Rank Correlation:

Suppose we have data in the form of n pairs of observations on X and Y:

$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

And suppose we do a scatterplot of these pairs with X on the horizontal axis and Y on the vertical axis

If the data fall perfectly along a (linear) line, the Pearson correlation would be ± 1

Otherwise, the extremes of ± 1 would not be reached even though there is a perfect monotone relationship among the variables

A monotone (strictly) increasing relationship is one in which when X goes up, so does Y

A monotone (strictly) decreasing relationship is one in which when X goes up, Y goes down

If the original observations on X and Y are changed to ranks from 1 to n, and if there is a perfect monotone relationship, the Pearson correlation between ranks (called the Spearman correlation and denoted by r_s) is a perfect ± 1

In other words, a scatterplot using the ranks would give the points falling perfectly along a (linear) line

Based on the usual randomization test for a (Spearman) correlation, an approximation would be:

$r_s \sim N(0, \frac{1}{n-1})$

Also, some tables based on untied ranks can be found in various sources

An alternative measure of rank correlation is called the Goodman-Kruskal Gamma (γ) Coefficient:

Considering the n pairs of observations on X and Y, choose two of the pairs, say, (x_i, y_i) and (x_j, y_j)

If $x_i > x_j$ and $y_i > y_j$, a rank *consistency* is said to exist;

If $x_i > x_j$ and $y_i < y_j$, a rank *inconsistency* is said to exist;

If there are ties on the x's and/or the y's, a decision as to a consistency or inconsistency cannot be made

Over the $\binom{n}{2}$ possible pairs (of pairs), let S_+ be the number of consistencies, and S_- be the number of inconsistencies

If all the x's and y's are untied, then $S_+ + S_-$ $= \binom{n}{2}$; otherwise, $S_+ + S_-$ is less than $\binom{n}{2}$

The Goodman-Kruskal γ is defined as

$$\gamma = \frac{S_+ - S_-}{S_+ + S_-} = \frac{S_+}{S_+ + S_-} - \frac{S_-}{S_+ + S_-}$$

As a probabilistic interpretation of γ with respect to the observed sample (i.e., what is called an operational definition), suppose I pick a pair (of the n pairs) at random but "throw it back" (and pick another pair) if I can't determine if I have a consistency or inconsistency because of ties on the x's or y's or both

γ = P(consistency | untied pairs) -

P(inconsistency | untied pairs)

Significance testing can de done using the usual randomization test

There are an enormous number of rank correlation variations depending on how ties are dealt with – Kendall's Tau (and numerous variations), Somer's d, among others.