Rank Correlation:

Suppose we have data in the form of $n$ pairs of observations on $X$ and $Y$:

$$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$$

And suppose we do a scatterplot of these pairs with $X$ on the horizontal axis and $Y$ on the vertical axis.

If the data fall perfectly along a (linear) line, the Pearson correlation would be $\pm 1$.

Otherwise, the extremes of $\pm 1$ would not be reached even though there is a perfect monotone relationship among the variables.
A monotone (strictly) increasing relationship is one in which when $X$ goes up, so does $Y$.

A monotone (strictly) decreasing relationship is one in which when $X$ goes up, $Y$ goes down.

If the original observations on $X$ and $Y$ are changed to ranks from 1 to $n$, and if there is a perfect monotone relationship, the Pearson correlation between ranks (called the Spearman correlation and denoted by $r_s$) is a perfect $\pm 1$.

In other words, a scatterplot using the ranks would give the points falling perfectly along a (linear) line.
Based on the usual randomization test for a (Spearman) correlation, an approximation would be:

\[ r_s \sim N(0, \frac{1}{n-1}) \]

Also, some tables based on untied ranks can be found in various sources
An alternative measure of rank correlation is called the Goodman-Kruskal Gamma ($\gamma$) Coefficient:

Considering the $n$ pairs of observations on $X$ and $Y$, choose two of the pairs, say, $(x_i, y_i)$ and $(x_j, y_j)$

If $x_i > x_j$ and $y_i > y_j$, a rank consistency is said to exist;

If $x_i > x_j$ and $y_i < y_j$, a rank inconsistency is said to exist;

If there are ties on the $x$’s and/or the $y$’s, a decision as to a consistency or inconsistency cannot be made
Over the \( \binom{n}{2} \) possible pairs (of pairs), let \( S_+ \) be the number of consistencies, and \( S_- \) be the number of inconsistencies.

If all the \( x \)'s and \( y \)'s are untied, then \( S_+ + S_- = \binom{n}{2} \); otherwise, \( S_+ + S_- \) is less than \( \binom{n}{2} \).

The Goodman-Kruskal \( \gamma \) is defined as

\[
\gamma = \frac{S_+ - S_-}{S_+ + S_-} = \frac{S_+}{S_+ + S_-} - \frac{S_-}{S_+ + S_-}
\]
As a probabilistic interpretation of $\gamma$ with respect to the observed sample (i.e., what is called an operational definition), suppose I pick a pair (of the $n$ pairs) at random but “throw it back” (and pick another pair) if I can’t determine if I have a consistency or inconsistency because of ties on the $x$’s or $y$’s or both

$$\gamma = P(\text{consistency} \mid \text{untied pairs}) - P(\text{inconsistency} \mid \text{untied pairs})$$

Significance testing can be done using the usual randomization test

There are an enormous number of rank correlation variations depending on how ties are dealt with – Kendall’s Tau (and numerous variations), Somer’s d, among others.