Horn, J. L. (1965). A rationale and test for the number of factors in factor analysis. *Psychometrika*, *30*, 179–185. (3362 citations in Google Scholar as of 4/1/2016)

Who would have thought that a paper published over 50 years ago would still have an impact today. But this one still does! Such longevity must be due to the method being used — the simulation of the random sampling base in factor analysis. Over the years, the journal *Psychometrika* has been a haven for what some might consider mathematical minutiae, but this paper is definitely not one of them. It has enjoyed almost 3,400 citations as of April, 2016; the question of "why" comes to mind.

The solution to the burning question of the correct number of common factors in a matrix of correlations is easy. Henry Kaiser used to say that "it was so simple that he solved it everyday before breakfast" (from J.L. Horn, Personal Communication, June 1977). Horn recognized that a solution was needed; but he thought he was proposing just a temporary fix, put in place for others to develop more formal statistical procedures (which they soon did! For a full review, see Lawley & Maxwell (1963); Horn & Engstrom (1979); Jöreskog & Sörbom (1979)). In Horn (1965), it was suggested that Guttman's latent-root-one lower bound estimate for the rank of a correlation matrix be accepted as a psychometric upper bound, following the proofs and arguments of Kaiser and Dickman. Kaiser shows that for a "principal component to have positive KR-20 internal consistency, it is necessary and sufficient that the associated eigenvalue be greater than one" (Kaiser, 1960, p. 6); this was the source of the root-one criterion. This rule, however, was for a population, so Horn tried to add some sampling considerations. In the statistical calculation of the correlations and the roots, the rank for a sample matrix should be estimated by subtracting out the component in the latent roots that can be attributed to sampling error and the least-squares "capitalization" on this error. Horn promoted a procedure, termed "Parallel Analysis" (PA) by later authors, based on the generation of random variables for estimating the component that needs to be subtracted.

Horn became an expert in the classical techniques of common factor analysis. He generally asked why any elegant mathematical-statistical theory should be based on specific assumptions when we know these key assumptions are wrong and untestable. Specifically, Horn suggested that the number of common factors should not be determined simply by using the well-known "eigenvalues greater than one" criterion defined by one of his favorite advisors, Henry Kaiser (1960; see Horn, 1965; and McArdle, 2007). Instead, Horn proposed the use of "computer simulation" techniques, mainly based on model assumptions that were random, and advocated for their use whenever possible (see Horn & McArdle, 1980).

Horn (1965) determined the number of common factors by selecting the number of the eigenvalues of a correlation matrix that were greater than or equal to those provided by data computer-simulated with known characteristics. In this very simple idea, all that was needed was to generate "random data of similar size"; one could then calculate the latent roots and vectors of these random data to provide a criterion tailored to the particular data set being analyzed. As an example, Horn (1965) implemented this procedure for 297 people measured on 65 ability variables; he found evidence for 16 common factors by the Kaiser criterion but only 9 by the PA criterion. Of course, we now recognize the need to consider statistical fluctuations in these latent roots, but it is noteworthy that these kinds of calculations were done some 50 years ago! Horn's random variable approach has more recently been found to be the most accurate for determining the number of unrotated common factors (e.g., Ledesma & Valero-Mora, 2007; Montanelli & Humphreys, 1976; Velicer, Eaton, & Fava, 2000; Zwick & Velicer, 1986; see also Dinno (2009), Hayton (2009), and Courtney (2013) for recent evaluation, and hearty affirmation). This success also marks the beginning of Horn's fascination with the use of computer simulated data to solve the most complex problems in mathematical statistics (see Horn & McArdle, 1980).

Notes on the Author

John Horn (1928–2006) was a pioneer in multivariate thinking and the application of multivariate methods to research on intelligence and personality. His key works on individual differences in the methodological areas of factor analysis and the substantive areas of cognition are briefly reviewed here. John was also my mentor, teacher, colleague, and friend. It is tempting to review John Horn's main contributions to the field of intelligence by highlighting his methods of factor analysis and his substantive debates about intelligence, but this is done elsewhere (in McArdle & Hofer, 2014).

As a leader in multivariate methodology, Horn tried to reach the incredible heights of his well-known mentor, Raymond Cattell. As illustrated here, John believed strongly in a multivariate scientific approach, and questioned the typical use of unweighted sum scores as if they represented the best scores of the psychological constructs of interest. On a substantive basis, John believed that there were important individual differences among adults within the domains of cognition and personality (see Horn & Knapp, 1974; Horn & Donaldson, 1977).

Some of Horn's early comments on the methods of factor analysis are worth repeating, especially the central concept of a "functional unity" (Horn, 1972, p. 161-162). He applied the ideas about a multivariate meta-theory to data on cognitive abilities, to create various testable hypotheses. This type of reasoning provides the basis for arriving at several substantive results on cognitive abilities, perhaps the most important being that Horn expanded on this initial work of his primary advisor, Raymond Cattell (see Horn, 1965; Horn & Cattell, 1966; 1967) to identify additional functional unities of primary mental abilities (see Woodcock, 1989).

For these reasons I think that John Horn's major contributions to psychology, only some of which have been discussed here, continue to be ahead of their time, and to have a profound influence on our thinking and critical approach to answering complex questions. His contributions to factor analysis and the structure of intelligence, the important methodological debates of the 1970s and 1980s regarding age and cohort effects and related issues of sample selectivity, the innovative ideas underlying his approach to evaluating state, trait and trait-change (Horn, 1972), and his willingness and encouragement to engage in critical evaluation of fundamental ideas and accepted scientific approaches (i.e., the g-theory; see McArdle, 2012) are and will remain important contributions. Through his research and teaching he forced people to question popular assumptions, evaluate all the data available, and consistently challenged us to think longer, harder, and better. His work will continue to inspire important research in the fields of multivariate analysis and human cognitive abilities for decades to come. The interested reader can see McArdle and Hofer (2014).

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