Multidimensional Scaling: Some Possibilities for Counseling Psychology

Louise F. Fitzgerald and Lawrence J. Hubert
Graduate School of Education, University of California, Santa Barbara

Although counseling psychologists conduct a great deal of research that attempts to reveal the structure of a given data set, rarely if ever do they utilize scaling procedures, preferring instead to rely on factor analytic strategies. In this article, we give a short introduction to the use of multidimensional scaling (MDS), with specific emphasis on applications in counseling and vocational psychology, including an example of one standard nonmetric scaling method. We conclude with a discussion of some conceptual and practical considerations associated with the use of MDS, along with a description of its possible applications to a variety of substantive issues in counseling and vocational psychology.

In their review of the use of multidimensional scaling (MDS) in vocational psychology, Rounds and Zevon (1983) note that there often exists a considerable lag between the introduction of a new statistical method and its application to substantive issues in subject matter disciplines. As they point out, “Although the algorithms for metric (Torgerson, 1952, 1958) and nonmetric (Kruskal, 1964a, 1964b) MDS have been available for approximately three decades, the intensive application of MDS to research in the behavioral sciences is a much more recent phenomenon” (p. 491). No where is this cultural lag more apparent than in counseling psychology research. With the exception of some work on the structure of interest models (i.e., those of Holland and Roe)—somewhat special case that has taken place almost completely within the noncounseling research on vocational behavior—only a handful of studies have appeared that utilize these techniques (e.g., Lewis, Lissitz, & Jones, 1975; Friedlander & Highlen, 1984; Hill & O’Grady, 1985; Ellis & Dell, 1986), although perusal of the back issues of the Journal of Counseling Psychology suggests a number of occasions when use of these techniques might have led to a more appropriate analysis strategy than that used by the authors.

The neglect of MDS as a standard analytic tool does not eman from a lack of interest in studying the interrelations present within a given data set, the area to which MDS is most applicable. Influenced perhaps by their historical roots in psychometric and differential psychology, counseling psychologists are great producers of inventories, surveys, tests, and measures, all of which are scrupulously examined for uses as to their structure or dimensionality. Almost invariably, the procedure of choice has been some variant of factor analysis. Rarely do counseling psychologists use MDS techniques to examine their data, although such techniques may reduce clearer and more parsimonious representations than factor analytic solutions, particularly under certain circumstances; for instance, lower dimensional representations would be expected when a simplex or circumflex structure accounts for the data (see MacCallum, 1974; Lingoes and Borg, 1979; Davison, 1985).

In this article, we give a short introduction to the use of multidimensional scaling, with specific focus on applications in counseling and vocational psychology. Following a brief overview, we illustrate one standard nonmetric scaling procedure with an example. Toward the end, we discuss some conceptual and practical considerations associated with the use of MDS, along with an explication of its possible application to a variety of current substantive problems in counseling psychology.

Overview

Multidimensional scaling is a general term for a set of procedures that can be used to represent spatially the interrelations among a set of objects. Typically, in the applications we envisage, the objects will have some specific psychological relevance. The usual data for this class of techniques is a collection of numbers that indicate object similarity or proximity, proximity being a general term referring to any numerical measure of relation. Examples include correlations, similarity judgments, co-occurrence frequencies from free sorts, amount of communication and interaction among people in a group, measures of stimulus confusability, and so forth. In other words, the data generally consist of a set of proximities (or numbers) that indicate how similar or dissimilar every object is to every other object. The results of an MDS analysis consist of a spatial configuration, or map, that graphically displays the relations among the objects as reflected through the original set of proximities.

The principal applied use of MDS is the attempt to uncover the spatially representable structure of the data and to interpret this structure, and possibly the underlying dimensions, in a substantively meaningful way. In this respect, MDS procedures bear a certain conceptual similarity to techniques such as factor analysis, but with the advantage of being generally more applicable to a wider variety of data, explicitly directed toward the task of spatial representation, and, in many cases, capable of providing lower dimensional solutions.
that are substantively interpretable (e.g., see Davison, 1985). The example that is probably most familiar to counseling psychologists is Holland's (1966, 1981) circular model of the structure of interests. A good deal of research has been undertaken, mostly by the Israeli group (e.g., Meir, 1973; Gati, 1982) to examine this configuration through the use of MDS procedures, typically with the Guttman-Lingoes Smallest Space Analysis, a particular implementation of what is referred to later as nonmetric MDS (Lingoes, 1965; Guttman, 1968). Additional research by Prediger (1982) suggests that, in addition to the circular arrangement of the six Holland types that might be obtained through MDS, two dimensions of data/ideas and people/things can be overlaid on the circular configuration of interests. An idealized version of these relations is displayed in Figure 1.

Figure 1 graphically suggests the two major purposes of multidimensional scaling mentioned by Rounds and Zevon (1983): configural verification and dimensional representation. In configural verification, the researcher examines the proximity data in light of theoretical expectations (in this case, the circular or hexagonal RIASEC arrangement postulated by Holland, 1966). Thus, configural verification can be thought of as a confirmatory strategy, although done in a somewhat informal manner. Dimensional representation, on the other hand, is typically more exploratory and involves the researcher's attempt to identify the attributes of the objects that have been scaled; for example, the things versus people and data versus ideas polarities. In a heuristic sense, then, the dimensions serve to explain the arrangement of the objects in the given space. One of the few nonvocational examples of this strategy in counseling psychology can be found in the work of Hill and O'Grady (1985), who used a particular nonmetric multidimensional scaling algorithm (ALSCAL; see Schiffman, Reynolds, & Young, 1981) to examine the structure among 19 therapist intentions on the basis of a proximity measure defined from co-occurrences of intentions over sessions. Using an (unspecified) clustering technique to assist in interpretation by embedding the clusters in the scaling (as discussed later in this article), they report the two-dimensional solution shown in Figure 2:

The foregoing general description of multidimensional scaling was intended to provide only the briefest overview of the purposes and applications of these techniques. We follow it with a more detailed discussion of what we call basic multidimensional scaling, that emphasizes one well-known and now prototypic approach, referred to as nonmetric MDS, usually attributed to Shepard and Kruskal (see Kruskal, 1964a, 1964b). Some more specific comments follow the presentation of a substantive illustration that relates to analyses derived from nonmetric MDS. Finally, we introduce the task of analyzing individual differences through the use of multiple proximity matrices that may be obtained, for instance, from identifiable subgroups or at the extreme, from individual subjects. The latter area may hold some of the best potential for the application of scaling methodologies to research issues in counseling psychology.

Basic Multidimensional Scaling

As stated above, multidimensional scaling refers to a general class of methods that seek to represent spatially a given set of

![Figure 1](image1)

Figure 1. An idealized hexagonal configuration of the six Holland (1966) personality types (R = realistic; I = investigative; A = artistic; S = social; E = enterprising; C = conventional).

![Figure 2](image2)

For the present, one technical issue to keep in mind is whether $p_{ij} = d_{i,j}$, for all object pairs $O_i$ and $O_j$; even this latter statement has to be operationalized in a more specific way. Although this statement may provide a succinct summary of the intent of multidimensional scaling methods, it does little to explain to a novice how this might proceed, for what reason, and for what type of data these strategies might be appropriate. Our intent is not to provide a comprehensive discussion of technical detail, but it still may help in providing a preliminary explanatory structure if we start with some very simple background notation. First, the objects that are of concern will be denoted by $O_1, O_2, \ldots, O_n$, and may refer to titles or characteristics, personality traits, and so forth; in this context, we will present only the barest representation (for instance, see Carroll and Arabic, 1980). The data available to the researcher are assumed to come in the form of a single numerically specified proximity measure between every object pair; for convenience, $p_{ij}$ will denote the proximity between objects $O_i$ and $O_j$.

Just like the term object, the term proximity can refer to a wide variety of ways of numerically indexing the similarity between object pairs, and, although there are standard exemplars, we are limited only by our own ingenuity as to the types of objects we might use and to the manner in which proximities among them may be defined. For example, if we start with a typical data set collected on a group of subjects over a set of variables (now considered our objects), which may be items, test scores, trait ratings (e.g., dominance, maturity) or the like, proximity could be defined by the usual correlations between the variables calculated over the subjects. Although this is probably the most common example, it is possible to reverse our interpretation and consider our subjects themselves as objects, with proximity being defined as some index of profile similarity.

In both of the cases just mentioned, proximity is derived by some means or other from a more basic data set. However, it is also possible to obtain a more direct index. For example, if the objects of interest were people, proximity could be defined as the frequency of interaction or communication; dyads that interact more frequently are psychologically closer, more involved, or more proximate to one another than those that interact less frequently. The study by Friedlander and Highlen (1984) investigating the interaction patterns of well-known family therapists with client families uses such a proximity measure. Alternatively, if stimuli of some sort were under study, proximity indexes might be generated by some index of confusability (e.g., the number of times subjects confused various pairs of tones, patterns, Morse code signals, and so forth). Another method, possibly closer to the experience of counseling psychologists, would be to have subjects conduct free sorts of the objects of interest (e.g., occupational titles or characteristics, personality traits, and so forth); in this case, proximity between objects would be indexed by the number of times an item (object) pair was sorted together. The only requirement that we will impose is one of symmetry, (that is, $p_{ij} = p_{ji}$ for all object pairs $O_i$ and $O_j$); even this latter constraint, however, can be relaxed for certain types of spatial representation (for instance, see Carroll and Arabic, 1980).

For the present, one technical issue to keep in mind is whether proximity refers to a similarity measure (in which small proximities refer to dissimilar objects) or to a dissimilarity measure (in which large proximities refer to dissimilar objects). This distinction will be important, at least mechanically, in implementing any method of spatial representation. The reader is referred to Shepard (1972), Sneath and Sokal (1973), Kruskal and Wish (1978), and Coxon (1982) for a more detailed discussion of possible proximity measures and how they might be derived or defined.

Given the objects and the associated proximity measure, the most common forms of multidimensional scaling can be characterized as implementing the following task: For some given number of dimensions, $k$, chosen by the researcher, arrange the $n$ objects in $k$-dimensional space so that the distances between objects in this space reflect, as closely as possible, the original proximities. Obviously, to obtain a clear understanding of the task itself, the terms that are only vaguely defined in this latter statement have to be operationalized in a more specific way. In fact, depending on how they are operationalized, a number of extant variations on the basic multidimensional scaling task can be defined (see Carroll and Arabic, 1980, for a thorough review). Although it might be of value to discuss all of these variations and attendant details, we will restrict ourselves to one particular approach, as implemented in the commonly available computer program KYST (see Kruskal and Wish, 1978, for a more complete presentation); even in this context, we will present only the barest outlines needed to understand an analysis.

**Adequacy of Fit**

We begin our discussion with how one might assess the adequacy of a particular spatial representation of $n$ objects in $k$-dimensional space, that is, how well the interpoint distances from the object placement reflect the given proximities (for the moment, we delay any mention of the process for initially obtaining this representation or of how it might be improved). For example, suppose the researcher obtains a two-dimensional arrangement by some means or other of the 19 therapist intentions discussed by Hill and O'Grady (1985) that were mentioned earlier in conjunction with Figure 2. How is the quality of this solution to be assessed? If the (Euclidean) distances between the objects are obtained, say $d_{ij}$ for objects $O_i$ and $O_j$, then the justification for the adequacy of the spatial representation must derive from the relation between $d_{ij}$ and $p_{ij}$ over all possible object pairs. For example, one immediately obvious measure of relation might be the simple Pearson product-moment correlation between $d_{ij}$ and $p_{ij}$, which would measure the adequacy of representation by the degree to which some linear transformation of the proximities approximate the actual distances. (Such a choice will eventually lead to a metric multidimensional scaling method, because interval level information in the proximities is being used to measure the adequacy of any given representation; this same interval level information would be used later to identify the best configuration.)

To formalize this notion of adequacy, suppose we obtain a regression equation for predicting $d_{ij}$ from $p_{ij}$ in the usual form, $d_{ij} = a + b p_{ij}$; here, $d_{ij}$ is the predicted value of $d_{ij}$, $a$ is the intercept, and $b$ is the slope; the latter two quantities are chosen to minimize the usual least squares criterion (or...
raw stress, as it is typically referred to in the multidimensional
cal literature):

$$\sum \sum (d_{ij} - \hat{d}_{ij})^2$$

Other things being equal, the closer raw stress comes to zero,
the more adequately the spatial configuration is thought to
represent the relations among the objects under study.

Because the $k$ dimensions can be arbitrarily stretched or
contracted, the raw stress measure, when reported, is typically
divided by a scaling factor, which has the effect of normalizing
the configuration (i.e., stretching or shrinking the axes, or
both), so that the sum of squared distances, $\sum \sum \hat{d}_{ij}$, is 1; a
square root merely places the resulting index in the same
units in which distance is measured, whatever units they may
be, producing the final form for a normalized stress measure
that can vary between 0 and 1:

$$\sqrt{\sum \sum (d_{ij} - \hat{d}_{ij})^2 / \sum \sum \hat{d}_{ij}}$$

As mentioned, the normalized stress measure assesses the
adequacy of any given spatial representation. Because our
ultimate concern is with the best possible representation,
however, some strategy must now be adopted to improve on
whatever spatial configuration is available. Starting with some
initial placement of objects in $k$-dimensional space that is
chosen by the researcher, possibly at random or through some
alternative means for producing a "good" starting configura-
tion, multidimensional scaling algorithms typically construct
better and better representations iteratively by minimizing
stress (or a similar adequacy function, such as Guttman's
coefficient of alienation) until the representation cannot be
altered to improve stress. The iterative procedure is actually
implemented by a numerical optimization procedure that
need not concern us directly here. For details see, for example,
Kruskal (1964a, 1964b).

There are a number of different ways in which the adequacy
of a given spatial representation might be evaluated and in
which the functions are chosen to relate proximities to dis-
tances through the construction of $\hat{d}_{ij}$. The most common
specification of the latter is not through a linear function as
used above but through a generalization of linear regression
to monotone regression. The latter leads to nonmetric multi-
dimensional scaling because only the rank order information
in the proximity measure is used. Explicitly, monotone regres-
sion produces a set of predicted values, $\hat{d}_{ij}$ for all $i$ and $j$, that
have the following property: they are in the same rank order
as the original proximities, and subject to this first constraint,
minimize raw stress. There are several issues to consider
regarding how ties might be handled in formalizing the notion
of "in the same rank order", but for our purposes they will be
ignored. The reader is referred to Schifman et al. (1981) for
some of the details as to how ties could be handled, as well as
for alternatives to using the best fitting monotone functions,
such as relying on Guttman rank images.

An Illustration

As a familiar example of how multidimensional scaling
operates, Figure 3 depicts the two-dimensional solution de-
ved from the correlations given in Table 1 among the six
scales of the Vocational Preference Inventory (VPI) and the
eight from the Vocational Interest Inventory (VII) obtained
by Lunneborg and Lunneborg (1975) with 235 college stu-
dents. This particular solution (without our circular connec-
tion of the points to facilitate interpretation) was obtained
from the computer program KYST by using its nonmetric
option (Kruskal, Young, & Seery, 1973). As discussed earlier,
the locations for each object (in this case, Holland and Roe
categories) are found in such a way that the distances in the
resulting space (computed by the Euclidean distance formula)
represent as well as possible, at least in terms of stress (which
in this case has a value of .12), the relation among the original
objects.

Figure 3 shows the familiar hexagonal structure (albeit in
the somewhat misshapen form produced by real data), with
the categories ordered as specified by Holland (1966), that is,
Riasec. Surrounding the hexagon is a circular Roe (1956)
configuration with the categories ordered Te, Sc, Od, AE, Sv,
GC, Bu, and Or. Although the latter departs to some degree
from that specified by Roe (1956), it is generally similar to
orderings found by other investigators and consistent with a
principal components analysis carried out by Lunneborg and
Lunneborg (1975) on the VII submatrix by itself. (It might be

![Figure 3](image-url)
Table 1

<table>
<thead>
<tr>
<th>Category</th>
<th>R</th>
<th>I</th>
<th>A</th>
<th>S</th>
<th>E</th>
<th>C</th>
<th>Te</th>
<th>Od</th>
<th>Sc</th>
<th>GC</th>
<th>AE</th>
<th>Sv</th>
<th>Bu</th>
<th>Or</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td></td>
<td>.43</td>
<td>.07</td>
<td>.00</td>
<td>.15</td>
<td>.33</td>
<td>.40</td>
<td>.30</td>
<td>.12</td>
<td>-.21</td>
<td>-.14</td>
<td>-.28</td>
<td>-.17</td>
<td>-.10</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td>.22</td>
<td>.10</td>
<td>-.15</td>
<td>-.01</td>
<td>.10</td>
<td>.26</td>
<td>.53</td>
<td>-.05</td>
<td>-.04</td>
<td>-.23</td>
<td>-.36</td>
<td>-.35</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>.36</td>
<td>.15</td>
<td>-.06</td>
<td>-.32</td>
<td>.13</td>
<td>-.25</td>
<td>.27</td>
<td>.60</td>
<td>.02</td>
<td>.16</td>
<td>-.26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td></td>
<td>.25</td>
<td>.21</td>
<td>-.36</td>
<td>-.18</td>
<td>-.17</td>
<td>.36</td>
<td>.00</td>
<td>.53</td>
<td>-.10</td>
<td>.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td>.64</td>
<td>.11</td>
<td>-.35</td>
<td>-.40</td>
<td>.11</td>
<td>-.04</td>
<td>-.08</td>
<td>.55</td>
<td>.46</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>.05</td>
<td>-.33</td>
<td>-.11</td>
<td>.03</td>
<td>-.26</td>
<td>-.13</td>
<td>.28</td>
<td>.57</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Te</td>
<td></td>
<td>.18</td>
<td>.10</td>
<td>-.43</td>
<td>-.25</td>
<td>-.40</td>
<td>-.09</td>
<td>.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Od</td>
<td></td>
<td>.03</td>
<td>.27</td>
<td>.00</td>
<td>-.21</td>
<td>-.41</td>
<td>-.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sc</td>
<td></td>
<td>-.33</td>
<td>-.17</td>
<td>-.19</td>
<td>-.41</td>
<td>-.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GC</td>
<td></td>
<td>.06</td>
<td>.21</td>
<td>.02</td>
<td>.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AE</td>
<td></td>
<td>-.08</td>
<td>-.15</td>
<td>-.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sv</td>
<td></td>
<td>-.12</td>
<td>-.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bu</td>
<td></td>
<td></td>
<td>.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Or</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note. R = realistic; I = investigative; A = artistic; S = social; E = enterprising; C = conventional; Te = technology; Od = outdoor; Sc = science; GC = general cultural; AE = arts and entertainment; Sv = service; Bu = business contact; Or = organization. (From "Factor Structure of the Vocational Interest Models of Roe and Holland" by C. E. Lunneborg and P. W. Lunneborg, 1975, Journal of Vocational Behavior, 7, p. 323. Copyright 1975 by Academic Press. Reprinted by permission.)*

We have discussed our example solely as configural verification of preexisting theory, but as noted earlier, it is common in the use of MDS to attempt dimensional identification as well; for example, by specifying the attributes individuals attend to when responding to a class of stimuli. The task of dimensional interpretation, or more generally, explicating the given pattern of spatial variation among the scaled objects, can be done either formally through a simple inspection of the objects and what they denote, or more formally. A formal strategy might involve the application of some alternative analysis method using the same proximities on which the scaling is based and an assessment of how consistent the latter is to the original scaling solution (e.g., embedding a set of clusters within the given spatial representation—an internal analysis—or, as discussed later, through the use of additional ancillary data collected on the scaled objects—an external analysis).

The dimensions derived from most MDS procedures are arbitrary and are not generally oriented optimally for ease of interpretation; thus, if a reasonable dimensional interpretation is desired, it may be necessary to rotate axes (or even loosen the restriction of orthogonality itself, or both). In our example, for instance, the given axes appear to have no convenient substantive meaning in their present orientation, but when rotated slightly, as depicted in Figure 4, the familiar data/ideas and people/things dimensions (Prediger, 1982) come into focus. In addition to an interpretation through axes rotation, Figure 4 illustrates the embedding of a complete-link clustering solution (Johnson, 1967), showing all clusters formed that have within-cluster intercorrelations greater than or equal to .21. We have a tight cluster of objects (Social—Service—General Cultural) anchoring one end of the People/Things dimension, with a smaller cluster (Realistic—Technology) anchoring the other. Similarly, a third cluster (Conventional—Enterprising—Business-Organization) appears at the Data pole of the Data/Ideas dimension with two separate clusters (Artistic—Arts and Entertainment—Science-Investigative) converging on the opposite pole. The Outdoor category appears as an isolate between the clusters of Ideas and Things, and remains, as in other investigations, somewhat of a theoretical problem.

Figure 4. Rotated two-dimensional scaling solution of the Holland (1966) and Roe (1956) categories. Complete-link clusters are shown. (See Figure 3 for explanation of abbreviations.)
Some Notes and Comments

There are a number of other points or issues that should be mentioned at least in passing, although we will not attempt to go into great detail here. For a more complete discussion of all these issues, the reader is referred to Kruskal and Wish (1978), Schiffman et al. (1981), and Coxon (1982). First, the choice of the number of dimensions to use in a scaling analysis has a status akin to finding the number of factors that should be extracted in a factor analytic study: thus, there is no mechanical strategy that will always lead to the optimal number. We might conduct several MDS analyses, calling each time for a different number of dimensions, and observe the patterning of goodness-of-fit values, such as stress, that are reported by the particular method being used. More practically, however, parsimonious solutions are preferred, and this typically implies a dimensionality less than or equal to three if graphic presentation is to be of much value. This point of view is reflected, somewhat tongue-in-cheek, in what we might refer to as the Shepard/Coxon “law,” “If a solution exists, it probably exists in two dimensions; if it doesn’t, then it certainly exists in three” (Coxon 1982, p. 87). There is the danger of picking up random variation in the data if too many dimensions are used, particularly if the number of objects is small. In fact, for stability considerations, a convenient empirical guideline with nonmetric scaling appears to require at least $4k + 1$ objects for a $k$-dimensional solution (see Kruskal & Wish, 1978, p. 34).

In general, substantive issues are dominant in the choice of the number of dimensions, such as the interpretability of each, the insights any dimension may offer concerning the data, and the consistency with alternative analyses (e.g., embedding clusters in the given solution). In our example, for instance, a third dimension (or greater) was not reported. Variation along additional dimensions was not substantively interpretable, the stress value in two dimensions was reasonably good (e.g., .12), and finally, the cluster solution embedded nicely into the configuration with no anomalies that might suggest the need for a third axis.

It is difficult to give rules that would justify these statements formally. At one time, certain guidelines (see Kruskal, 1964a, 1964b) were offered as to what “good” stress values should be in the typical nonmetric application (e.g., .20 = poor, .10 = fair, .05 = good, .025 = excellent, and 0.0 = perfect), but their justification is somewhat suspect. One hesitates to even mention rules because those that may exist are so heavily dependent on the number of objects versus the number of dimensions as well as the amount of “error” in the proximities (see the Monte Carlo studies reported by Spence and Graef, 1974). It appears that stress values above, say .15, might be considered problematic, but even this weak guideline is open to challenge in the presence of a substantively interpretable representation. Probably a more reasonable approach to assessing solution adequacy, apart from merely considering the actual stress value, is their consistency with alternative analyses, such as those mentioned above in connection with embedding the results of a particular cluster solution. If the spatial placement of the points requires that clusters be “worm-like” by connecting distant points, for which an intervening cluster is already present, then the issue arises as to whether what is being interpreted is more dependent on method than on the actual data.

Secondly, the user of any iterative MDS program must contend with the possibility that the optimization method has identified a local optimum; that is, a solution that may not be the best possible because of the initial configuration chosen and because the iterative method proceeds in small steps until the particular measure of adequacy being used cannot be improved upon. This problem is particularly pernicious in one dimension (e.g., Shepard, 1974), and good advice might be to avoid any attempt to interpret one-dimensional solutions using any of the standard nonmetric routines. Although it is generally a less troublesome problem, the possibility of obtaining local optima does exist in two or more dimensions. If such a situation is suspected from, for example, difficulty in interpretation or inconsistencies with alternative analyses, many programs, such as KYST, offer options that would allow restarts of the iterative process at different beginning configurations; for example, from different random placements. Generally, if several starts lead to essentially the same object placement, the possibility of the analysis being “trapped” in a local optimum is slight. A related issue arises when nonmetric scaling is being used if the scaling results in what appears to be very tight clusters of objects around a few nodes, suggesting that a degenerate solution may be present. As Carroll (1985) notes:

A degenerate solution can be defined as one with a zero stress value, or a “perfect” value of whatever the goodness or badness of fit measure being optimized, but which retains minimal structural information about the data—e.g., the classical degeneracy in which the stimulus points coalesce into two points with no structural information about the interrelations of stimuli clustered into those two points. (p. 135)

Usually, the original data in such cases demonstrate a very strong clustering structure such that proximities between objects in the same group are (almost) always less (in the case of dissimilarities) than proximities defined between groups. Although stress may be zero or very close to it, the method merely places the objects at a few nodes, producing a representation that is essentially uninterpretable as is. If one has enough objects, separate scalings within clusters are a possibility. Others include the use of some form of metric MDS analysis, or some compromise between metric and nonmetric solutions (see Kruskal & Wish, 1978).

Interpreting a Scaling Solution—External Analyses

The issue of interpreting a given multidimensional scaling, particularly through the concomitant use of another analysis method, has been mentioned in connection with our numerical example. As indicated in Figure 4, we might begin by deriving a spatial configuration for the structure of interests from a set of VII/VPI correlations and attempting an internal analysis of this structure, assisted by the use of a clustering (or other) technique. In addition to such internal analyses, and if the researcher has access to other data about the scaled
objects, there are several more precise approaches to explaining the spatial variation among the objects and identifying what dimensions might be present. For instance, if a set of ratings were available for each Holland (1966) or Roe (1956) category on one or more attributes (e.g., status, degree of interaction with people, masculinity–femininity), it would be possible to map this information onto the configuration itself in an attempt to relate the spatial placements of the objects to this auxiliary data, and ultimately, help interpret whatever dimensions might underlie the representation. Schiffman et al. (1981) discuss such property fitting in detail although they do not use the terminology; such strategies are good examples of the application of external analysis for dimensional interpretation. In much the same way, information about subjects' preferences (e.g., for the Holland/Roe categories) might be embedded into the spatial configuration. Here, the external analysis (of preference) is not so much concerned with identifying dimensions but with how preferences may themselves be represented in the multidimensional scaling.

To address such “property fitting” in more formal terms, suppose we have a set of attribute values on our n objects, say $A_1, A_2, \ldots, A_n$ (attribute being a general term used to denote any class of additional data collected on the objects). We wish to use this set of attribute values to interpret the spatial variation present in the k-dimensional solution. Suppose, for instance, that we try to predict the value $A_i$ for object $O_i$ from the k coordinates for the objects in the scaling solution through, say, multiple regression. The coefficients in the equation can be used to produce a particular directed line through the origin in the k-dimensional space with the following property: The projection of each of the n objects onto the line produced from multiple regression induces values along it that correlate maximally with the given attribute values (as compared with all other possible directed lines). (For a technical discussion and specific details, see Kruskal and Wish, 1978.) In effect, a dimension is produced in the k-dimensional space that may help “explain” the spatial variation in the configuration to the degree that the multiple correlation is high. In fact, if the latter correlation is large and the regression weights are large for only a single dimension, that dimension might be named in accordance with whatever the original attribute values refer to. If more than one attribute were available, they could be handled in the same way, each providing a directed line of best fit in the given spatial representation.

A recent example in counseling psychology is provided by Ellis and Dell (1986), who used a metric individual differences MDS program (INDSCAL) discussed in the next section to assess the salient dimensions that counselor supervisors rely on in their perceptions of supervisor roles. Basing their design primarily on Bernard’s (1979) two-dimensional model of supervision, Ellis and Dell had 19 supervisors rate nine supervision role-functions for perceived dissimilarity, using a 9-point bipolar Likert-type scale. To facilitate interpretation of their three-dimensional solution, they performed a series of multiple regressions, using data collected from a pilot group of supervisors who listed the criteria they had used in evaluating the similarity of the stimulus pairs. This pilot study generated a set of attributes, including cognitive, emotional, supportive, degree of supervisor control, and so forth, which were then represented as directed lines in the group stimulus space, and the dimensions were defined (at least partially) in terms of these attributes.

Using a similar interpretation strategy, Weinberg and Richardson (1981) had 38 parents of young children rate 14 stressful experiences relevant to early parenting (e.g., minor illnesses, added financial burden, lack of freedom, and so forth). The stimuli were arranged in pairs, and each pair was rated in terms of similarity on a 9-point Likert-type scale. In addition, each participant made judgments about each of the experiences on each of eight bipolar scales designed to measure qualitative differences among unpleasant events (e.g., stressful, traumatic, frustrating, upsetting, and the like). Their INDSCAL analysis yielded four meaningful dimensions (major versus minor child problems, immediate versus long-range problems, child welfare versus self-welfare, and restriction of self). To further interpret the dimensions and the spatial variation present in the solution, mean ratings of the stressful events on each of the eight bipolar scales (i.e., properties) were regressed on the stimulus coordinates of the four dimensions. As an example, Figure 5 shows the best fitting directed lines for the properties stressful and frustrating for Dimension 2 (for which they had the highest weights) against Dimension 4, i.e., in moving from long-range to short-range problems, they tend to be rated as more stressful and frustrating. Figure 6 depicts the properties traumatic (which had the highest weight on Dimension 1) and infuriating (which had its highest weight on Dimension 3) plotted against these dimensions. As can be seen from Figure 6, the feelings of anger (infuriating) tend to be associated with the problems of self-welfare as opposed to the welfare of the child, given the directionality of the line; similarly, higher traumatic ratings are associated with the seriousness of the problem. (Because of graphic presentation, the four properties plotted in Figures 5 and 6 are projections in two dimensions of directed lines defined originally in four; the multiple correlations with respect to all four dimensions are fairly large (e.g., traumatic: .89, infuriating: .73, stressful: .95, and frustrating: .93). We do note, however, that properties not shown in a certain two-dimensional space may still help to interpret the spatial variation implicit in that representation; for instance, the attribute stressful, if plotted in Dimensions 1 versus 3, would produce a line very close to the horizontal axis, which has been interpreted as “major versus minor problems”, and directed positively so that major problems would tend to be rated as more stressful. Thus, one should use care in using a property to name a specific dimension if only a two-dimensional projection is observed—a particular property could also be highly related to a third dimension that is not present in the projection under consideration.)

An alternative approach to the one used by Ellis and Dell (1986) and Weinberg and Richardson (1981) would be to visually inspect the stimulus configuration, including any clustering results that may have been embedded, and attempt to define the nature of the dimensions subjectively, ideally but not necessarily drawing from preexisting theory. Such subjective or theoretical interpretations could then be tested by having an appropriate group of subjects rate or sort the
stimulus objects in terms of the hypothesized attribute. Consider, for example, Figure 3, which depicts the two-dimensional solution in its original unrotated form derived from the correlations among the six scales of the VPI and the eight from the VII. It would be possible to formally test Prediger's (1982) hypothesis (that two dimensions of data/ideas and people/things can be overlaid on the circular configuration of interests) by asking subjects familiar with the Holland/Roe categories to sort these categories in rank order on the degree to which they represent involvement with people versus things, on the one hand, and data versus ideas on the other. These data could then be fit to the stimulus configuration to empirically test Prediger's notion. Of course, these results would likely be nonprovocative, given the enormous body of theory and data supporting the people/things and data/ideas continua in vocational psychology. However, the strength of this version of property fitting lies in less well-developed areas in which theory is not as strong. Here, when the researcher's conjectures about the dimensional nature of the data may be less than completely accurate, subjects can be expected to have difficulty with the sorting procedure, or their results may show little consistency, leading to revision of the theoretical interpretation, which again can be put to empirical test. This procedure is more systematic and elegant than many of the "shotgun" approaches often seen in the MDS literature, and at its best, may approximate Marx's (1963) notion of functional theory construction with a procedure that is extremely economical in terms of time, data collection requirements, and subjects.

As a generalization of fitting directed lines through multiple regression or other means (implementing what is called the vector model), each ancillary attribute might also be fit as an ideal point in the given configuration. Here, the typical concern is with the representation of preference and not with dimensional identification; the latter is best approached with some version of a vector model. In this ideal point model, a location is identified in the \( k \)-dimensional space in such a way that the distances to the location from the \( n \) objects reflect as well as possible the attribute values \( A_1, A_2, \ldots, A_n \). Theoretically, the ideal point model generalizes the use of directed lines to represent an attribute, because an ideal point placed at an extreme of the configuration in effect produces a directed line in the \( k \)-dimensional configuration. The reader is referred to Schiffman et al. (1981) for a more complete presentation. For an illustration of this strategy in vocational psychology, though using a spatial representation obtained from principal components rather than MDS, see Soutar and Clarke (1983).

**Individual Differences Multidimensional Scaling**

As a perusal of any recent volume of, say, *Psychometrika* will attest, a substantial amount of research on the basic topic of nonmetric MDS and various alternatives, extensions, and
refinements is continuing (e.g., embedding the topic of scaling, which has been used primarily as a descriptive tool, into a more traditional statistical framework; for instance, by placing confidence regions around the possible spatial locations for each scaled object). We cannot hope to discuss all of these advances here, but we would be remiss if the important topic of weighted Euclidean (or individual differences) scaling models was not touched upon, at least briefly, as this area offers a potentially rich source of analysis methods especially pertinent to research in counseling psychology.

Weighted Euclidean or individual differences models may be appropriate wherever more than a single proximity measure can be defined for our set of objects. For example, multiple measures may be generated if we treat each subject's data as a separate proximity matrix when they represent direct judgments about the perceived similarity among the objects of interest. Or, we might group our subjects on the basis of some salient variable, either demographic (e.g., sex, race, or age) or psychological (e.g., intelligence, personality traits, and so forth) and construct proximity measures for each of the identified groups. Alternatively, separate proximity functions could be constructed for data collected at different times or in different settings. Whatever the basis of their generation, the existence of such a collection of measures allows us to extend our representation task to individual differences multidimensional scaling (Carroll & Chang, 1970). The latter provides an extension of the MDS strategies we have mentioned thus far to allow not only for a representation of the interrelations among objects and a determination of the (fixed) dimensions that underlie and describe these relations, but it also permits the examination of individual or group differences according to the importance of each dimension. Thus, the researcher can produce a spatial representation not only of the relations among objects but also of the variance among subjects in how that structure is viewed. A classic example, which is often cited in texts on multidimensional scaling, is reviewed by Kruskal and Wish (1978). Subjects who were classified as hawks, in terms of their views of the Vietnam war, were likely to attend to a “political” dimension when asked to complete similarity ratings of various nations. Subjects who were classified as doves, however, placed more weight on an “economic development” dimension.

Very few investigations of this sort can be found in the mainstream counseling psychology literature, although extensive work has been done in occupational sociology (see, for example, Coxon & Jones, 1978). In one of the few studies in counseling psychology, Weinberg and Richardson (1981) examined their data for group differences on the four dimensions of parental stress that emerged from their INDSCAL analysis. They note, for example, that working parents weighted Dimension 4 (restriction of self) more heavily than did nonworking parents; similarly, the distinction between immediate and long-range problems was more salient for nonworking mothers than for working mothers and for parents with more than one child as opposed to those with only one.

Ellis and Dell (1986) also examined their data in this manner to determine whether level of experience (of either supervisor or trainee) influenced the salience for supervisors of the various dimensions of counselor supervision, but found no differences, whereas Lewis et al. (1975) found in an early study that the leader of the T-group they studied emphasized a traditional rather than a radical values dimension in assessing similarity among group members. This distinction did not emerge as an important one for other members of the group.

Because of the traditional importance of individual differences research to counseling psychology and the rich promise of individual differences MDS (sometimes referred to as three-way MDS) for such research, we have constructed a hypothetical example to more graphically depict the results that might be obtained from this class of analyses. We begin our example by recalling that Gottfredson (1981) has proposed that much of the variance in career choice can be accounted for by an individual's need to select an occupation that falls within a zone of acceptable alternatives, defined (in part) by status needs and gender role considerations (i.e., the degree to which an occupation is considered traditional or nontraditional). The actual range of acceptable levels of status and gender-related traditionality is assumed to vary from individual to individual. Although it is contrary to Gottfredson's theory, which posits that gender considerations are always primary, we propose for purposes of the present example that individuals also vary in terms of the degree to which status or gender considerations are primary.

Let us assume that we have assembled a sample of 15 high school seniors and asked them to rate the similarity of all possible pairs of 15 occupations, thus producing a 15 X 15 similarity matrix which, when subjected to an INDSCAL analysis, produces the hypothetical two-dimensional solution depicted in Figure 7. With the assistance of the (also hypothetical) three-cluster solution embedded in Figure 7, we interpret

---

**Figure 7.** Hypothetical representation in two dimensions of occupational gender type (Dimension 1) and occupational status (Dimension 2).
our dimensions as occupational gender type (Dimension 1) and status (Dimension 2). We further assume that by some means (interview, questionnaire, rating scale) we determine that one third of our subjects believe occupational status to be more important than gender role appropriateness in their personal choice of a career, one third believe the opposite, and one third rate these factors as equally important. If we were to plot the weights each of our fictional subjects could be expected to place on the two dimensions when performing their rating tasks, the results might be expected to resemble those displayed (in highly stylized fashion) in Figure 8.

Finally, for those with an ideographic bent, the dimension weights for any subject can be plotted in still another space, generally referred to as subject space. The usual individual scaling models assume that subjects have their own unique perceptual space defined by the relative importance of each dimension for that subject, that is, the weights indicate the degree to which the fixed dimensions underlying the scaling of the objects (the group space) have to be stretched or shrunk to represent the data for that particular subject. For example, in the Holland (1966) model, it is likely that some individuals place more importance on the people/things dimension than on the data/ideas dimension, and vice versa. These relative emphases can be depicted graphically by generating the unique spatial configuration for any individual subject. Several computer programs are available for the researcher interested in these extensions of basic multidimensional scaling, of which INDSCAL, SINDSCAL, and ALSCAL are probably the most widely known (see Schiffman et al., 1981).

Discussion of Applications

In the discussion thus far, we have attempted to outline the task of multidimensional scaling and, in the process, suggest a few situations and types of data for which it might be appropriate. It remains, however, to discuss a few prototypic substantive applications to some current problems in counseling and vocational psychology and explore the insights that such procedures might offer.

Vocational Psychology

One of the more enduring topics of interest in vocational psychology is that of why people work. Variously referred to as work values, needs, preferences, job orientation, and so forth, this class of variables is considered to be motivational in nature, and individual and group differences are thought to be important for the prediction of vocational satisfaction, one of the twin criteria of vocational adjustment. For example, it was for many years assumed that women, largely because of the demands of their sex role, worked for different reasons than did men. Men were thought to work for achievement, success, prestige, and the intrinsic value of the work itself; women, on the other hand, were thought to value the opportunity to interact with others, make friends, be socially useful, and enjoy pleasant surroundings.

The findings of the research on this topic are both conflicting and confusing, and support can be found for almost any position. For example, a sizeable subset of studies exists that support the traditional stereotypes (Wagman, 1966; Manhardt, 1972; Singer, 1974; Schuler, 1975; Bartol & Manhardt, 1979) whereas a similarly sizeable subset of investigations has failed to detect any meaningful gender differences (Saleh & Lalljee, 1969; Brief & Aldag, 1975; Brief & Oliver, 1976; Brief, Rose & Aldag, 1977; Lacy, Bokemeier & Shephard, 1983). This ambiguity is also present in some very recent work; for example, Beutell and Brenner (1986) demonstrated that there were fairly clear gender differences on 18 out of 25 items of Manhardt’s (1972) survey of work values; however, many were in a counterstereotypic direction (e.g., the women placed higher value on accomplishment and knowledge development, whereas the men placed higher value on leisure and security). Obviously, this is an area where clarification is sorely needed. We suggest that a reframing of the basic research issues in a form amenable to a scaling analysis might prove helpful in resolving some of the confusion surrounding this area, such as what the basic dimensions (structure) of work values are and whether there are differences in the emphasis placed on these dimensions by men and women. To address these questions, the researcher might first administer a standard measure of work values (say, the Minnesota Importance Questionnaire or the Work Values Inventory) to a sample of men and women matched to some reasonable degree on age, education, and socioeconomic status—variables that have been shown to exert influence on work values—and, concurrently, collect information on subject variables thought to be important in this context (e.g., career salience, sex role traditionality). The Work Values Inventory data can be used to
generate correlation (proximity) matrices for men and women, which can then be subjected separately to MDS analyses using KYST (or a similar procedure). It may be generally prudent to construct separate spatial representations for the sexes, and examine them for structural consistency before combining them into a common individual differences scaling. Such an examination might be an informal one, in which the spatial representations, or attendant analyses such as clustering, are merely inspected for consistency, or a more formal one, in which one solution is rotated to a best fitting approximation to the other and the degree of fit is assessed. If the investigator is satisfied that the spatial configurations are reasonably consistent, the issue of gender difference and dimensional salience can be approached through one of the individual differences procedures (e.g., INDSCAL, SINDSCAL, or ALSCAL—see Schiffman et al., 1981). Similar analyses could be performed by classifying subjects on other variables of interest, for example, high and low career salience, and using these latter sub-configurations to construct the set of proximity matrices subjected to an individual differences analysis. Other substantive issues in vocational psychology that should prove amenable to a scaling approach include the structure of job satisfaction, the dimensions of career maturity, and a wide range of questions that address either structural considerations or the perennial issue of individual differences.

Counseling Psychology

It may seem at first glance that scaling procedures have less to offer the researcher interested in the process and outcome of counseling than the investigator working in vocational psychology. And it is true that no theoretical structures currently exist in the process and outcome literature that lend themselves to configurational verification procedures in the manner of the Holland and Roe models. However, when this literature is examined with an eye to the other purposes of multidimensional scaling (i.e., dimensional interpretation and in particular, individual differences scaling procedures), another perspective emerges.

In the article by Hill and O'Grady (1985) discussed earlier, they provide an excellent example of the use of MDS for the purpose of dimensional interpretation in research on counseling process. In addition, their use of a clustering technique illustrates nicely what was previously alluded to as internal analysis as an aid to interpretation. Although they do not apply scaling procedures to their study of differing therapeutic orientations or within-session intention shifts, these topics actually represent excellent candidates for individual differences scaling analysis, a procedure that might prove to be an attractive and telling alternative to the rather complex methodology chosen by the authors. Other promising applications of this procedure are to the Counselor Verbal Response Category System (Hill, 1978), and the ongoing work in the structure of empathy (Elliott et al., 1982). Finally, the ability of these procedures to allow researchers to examine individual differences in perceptual emphases may offer the best available opportunities to apply formal quantitative methodology to the investigation of the private phenomenological worlds of the individual counselor and client.

References


Call for Nominations

The Publications and Communications Board has opened nominations for the editorship of the Journal of Experimental Psychology: General for the years 1990-1995. Sam Glucksberg is the incumbent editor. Candidates must be members of APA and should be available to start receiving manuscripts in early 1989 to prepare for issues published in 1990. To nominate candidates, prepare a statement of one page or less in support of each candidate. Submit nominations no later than February 15, 1988 to

Donald J. Foss
Department of Psychology
University of Texas
Austin, Texas 78712

Revision received November 20, 1986
Accepted November 26, 1986

---


---

Revision received November 20, 1986
Accepted November 26, 1986