Psychology 407
Matrix Algebra Assignment

1) For the matrices below, find: a) $\mathbf{A}+\mathbf{C}$, b) $\mathbf{A}-\mathbf{C}$, c) $\mathbf{B}'\mathbf{A}$, d) $\mathbf{C}'\mathbf{A}$. State the dimensions of each resulting matrix as well.

$$\mathbf{A}=\begin{pmatrix} 4 & -1 \\ 4 & 6 \\ 7 & 2 \\ 5 & -2 \end{pmatrix} \quad \mathbf{B}=\begin{pmatrix} 1 \\ 2 \\ 4 \\ -4 \end{pmatrix} \quad \mathbf{C}=\begin{pmatrix} 3 & 1 \\ 0 & 2 \\ 1 & 4 \\ 5 & -7 \end{pmatrix}$$

2) Show how the following expressions are written in terms of matrices: (a) $Y_i - \hat{Y}_i = e_i$, (b) $\Sigma X_i e_i$. Assume $i = 1, \ldots, 4$

3) The data below show for a consumer finance company operating in six cities, the number of competing loan companies operating in the city (X) and the number per thousand of the company's loans made in that city that are currently delinquent (Y):

i:	1	2	3	4	5	6
X_i:	4	1	2	3	3	4
Y_i:	10	5	10	15	13	22

Assume that the usual simple linear regression model is applicable. Using matrix methods, find, $\mathbf{Y}'\mathbf{Y}$, $\mathbf{X}'\mathbf{X}$, $\mathbf{X}'\mathbf{Y}$, and $(\mathbf{X}'\mathbf{X})^{-1}$.

4) Let $\mathbf{B}$ be defined as follows:

$$\mathbf{B}=\begin{pmatrix} 1 & 5 & 0 \\ 1 & 0 & 5 \\ 1 & 0 & 5 \end{pmatrix}$$

(a) Are the column vectors of $\mathbf{B}$ linearly dependent?

(b) What is the rank of $\mathbf{B}$?
(c) What must be the determinant of $\mathbf{B}$?

5) Find the inverse of each of the following matrices (and check in each case that the resulting matrix is indeed the inverse):

$$\mathbf{A} = \begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 4 & 3 & 2 \\ 6 & 5 & 10 \\ 10 & 1 & 6 \end{pmatrix}$$

6) Consider the simultaneous equations:

$$\begin{aligned} 8 &= 5y_1 + 2y_2 \\ 28 &= 23y_1 + 7y_2 \end{aligned}$$

a) Write these equations in matrix notation.
b) Using matrix methods, find the solutions for y_1 and y_2.

7) Consider the following functions of the random variables Y_1, Y_2, and Y_3:

$$\begin{aligned} W_1 &= Y_1 + Y_2 + Y_3 \\ W_2 &= Y_1 - Y_2 + Y_3 \\ W_3 &= Y_1 - Y_2 - Y_3 \end{aligned}$$

a) State the above in matrix notation.
b) Find the expectation of the random vector $\mathbf{W}$.
c) Find the variance-covariance matrix of $\mathbf{W}$.

8) Find the matrix $\mathbf{A}$ of the quadratic form:

$7Y_1^2 - 8Y_1Y_2 + 8Y_2^2$

9) For the data in (3), (a) using matrix methods, obtain the following:

(1) vector of estimated regression coefficients,
(2) vector of residuals,
(3) SSR,
(4) SSE,
(5) estimated variance-covariance matrix of $\mathbf{b}$,
(6) estimated point estimate for $E(Y_h)$ when $X_h = 4$,
(7) estimated variance of a "new" observation Y_h when $X_h = 4$.

(b) From your estimated variance-covariance matrix for $\mathbf{b}$, obtain
(1) covariance of b_1 and b_0,
(2) variance of b_1 and of b_0.

(c) Find the hat matrix $\mathbf{H}$.