and use them to draw the best fitting monotone step function through the scatterplot. This shows the **optimal scaling** of the data, in this case the monotone transformation of the data which best fits the fitted model values. The scatterplot with y and f, and $\hat{y}$ drawn in, is called the Shepard diagram.

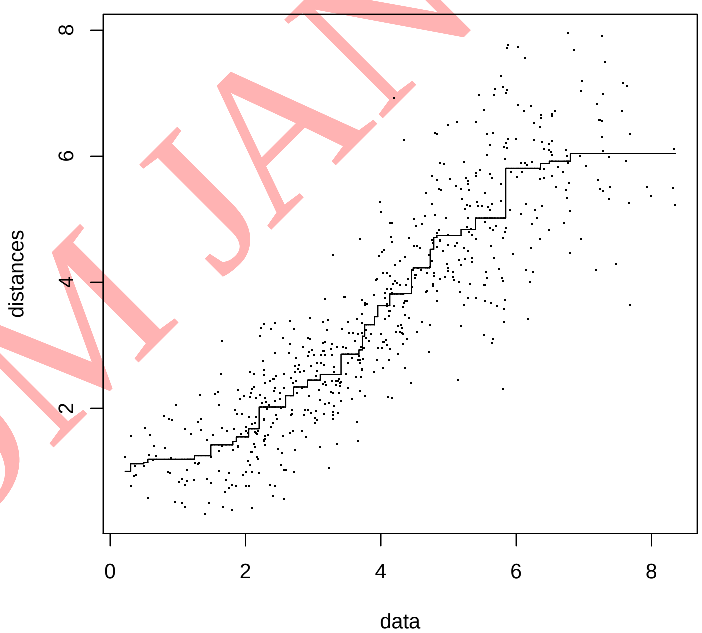

FIGURE 1. Shepard Diagram Morse Code Data

In Figure 1 we show an example from a nonmetric analysis of the classical Rothkopf Morse code confusion data [2]. Stimuli are 36 Morse code signals. The raw data are the proportions p_{ij} which signals i and j were judged to be the same by over 500 subjects. Dissimilarities were computed using the transformation

$$\delta_{ij} = -\frac{1}{2}\log\frac{p_{ij}p_{ji}}{p_{ii}p_{jj}},$$

which is suggested by both Shepard's theory of stimulus generaliztion and by Luce's choice model for discrimination (see [1] for details). A nonmetric scaling analysis in two dimensions of these dissimilarities gives the Shepard diagram in Figure 1.

References

[1] R.D. Luce. Detection and Recognition. In R.D. Luce, R.R. Bush, and E. Galanter, editors, *Handbook of Mathematical Psychology*, volume 1, chapter 3, pages 103–189. Wiley, 1963.

[2] E.Z. Rothkopf. A Measure of Stimulus Similarity and Errors in Some Paired Associate Learning. *Journal of Experimental Psychology*, 53: 94–101, 1957.

DEPARTMENT OF STATISTICS, UNIVERSITY OF CALIFORNIA, LOS ANGELES, CA 90095-1554

E-mail address, Jan de Leeuw: deleeuw@stat.ucla.edu

URL, Jan de Leeuw: http://gifi.stat.ucla.edu